



Global Time Echoes: MGEX Multi-GNSS Clock Replication, 2025–2026

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Code Availability: github.com/matthewsmawfield/TEP-GNSS-MGEX

Abstract

The Temporal Equivalence Principle (TEP) predicts correlated phase-coherent disturbances in GNSS timekeeping with a spatial correlation length of order thousands of kilometres, an east–west anisotropy, coupling to Earth orbital velocity, and an annual-phase direction near Earth's aphelion velocity tangent. The directed CMB-apex template is anti-phase rather than detected as a rest frame. Papers 1 and 2 examined these signatures in GPS precise point-positioning products from three analysis centres spanning 2000–2025; Paper 3 used raw RINEX single-point positioning.

This paper is a product-type independence stress test using the public CODE COD0MGXFIN combined multi-GNSS receiver-clock solution distributed by NASA CDDIS from 2025-01-01 to 2026-05-01. Receiver-clock offsets for 418 globally distributed stations are read directly from daily 5-minute CLK files. The combined solution provides no per-constellation independence, but supplies a distinct product family and a largely held-out 2025–2026 epoch.

The isotropic correlation length is $\lambda = 1862$ km ($R^2 = 0.699$; 1.83 million pair-day observations) with a formal within-fit error of ± 112 km and a cluster-robust statistical uncertainty of ± 155 km obtained by resampling whole epochs (month-block bootstrap over 17 calendar months; day-block ± 86 km; leave-one-month-out jackknife ± 94 km). The scale is epoch- and product-dependent at the ~ 30 – 40% level: single-window realisations span 1,050–1,935 km, and matched-day estimates are 2,744 km for CODE against 1,933 km for the independently processed GRG product chain (same tracking network) — the latter landing within 4% of the pooled scale — so the defensible content is the thousands-of-kilometres coherence rather than the precise value. It is stable at 1,819–2,000 km across minimum-baseline cutoffs of 50–2,000 km and persists on geomagnetically quiet days ($\lambda = 1708$ km). The full-range anisotropy is not robust (ratio 1.29, station-clustered $p = 0.49$); the longitude-matched subset is suggestive by pair bootstrap but not spatially clustered resampling. The primary monthly λ and EW/NS orbital metrics are null (Bonferroni $p > 0.5$). A model-free PA-difference statistic gives associations with orbital speed ($r = -0.716$, $p = 0.0018$) and CMB-dipole velocity projection ($r = -0.767$, $p = 0.0005$) across 16 chronological full months. Autocorrelation-aware corrections temper but do not erase the association: the effective-degrees-of-freedom values are $p = 0.072$ (speed) and $p = 0.018$ (v_{proj}), matched AR(1) surrogates give $p = 0.019$ and $p = 0.003$, the phase-preserving circular-shift null reaches its 1/16 resolution floor for the projection channel, and leave-one-month-out correlations stay within $r = -0.66$ to -0.86 , so no single phase alignment dominates. The association also survives operational-asymmetry controls: a composition-matched EW–NS contrast over distance $\times$ latitude cells gives $r = -0.65$ ($p = 0.006$) for the projection channel,

and a uniformly-tracked-station subset gives $r = -0.74$ ($p = 0.001$). Hemisphere-split checks return the same signed anticorrelation in northern, southern, and cross-hemisphere pair classes ($r = -0.45, -0.88, -0.49$) at a common annual phase ($\Delta\phi = 22^\circ \pm 44^\circ$), favouring a global heliocentric driver over hemisphere-local seasonal systematics. The physical hypothesis and sign were motivated by earlier TEP-GNSS papers, making this cross-product corroboration; the operationalisation remains exploratory within MGEX because it was selected after the primary estimators were unstable. A 37×19 directional grid finds strong within-grid modulation (200-permutation plus-one $p = 0.005$). Its nominal maximum is 82.7° from the CMB dipole. Orbital velocity constrains only an axis's projection onto the orbital plane, and that projection is highly degenerate (the CMB and nominal-maximum projected predictors correlate at $r = 0.984$, and only 61 of 703 grid directions meet or exceed the direct-CMB $|r|$). The CMB-labelled projection is therefore one member of a broad family of annual templates, not evidence for a CMB rest frame or a unique three-dimensional axis. All four null controls pass under the robust analysis (minimum baseline 500 km); under the primary short-baseline parameters the solar-rotation reassignment null retains $R^2 = 0.50$, traced to a small-sample bin artifact at 100–152 km, so the robust analysis is reported as the clean result (the fitted scale is stable, 1,862 vs 1,819 km). The two leading mundane attribution channels are explicitly modelled and strongly disfavoured by the tested controls rather than assumed absent: a forward-modelled shared satellite-error kernel is flat through the signal band and decays only on the constellation-footprint scale, and the weather-organized partitions contradict the atmospheric signature — cross-hemisphere pairs retain the cleanest decay ($\lambda = 2101$ km) while same-latitude-band pairs beyond 8,000 km return -0.118 rather than the positive zonal coherence an atmospheric driver requires. A supplementary satellite-clock analysis does not recover the exponential structure, consistent with the shared reference-time architecture of the combined MGEX product: because satellite clocks are not independently estimated in that product, it tests only the receiver-clock channel, and the satellite-clock null is a product limitation rather than a prediction test.

Keywords: Temporal Equivalence Principle, MGEX, multi-GNSS, receiver clock, correlation length, anisotropy, orbital velocity, annual phase, ionosphere, temporal shear, temporal topology

1. Introduction

Papers 1 and 2 of this series established correlated phase-coherent disturbances in GPS-only precise point positioning (PPP) products from CODE, IGS and ESA, spanning 2000–2025. The disturbances exhibit a spatial correlation length of order thousands of kilometres, an east–west anisotropy exceeding the north–south counterpart, coupling to Earth orbital velocity, and a recovered annual-phase direction near Earth's aphelion velocity tangent. In the corrected directed convention, the CMB-apex template is anti-phase and is not a detected rest frame. Paper 3 moved to raw RINEX single-point positioning (SPP), demonstrating that the same signatures persist without reliance on analysis-centre orbit or clock products. Between them, those papers already provide multi-centre, multi-decade, raw-observation, and GPS-only product-family independence. The principal forms of robustness they do not yet supply are consistency across the historical training epoch and consistency in a combined multi-GNSS clock product family.

This paper (Paper 14) conducts a stress test across those two dimensions. The analysis uses the public MGEX combined multi-GNSS receiver-clock solution—the CODE COD0MGXFIN CLK product distributed by NASA CDDIS—for the window 2025-01-01 to 2026-05-01. The present MGEX analysis uses a combined multi-GNSS clock product distinct from the GPS-only PPP and raw RINEX products analysed earlier, and a largely held-out 2025–2026 window, though it is not fully temporally disjoint from all earlier analyses. The MGEX clock product is generated by combining GPS, GLONASS, Galileo and BeiDou observations into a single receiver-clock estimate per station, using software, orbit and combination strategies that differ from the GPS-only PPP of Papers 1–2. It therefore tests cross-product consistency using a distinct combined multi-GNSS clock solution over a largely held-out epoch. Receiver-clock offsets for 418 globally distributed stations are read directly from the daily 5-minute CLK files; no positioning is performed.

It is emphasised at the outset what this product tests and what it does not. Because the MGEX clock files contain one combined multi-GNSS solution per station, they do not permit an independent per-constellation comparison; that form of independence is the province of raw per-system processing, which lies outside the scope of this combined-product analysis. The independence offered here is temporal (a largely held-out epoch) and product-type (a combined multi-GNSS clock product line from the same analysis centre, CODE, whose IGS-final products anchor Papers 1–2), not per-constellation.

The measured $\lambda \approx 1862$ km is a clock-sector covariance scale of C_A . It is not S_S ; in the near-Earth environment, this correlation length traces the macroscopic geometric signature of the continuous saturation of Temporal Topology, anchoring the differential clock correlations independently of the specific multi-GNSS processing strategy. The precise relationship of the covariance structure to the solved scalar field remains to be derived.

2. Data and Product Scope

2.1 MGEX Combined Clock Product

All data are public and freely available from NASA CDDIS (cddis.nasa.gov) and the International GNSS Service (IGS). The analysis uses the Multi-GNSS Experiment (MGEX) combined receiver-clock product COD0MGXFIN produced by the Center for Orbit Determination in Europe (CODE), with the Wuhan (WUM) and CNES (GRG) MGEX products as fallbacks on the few days CODE is unavailable (WUM 20 days, GRG 9 days out of 486 calendar days in the window, i.e. $\approx 6\%$ fallback usage). These are daily clock (CLK) files at a 5-minute sampling interval. Each file reports a single combined multi-GNSS receiver-clock estimate per station, derived by CODE from GPS, GLONASS, Galileo and BeiDou observations; the combined solution is the quantity analysed. No positioning is performed and no raw RINEX observation files are used.

2.2 Analysis Window and Station Set

The analysis period is 2025-01-01 to 2026-05-01 (462 days with usable data out of 486 calendar days in the window), yielding approximately 16 months of daily files. The window is chosen to provide a largely held-out 2025–2026 epoch, though it is not fully temporally disjoint from Papers 1–2, which span 2000–2025. Receiver-clock offsets are extracted from the AR (receiver-clock) records of each CLK file for 418 globally distributed stations, converted to nanoseconds, and assembled into per-station daily time series.

2.3 Product Limitation: No Per-Constellation Separation

Unlike Papers 1–3, which can isolate individual systems, the MGEX CLK product contains one combined receiver clock per station rather than per-constellation clocks. The four constellations therefore cannot be separated in this product, and the analysis is performed once on the combined solution. The independence offered here is temporal (a largely held-out epoch) and product-type (a combined multi-GNSS clock product line), not per-constellation and not analysis-centre: COD0MGXFIN is produced by CODE, the same centre whose IGS-final products anchor Papers 1 and 2.

2.4 Data Provenance and Shared Error Budgets

The provenance matrix across the GNSS papers makes explicit which analyses share an error budget:

Paper	Data layer	Product / processing chain	Epoch	Stations	Error budget shared with
1	AC precise clocks	CODE final, IGS combined, ESA final (Bernese / IGS combination / NAPEOS)	2023-01 to 2025-06	364	CODE family shared with Papers 2, 14; AC ecosystem shared with Paper 33's centre arms
2	AC precise clocks	CODE IGS-final 30 s clocks (Bernese)	2000-03 to 2025-06	474	Same CODE product family as Papers 1 and 14; epoch superset of Paper 1
3	Raw RINEX	Broadcast ephemerides + own RTKLIB SPP	2022 to 2024	539	Estimation-chain independent; shares the physical tracking network
14 (this study)	AC combined clocks	CODE COD0MGXFIN (WUM/GRG fallback $\approx 6\%$ of days)	2025-01 to 2026-05	418	Same CODE centre as Papers 1–2; epoch largely held out
33	Mixed	Own RTKLib SPP arms + AC receiver clocks (COD/GFZ/GRG/JPL)	2022 to 2025	5–160 per arm	AC arms share the IGS/MGEX ecosystem; SPP arms are estimation-independent

Agreement with Papers 1–2 is therefore corroboration within a shared product ecosystem — a different product line and a largely held-out epoch over substantially the same tracking network — rather than replication on independent data. The GRG and WUM products used for fallback and matched-day comparison run different processing software on that same network, so they bound software- and implementation-specific artifacts but not network-level or datum-level common modes. The estimation-chain-independent axis in the corpus is Paper 3 (raw RINEX, broadcast ephemerides, own processing); Paper 33's broadcast-SPP arms provide a second such axis, while its analysis-centre arms share this ecosystem.

2.5 Evaluated Signatures

The analysis evaluates the following signatures, consistent with the approach in earlier papers: correlation length, azimuthal anisotropy, orbital-velocity coupling, annual-phase directional structure, fixed-frame controls, ionospheric independence, and geometric robustness.

3. Methods

3.1 Receiver-Clock Extraction

For each daily CLK file the AR records are parsed, which report the combined multi-GNSS receiver-clock offset for each station at the 5-minute sampling interval. The offsets are converted from seconds to nanoseconds and ordered in time, giving one clock time

series per station per day. No positioning is performed and no coordinate residuals are computed; the receiver clock is the sole observable.

Screening in TEP is represented at the theory level by the environmental operator $S_{\Sigma}(\mathcal{E})$. Quantities such as ρ_T , $R_T(M)$, $S_{\oplus}(r)$, compactness Φ/c^2 , local stellar density, geometric coherence length, and channel-specific response coefficients are domain-specific projections of $\mathcal{E}$, not independent screening mechanisms and not interchangeable universal thresholds. Each is an observational transfer model that parameterizes the same underlying operator in a regime-appropriate form.

3.2 Spectral Pre-processing

Each daily series is linearly detrended to remove the dominant clock drift. Cross- and auto-spectra are estimated with Welch's method (Hann window, segment length up to 96 samples). Spectral estimates are restricted to the nominal TEP-sensitive band [10 μ Hz, 500 μ Hz], but with the daily Welch segmentation used here the effective lowest non-zero resolved frequency is set by the segment length, approximately 35 μ Hz for 96 five-minute samples. The practical analysed band is therefore approximately 35–500 μ Hz, corresponding to periods from about 8 hours down to 33 minutes, which lies safely within the 5-minute Nyquist limit.

3.3 Spectral Phase-Alignment Metric

For every station pair, the magnitude-weighted circular mean of the cross-spectral phase is computed across the selected frequency band. The phase-alignment index is the cosine of the weighted mean phase, ranging from -1 (anti-phase) to $+1$ (in-phase). It measures signed mean phase-offset alignment, not phase concentration; the latter requires the magnitude of the complex resultant. The index is bounded and saturating — it reports the directional dominance of the in-phase component of the band-integrated cross-spectrum rather than the correlated-power fraction — so its fitted exponential amplitude is a phase-direction measure, not a calibrated coupling, and is decomposed against a linear coherence estimator in §4.5b. The established phase-alignment statistic is retained consistently with the earlier GNSS analyses, and is the quantity carried through all subsequent tests. Pairs are binned by great-circle distance into 40 logarithmically spaced bins from 50 km to 13,000 km, with a minimum of 10 pairs per bin.

3.4 Correlation-Length Model

The distance-binned phase alignment is modelled as

$$C(r) = A \exp(-r/\lambda) + C_0$$

where A is the amplitude, λ is the correlation length, and C_0 is an incoherent offset. Fitting uses bounded non-linear least squares (Trust Region Reflective), weighting each bin by the inverse of its standard error of the mean ($1/\text{SEM}$, where $\text{SEM} = \sigma_{\text{bin}} / \sqrt{\text{count}}$). Fit quality is quantified by R^2 , and the uncertainty on λ is propagated from the covariance matrix. Because pair-day records share stations within each day and orbit/product states across consecutive days, the formal covariance error is supplemented by cluster-robust resampling of whole epochs: a day-block bootstrap, a calendar-month-block bootstrap, and a leave-one-month-out jackknife, each refitting Eq. (1) on the resampled record pool. The choice of morphology is likewise tested rather than assumed: on identical bins and weights the exponential is compared by AIC against a stretched exponential with free exponent, the algebraic family $A/[1+(r/r_t)^k] + C_0$ with free k and at the derived Paper-0 pairwise exponents $k = 4$ and $k = 4/3$, and a Gaussian. Sensitivity rows at raised minimum-baseline cutoffs are flagged when the fitted λ contacts the lower parameter bound (which equals the cutoff), since a boundary-pinned value is not an interior measurement.

3.5 Anisotropy, Orbital-Phase, Fixed-Frame, Ionospheric, and Null Tests

The $\text{EW} > \text{NS}$ test fits λ separately in eight 45° -wide azimuth sectors and compares the mean of the east and west sectors with the mean of the north and south sectors. Because ionospheric decorrelation inverts the anisotropy at long baselines, a longitude-matched subset is also reported (station-pair longitude difference below 30°), which, following Paper 3, isolates pairs at similar local solar time. Significance and confidence intervals for the ratio are obtained from 500 bootstrap resamples. For anisotropy, pair-bootstrap results are reported as sensitivity tests, while station/spatially-clustered resampling is treated as the more conservative inference because station pairs are not independent. The ratio is additionally reported in three distance strata (0–500, 500–1,000, >1,000 km).

The orbital-coupling test correlates the monthly λ and the monthly EW/NS ratio with the projection of Earth's orbital velocity onto the CMB dipole direction, computed from a sinusoidal annual model; the two correlations are Bonferroni-corrected. Because exponential-fit noise on short baselines can degrade these metrics, an exploratory PA-difference metric is also computed: the monthly mean EW minus NS phase alignment, which avoids fitting entirely and directly measures anisotropy strength. The physical hypothesis and sign were motivated by earlier TEP-GNSS analyses, but this operationalisation was selected after the primary MGEX estimators proved unstable; it is therefore exploratory within MGEX and excluded from confirmatory family-wise inference. Because the 16-month series is short and its monthly values are plausibly autocorrelated, the nominal Gaussian p-values for the exploratory PA-difference associations are accompanied by four robustness diagnostics computed in the same step: an effective-degrees-of-

freedom correction from the residual lag-1 autocorrelation ($n_{\text{eff}} = n(1-\rho)/(1+\rho)$, t-test on $n_{\text{eff}}-2$), a Monte-Carlo ensemble of AR(1) surrogates matched to the observed residual autocorrelation, a phase-preserving circular-shift permutation of the monthly series against the fixed periodic predictor (resolution floor $1/n$), and a leave-one-month-out correlation range testing whether a single phase alignment dominates. Two operational-asymmetry controls further price the vulnerability of the raw EW – NS contrast to unequal east/west station populations: a composition-matched difference computed as the equal-weight mean of cell-level EW – NS contrasts over ($\text{distance} \times |\text{midpoint-latitude}|$) cells, and a uniformly-tracked-station subset restricted to pairs whose two endpoints both sit above the median pair-day participation count; each variant is correlated with the same two predictors under the same diagnostics. Months are retained chronologically (YYYY-MM) and a partial trailing month is excluded. The CMB-frame test performs a full-sky grid search for the anisotropy axis on a 37×19 grid in right ascension and declination (10° steps), correlating the monthly EW–NS phase-alignment difference with the projection of Earth's orbital velocity onto each candidate axis. The difference metric is used in place of the EW/NS ratio because phase alignment ranges from -1 to $+1$, making the ratio unstable when the NS mean is near zero; the difference is bounded and requires no sign-based selection filter. Monthly aggregation reduces daily noise, while the daily analysis is retained as a supplementary diagnostic. The grid search tracks both the most positive and most negative correlation extrema. Because orbital velocity samples essentially the ecliptic plane, the scan is interpreted as a test of a plane-projected annual predictor, not as a unique three-dimensional axis localization; a nominal sky-angle threshold is reported descriptively only. A permutation null performs a complete grid search over all 703 directions for each of 200 permutations, accounting for the look-elsewhere effect directly. Ionospheric controls stratify the fit by geomagnetic Kp index and exclude storm days; geometry controls repeat the fit on hemisphere-balanced and distance-matched subsets; a forward-modelled product-error kernel tests the shared satellite-error projection channel directly; hemisphere-class and latitude-band within-product partitions test the weather-organized atmospheric channel; and four null tests (temporal shuffle, spatial shuffle, phase randomisation, and solar-rotation reassignment) test whether the recovered structure vanishes under label permutation.

3.6 Sensitivity and Robustness Analysis

Because the 418-station network contains dense regional clusters that disproportionately populate short baselines, a sensitivity analysis is performed to verify that the reported results are not artifacts of short-baseline fitting noise. For each primary result the analysis is repeated at five minimum-baseline cutoffs (50, 200, 500, 1,000, and 2,000 km), tabulating the correlation length, anisotropy ratio, and CMB correlation as functions of the cutoff. A robust supplementary analysis uses a 500-km minimum baseline and a 50-pair minimum bin count, eliminating bins with fewer than 50 pairs where small-sample fluctuations can produce extreme coherence means. For the anisotropy ratio a station-declustered point estimate is also computed, weighting each pair by the inverse square root of the product of its two endpoints' pair counts so that stations in dense clusters do not dominate the exponential fit. The robust null tests apply the same elevated thresholds to all four shuffle controls. These sensitivity analyses do not replace the pre-specified primary analysis but provide a conservative supplementary assessment of each result's stability.

4. Results

The analysis comprises 1,827,555 station-pair-day observations drawn from a network of 418 stations over 462 usable days. Because the MGEX product is a single combined solution, the results are reported once rather than per constellation. The analysis evaluates the core correlation-length signature, directional anisotropy, orbital-velocity coupling, annual-phase templates, and ionospheric/geometric robustness controls. The correlation length and ionospheric persistence are observed; the longitude-matched anisotropy is not robust under spatially clustered resampling. The primary pre-specified monthly λ and EW/NS orbital metrics are null (Bonferroni $p > 0.5$). A model-free PA-difference metric (mean EW minus NS phase alignment) gives a suggestive orbital association across 16 chronological full months that survives autocorrelation-aware corrections for one CMB-labelled projection channel (corrected $p = 0.018$; AR(1) $p = 0.003$; circular-shift floor), but it is exploratory within this MGEX study because it was adopted after the primary fits proved unstable. The full-sky scan detects annual directional modulation, but orbital velocity constrains only a plane-projected phase: the nominal sky axis is not uniquely localizable and the CMB template is not uniquely selected.

4.1 Correlation Length

The isotropic fit gives a well-constrained correlation length.

λ (km)	σ_λ (km, formal)	A	C_0	R^2	Pair-day observations
1862	112	0.827	−0.065	0.699	1,827,555

This λ is shorter than the 3,000–5,000 km reported for GPS PPP in Papers 1–2, consistent with the change to the phase-alignment metric and the combined multi-GNSS clock product. The fitted amplitude $A = 0.827$ is likewise a bounded phase-direction index rather than a calibrated correlation amplitude; its $\sim 7\times$ offset from the IGS-final family is decomposed estimator-axis by estimator-axis in §4.5b (Step 3.6). Until the transfer map from $\mathcal{S}_\Sigma(\mathcal{E})$ to λ is derived, the fitted correlation length is therefore a product- and metric-dependent phenomenological observable: the physical TEP signature carried across products is the order-of-magnitude (thousands-of-kilometres) spatial coherence, not the precise numerical scale, which responds to the observable definition and clock-product construction. The ± 112 km uncertainty is the formal covariance-based fit error, which treats the 1.83 million pair-day records as

independent within-bin samples. Because pair-days share stations within each day and share orbit/product states across consecutive days, cluster-robust resampling of whole epochs was used to measure the true statistical scatter: a day-block bootstrap over the 462 days gives $\sigma_\lambda = \pm 86$ km (95% CI 1,639–1,932 km), a month-block bootstrap over the 17 calendar months gives $\sigma_\lambda = \pm 155$ km (95% CI 1,363–1,971 km), and a leave-one-month-out jackknife gives $\sigma_\lambda = \pm 94$ km with leave-out values confined to 1,794–1,896 km. The within-product statistical precision is therefore about $\pm 8\%$, only modestly above the formal error; the dominant scale uncertainties are the epoch and product systematics quantified below. A sensitivity analysis demonstrates that the correlation length is stable across minimum-baseline cutoffs from 50 to 2,000 km, with fit quality improving substantially as the shortest baselines are excluded ($R^2 = 0.699$ at 50 km to approximately 0.92).

Min. baseline (km)	λ (km)	R^2
50 (primary)	1862	0.699
200	1893	0.791
500	1819	0.848
1000	1850	0.917
2000	2000 [†]	0.912

[†] The 2,000 km cutoff row is a boundary contact: the fitted value (2000.000 km) sits on the lower parameter bound ($\lambda \geq$ minimum baseline) rather than interior to it, so it indicates only that the data prefer $\lambda \geq 2,000$ km at that cutoff — it is flagged as pinned in the archived output and is not an independent measurement of the scale.

The correlation length varies by less than 8% across the full range of cutoffs ($\lambda = 1,819$ –2,000 km), while R^2 improves from 0.70 to 0.92. The stability of the fitted correlation length across these cutoffs demonstrates that the recovered distance-decay scale is not driven solely by the shortest-baseline bins.

Two further stability checks bound the epoch and processing dependence of λ (Step 3.1). First, splitting the 462-day pair record into quarterly windows returns $\lambda = 1,050$ –1,935 km (R^2 from -0.18 to 0.80), with $\lambda = 1,320$ km for 2025 alone and 1,935 km for the 2026 subset: the fitted scale is epoch-dependent at the ~ 30 –40% level, and any single ~ 1 -year estimate — including the 1,396 km figure reported for the original ~ 1 -year span — is an unstable realisation rather than a product-fixed value. Second, on strictly matched days the independently processed CNES (GRG) product chain — a different code base on the same tracking network — returns pooled $\lambda = 1,933$ km ($R^2 = 0.58$) against 2,744 km for the CODE product on verified-independent matched days ($\sim 44,000$ pairs each; same-product fallback days excluded by epoch-aligned detrended-residual comparison). The GRG estimate lands within 4% of the pooled MGEX scale (1,862 km), while the $\sim 30\%$ residual offset to the CODE chain on identical days is the direct product-conditioning signature decomposed in §4.5b: the recovered scale is a property of the data rather than of one center's constellation weighting or software. These numbers fold in the cadence correction of §4.5b — GRG products are delivered at 30-s sampling and per-pair series are now built by intersecting station epochs on the shared 300-s grid; under the earlier length-truncated handling the same diagnostic returned 3,257 km against 2,514 km, so the correction tightens the GRG-pooled agreement from $\sim 75\%$ to $\sim 4\%$. The dominant systematic on the λ scale is therefore epoch/window composition (~ 30 –40%), with analysis-center processing contributing a further $\sim \pm 600$ km, both well above the ± 155 km cluster-robust statistical error of the pooled estimate.

The choice of decay morphology is tested rather than assumed. On identical log-spaced bins and identical SEM weights, six functional forms are compared by AIC: the exponential (AIC = 1,664.7, $\lambda = 1,862$ km), a stretched exponential with free exponent (AIC = 1,665.2; the fit selects $b = 0.98$, i.e. the pure exponential is recovered rather than assumed), the algebraic suppressor family $A/[1 + (r/r_t)^k] + C_0$ with free exponent (AIC = 1,893.8 at $k = 1.78$), the same family at the derived Paper-0 pairwise exponents $k = 4$ (AIC = 4,209.0) and $k = 4/3$ (AIC = 2,125.1), and a Gaussian (AIC = 3,355.2). The exponential is the AIC winner and the nested stretched-exponential family independently returns $b \approx 1$: the measured profile is a finite-range exponential correlator — the two-point morphology of a screened spatial field — rather than an algebraic tail. The derived suppressor operators $\mathcal{S}_\Sigma$ and $y(s)$ describe the response as a function of the ambient environment, a different observable; the correlation kernel measured here is the field's spatial covariance, and it is cleanly exponential.

4.2 EW / NS Anisotropy

The unfiltered full-range ratio is modestly east–west dominated (ratio 1.29) but does not reach significance under the station-clustered bootstrap ($p = 0.49$). A suggestive east–west excess emerges once pairs are matched in longitude (fitted ratio 1.32, bootstrap mean 1.71, pair-bootstrap $p = 0.058$), though the significance dissolves under spatial-clustered resampling (95% CI [0.17, 13.28], $p = 0.214$). The 500–1,000 km stratum also shows the expected excess.

Subset	λ_{EW} (km)	λ_{NS} (km)	Ratio	95% CI	p
Full range	1810	1402	1.29	[0.18, 2.60] (station-boot)	0.49

Subset	λ_{EW} (km)	λ_{NS} (km)	Ratio	95% CI	p
Longitude-matched ($< 30^\circ$)	1753	1327	1.32	[0.91, 2.65] (pair-boot, mean 1.71)	0.058

Distance stratum	EW/NS ratio
0–500 km	0.73
500–1,000 km	1.38
>1,000 km	0.82

A sensitivity analysis reveals that the directional ratio is fragile under conservative analysis choices. The ratio declines from 1.29 at the primary 50-km cutoff to 1.08 at 500 km and 0.89 at 1,000 km, with station de-clustering yielding a ratio of 0.99. The anisotropy is therefore classified as not robust; the directional signal is interpreted as null in the combined MGEX product, consistent with the non-significant station-clustered bootstrap ($p = 0.49$).

Analysis	λ_{EW} (km)	λ_{NS} (km)	Ratio
Primary (min 50 km)	1810	1402	1.29
Min 200 km	2098	1384	1.52
Min 500 km	1880	1734	1.08
Min 1000 km	2211	2479	0.89
Min 2000 km	2496	2671	0.93
Station de-clustered	—	—	0.99
Robust (min 500 km, bin 50)	1878	1733	1.08

4.3 Orbital-Velocity Coupling

The traditional monthly λ and EW/NS ratio do not correlate significantly with Earth's orbital speed or its projection onto the CMB dipole (Bonferroni $p > 0.5$ for both). These primary pre-specified metrics are degraded by exponential-fit parameter degeneracy on the short 16-month baseline, where bounded non-linear least squares fitting of $A \exp(-r/\lambda) + C_0$ suffers from severe variance inflation with too few monthly points. The monthly means retain January 2025 through April 2026 as 16 separate chronological observations; the one-day May fragment is excluded. A model-free PA-difference metric (monthly mean EW – NS phase alignment) avoids exponential fitting and has a negative association with orbital speed ($r = -0.716$, $p = 0.0018$) and with the velocity projection onto the CMB dipole ($r = -0.767$, $p = 0.0005$). The nominal p-values assume independent monthly points; under autocorrelation-aware diagnostics the associations are tempered but not erased. With effective degrees of freedom from the residual lag-1 autocorrelation ($\hat{\rho} = 0.40$ and 0.29), the corrected values are $p = 0.072$ (speed) and $p = 0.018$ (v_{proj}); matched AR(1) surrogates give $p = 0.019$ and $p = 0.003$; the phase-preserving circular-shift null gives $p = 0.125$ and $p = 0.063$ — the 1/16 resolution floor for the CMB-projection channel. Leave-one-month-out correlations stay within $r = -0.66$ to -0.86 for both predictors, so the association is not carried by a single favourable phase alignment. The physically motivated velocity-projection channel is thus the one that survives every correction, while the scalar-speed control is marginal. The physical hypothesis and expected sign were motivated by the earlier TEP-GNSS papers, which reported comparable orbital associations; this is therefore corroborative across products. It remains exploratory within the MGEX analysis because this particular operationalisation was selected after the primary λ and EW/NS estimators proved unstable. It is not included in the family-wise correction applied to confirmatory tests and should be tested prospectively in a longer MGEX series. The raw EW – NS contrast is the estimator most sensitive to any operational asymmetry between the east and west station populations (network density, tracking schedules); two controls price that channel directly. A composition-matched difference — EW – NS of cell means averaged with equal weight over (distance $\times$ |latitude|) cells — retains the association ($r = -0.586$, $p = 0.017$ vs speed; $r = -0.650$, $p = 0.006$ vs v_{proj} , $n = 16$), and a uniformly-tracked-station subset (both endpoints above the median pair-day participation) retains it more strongly ($r = -0.685$, $p = 0.003$; $r = -0.739$, $p = 0.001$; corrected $p = 0.029$ and AR(1) $p = 0.007$ for the v_{proj} channel). The association is therefore not carried by population or scheduling asymmetry between the sectors. Two further diagnostics bound the interpretation. First, the fitted-scale nulls are consistent with estimator dilution rather than signal absence: the monthly λ sequence scatters with $CV \approx 21\%$ about its mean while remaining uncorrelated with the PA-diff sequence ($r = -0.01$), and the per-month per-sector fits that enter the EW/NS λ ratio carry median propagated ratio errors of $\approx 27\%$ — and one month (2025-08) has a sector fit pinned at the λ upper bound, rendering that month's ratio formally uninformative — while the resulting ratio sequence is uncorrelated even with the fit-free anisotropy it is meant to measure (ratio vs PA-diff $r = -0.20$, $p = 0.45$) and remains null after dropping the degenerate month ($r = -0.10$ vs speed, $r = -0.17$ vs v_{proj}). A velocity-driven modulation at the observed contrast amplitude cannot propagate through the fitted decay length on a 16-month baseline — the unfitted contrast is the sensitive channel by construction. Second, a hemisphere decomposition of the same contrast (Step 3.4) returns the signed orbital-

speed anticorrelation in all three pair classes — intra-northern ($r = -0.45$), intra-southern ($r = -0.88$), and cross-hemisphere ($r = -0.49$) — with the northern and southern annual sinusoid phases consistent with a common value ($\Delta\phi = 22^\circ \pm 44^\circ$). A hemisphere-local seasonal mechanism would appear in anti-phase between the hemispheres; the observed common phase instead indicates a global heliocentric driver.

Quantity	r	p (raw)	p (corr.)	Interpretation
λ vs speed	-0.16	0.554	1.000	Primary (pre-specified): not significant
EW/NS ratio vs speed	0.14	0.610	1.000	Primary (pre-specified): not significant
λ vs v_{proj}	-0.05	0.850	1.000	Primary (pre-specified): not significant
EW/NS ratio vs v_{proj}	-0.03	0.911	1.000	Primary (pre-specified): not significant
PA_diff vs speed	-0.716	0.0018	0.072	Exploratory; $n_{\text{eff}} = 6.9$; AR(1) $p = 0.019$; shift $p = 0.125$; LOO $r \in [-0.84, -0.66]$
PA_diff vs v_{proj}	-0.767	0.0005	0.018	Exploratory; $n_{\text{eff}} = 8.8$; AR(1) $p = 0.003$; shift $p = 0.063$ (floor); LOO $r \in [-0.86, -0.72]$
PA_diff (cell-matched) vs speed	-0.586	0.0170	—	Operational-asymmetry control; composition-matched over distance $\times$ latitude cells
PA_diff (cell-matched) vs v_{proj}	-0.650	0.0064	—	Operational-asymmetry control; $n_{\text{eff}} p = 0.065$; AR(1) $p = 0.028$; shift $p = 0.063$ (floor)
PA_diff (uniform-tracking) vs speed	-0.685	0.0034	—	Operational-asymmetry control; both endpoints above median participation
PA_diff (uniform-tracking) vs v_{proj}	-0.739	0.0011	—	Operational-asymmetry control; $n_{\text{eff}} p = 0.029$; AR(1) $p = 0.007$; shift $p = 0.063$ (floor)

4.4 Projection-Degenerate Annual-Direction Test

This analysis uses the same PA-difference observable as Section 4.3 and is therefore a directional characterization, not an independent test of a cosmic rest frame. With 16 chronological full-month observations, the direct CMB-projection correlation is $r = -0.733$, computed from daily EW–NS differences averaged equally within each calendar month. This differs from the -0.767 result in Section 4.3, which averages all pair observations within each month (so months are pair-count weighted) before correlating PA difference with the CMB-projected velocity. The latter is the stronger weighted PA-difference result ($p = 0.0005$). A 37×19 scan gives a nominal maximum at RA = 230° , Dec = $+60^\circ$ ($r = -0.745$), 82.7° from the CMB dipole. Earth’s orbital velocity samples essentially one plane, so the scan identifies only a direction’s projection onto that plane, not its full three-dimensional vector. The CMB and nominal maximum projections correlate at $r = 0.984$, and 61 of 703 grid directions have $|r|$ at least as large as the direct CMB value. The 30° three-dimensional threshold is consequently not an identifiable test in this geometry. The grid detects directional annual modulation (plus-one permutation $p = 0.005$ from 200 complete-grid shuffles), but it does not select the CMB projection over the many compatible annual templates. Widening sectors to 60° gives $r = -0.791$ at another member of the same projection-degenerate family. The short series further limits precision, so this is evidence only for exploratory annual directional structure, not a CMB-frame detection or a unique sky-axis localization.

Min. baseline (km)	Direct r(CMB)	Months
50 (primary)	-0.733	16
200	-0.731	16
500	-0.730	16
1000	-0.723	16
2000	-0.697	16

The daily EW–NS difference, retained as a supplementary diagnostic, gives a direct CMB correlation of $r = -0.495$ ($n = 462$). Its agreement with the monthly result supports the directional temporal pattern. Neither cadence can identify a unique three-dimensional direction from Earth-orbital velocity alone.

Cadence	Sector width	Nominal RA (°)	Nominal Dec (°)	r	$\Delta\theta_{\text{CMB}}$ (°)	p (perm.)	Interpretation
Monthly	45°	230	60	−0.745	82.7	0.005	Annual template detected; CMB not uniquely selected
Monthly	60°	230	60	−0.791	82.7	0.005	Annual template detected; CMB not uniquely selected
Daily	45°	230	60	−0.505	82.7	—	Axis not uniquely identifiable

4.5 Ionospheric and Geometry Controls

The correlation length persists on geomagnetically quiet days ($\lambda = 1708$ km, $K_p \leq 2$, 712,186 pairs, $R^2 = 0.743$). Active days ($K_p \geq 5$, 39,093 pairs) give a longer $\lambda = 1931$ km ($R^2 = 0.295$); the smaller sample increases the variance and the value awaits confirmation with additional active-period data. Storm-excluded days ($K_p < 5$) give $\lambda = 1858$ km ($R^2 = 0.705$, 1,788,462 pairs), consistent with the base value. The distance-matched subset yields a comparable $\lambda = 1889$ km ($R^2 = 0.599$), indicating the signal is not a product of network geometry. Hemispheric λ differs north to south, as expected from the uneven station distribution; the distance-matched control is the geometry-robust comparison.

Control	λ (km)	R^2
Base (all days)	1862	0.699
Quiet days ($K_p \leq 2$)	1708	0.743
Active days ($K_p \geq 5$)	1931	0.295
Storm-excluded ($K_p < 5$)	1858	0.705
Distance-matched	1889	0.599
Northern hemisphere	1365	0.678
Southern hemisphere	2353	0.038

4.5a Product-Error and Atmospheric-Channel Controls

Two mundane channels could in principle mimic a distance-ordered clock correlation; each is evaluated against a computed forward model or a within-product partition rather than by label-permutation nulls alone. The leading channel is shared satellite orbit and clock error projecting into each station's receiver-clock estimate. The forward model (Step 3.2) constructs the expected kernel directly: the pairwise cosine similarity of the stations' satellite-weight vectors, built from real CODE SP3 orbits (122 satellites, 300 s cadence, 97 epochs spanning a full day) under standard elevation weighting and the 513 MGEX station coordinates. The kernel is flat above 0.93 through the 0–2,000 km band where the measured coherence carries most of its structure, decays only on the constellation-footprint scale (0.70 at 3–5,000 km, 0.45 at 5–8,000 km, 0.18 at 8–12,000 km — an effective scale an order of magnitude longer than the observed $\lambda = 1862$ km), and is everywhere non-negative whereas the measured profile crosses zero. A shared satellite-error projection can weight this kernel but cannot steepen it, so the geometric product-error channel cannot supply the measured decay.

The remaining mundane candidate is the spatially-correlated atmospheric component — tropospheric delay terms absorbed into the per-station clock estimates. An atmospheric driver is weather-organized rather than distance-organized: synoptic fields are hemisphere-confined, and the large-scale modes (annular modes, jet-stream teleconnections) are coherent along latitude bands over $\sim 10^4$ km, not across them. Three within-product controls on true station latitudes (Step 3.3) test this directly. First, refitting the profile within hemisphere classes, the cross-hemisphere subset — pairs that cannot share any synoptic weather — returns the cleanest structure in the archive ($\lambda = 2101$ km, $R^2 = 0.769$, 655,068 pair-days), against intra-NH $\lambda = 1308$ km ($R^2 = 0.653$). The intra-SH binned profile likewise decays monotonically (+0.63 to −0.14); its weak exponential-fit quality ($R^2 = 0.032$) reflects the long-baseline-dominated coverage of the southern network rather than absent structure. Second, at $d > 4000$ km, same-latitude-band pairs ($|\Delta\text{lat}| < 10^\circ$) return mean phase alignment −0.096 while different-band pairs ($|\Delta\text{lat}| > 30^\circ$) return −0.026 — the opposite ordering of the zonal-teleconnection prediction, and not a distance artifact because the same-band subset is the closer one (median 7,963 vs 9,129 km). Third, the decisive cell: same-band pairs at $d > 8000$ km — where a zonal atmospheric kernel must retain positive coherence — return mean phase alignment −0.118 on 107,013 pair-days, opposite in sign to the atmospheric requirement. The weather-organized component is therefore excluded at the level of its required structure; what remains of the tropospheric channel is per-station incoherent residual error, which cannot generate a distance-ordered pair correlation by construction.

Control subset	λ (km)	R^2	Pair-days	Mundane prediction
Intra-NH pairs	1308	0.653	850,240	Structure expected
Intra-SH pairs (profile decays $+0.63 \rightarrow -0.14$)	2437	0.032	322,247	Structure expected
Cross-hemisphere pairs (no shared weather)	2101	0.769	655,068	No structure expected — cleanest fit observed

Latitude-band control	Mean PA	Pair-days	Median d (km)	Atmospheric requirement
Same band, $ \Delta\text{lat} < 10^\circ$, $d > 4000$ km	-0.096	217,430	7,963	Positive zonal coherence — contradicted
Different band, $ \Delta\text{lat} > 30^\circ$, $d > 4000$ km	-0.026	899,956	9,129	Lower coherence than same band — contradicted
Same band, $ \Delta\text{lat} < 10^\circ$, $d > 8000$ km	-0.118	107,013	—	Positive coherence required — contradicted

4.5b Estimator Decomposition of the Fitted Scale

The pooled MGEX scale ($\lambda = 1862$ km) sits below the IGS-final family reported in Papers 1–2 (3,330–4,549 km). Both analyses measure the same statistic — the cosine of the magnitude-weighted circular mean cross-spectral phase — so the offset can be decomposed estimator-axis by estimator-axis on identical real station-days (Step 3.5). Three documented settings differ between the pipelines: the Welch segment length (96 samples here, $\Delta f \approx 34.7$ μHz , versus a full-day segment in Papers 1–2, $\Delta f \approx 11.6$ μHz), the per-epoch common-mode removal (applied here; not applied in Papers 1–2), and the clock products themselves.

On a 58-day sample spanning the archive (211,012 pair-days), the factorial decomposition returns: canonical settings $\lambda = 1285 \pm 233$ km (day-block bootstrap; the full-archive estimate on all 462 usable days is 1,862 km); common-mode removal switched off, $\lambda = 1368$ km; full-day spectral resolution with common-mode, $\lambda = 1363$ km; and the Papers-1–2 estimator combination (full-day resolution, no common-mode removal), $\lambda = 1598$ km. Restricting the full-day spectrum to the resolved MGEX floor (≥ 34.7 μHz) returns $\lambda = 1045$ – 1049 km, confirming that the two lowest resolved bins carry the longest-range content — as required if the correlation is band-ordered. The estimator axes thus move the fitted scale over 1.0 – 1.6×10^3 km on identical data ($\approx \pm 25\%$ around the canonical value) and cannot supply the $\sim 2\times$ offset to the IGS-final family; the residual resides in the clock products themselves — their multi-GNSS combined construction, station complement, and epoch coverage — rather than in the correlation estimator.

The same ordering is recovered per sub-band: on the full-day matrix the fitted scale declines from $\lambda \approx 1.8$ – 2.0×10^3 km below 30 μHz to ≈ 0.5 – 1.2×10^3 km above 50 μHz , mirroring the IGS-final profile, which declines from ≈ 3.9 – 5.9×10^3 km in the tidal bands to ≈ 1.0 – 1.4×10^3 km at intermediate frequencies (Paper 1 §3.5.1). Both products therefore return the same band-ordered, distance-structured correlation; the MGEX profile is a scaled-down version of the IGS-final one. The defensible cross-product statement is a thousands-of-km coherence scale with a shared band ordering. The fitted scale is product- and estimator-conditioned (product-level values ~ 1.0 – 4.8×10^3 km; raw-SPP channels ~ 0.7 – 1.1×10^3 km and the raw ionofree channel returns $4,767 \pm 835$ km; Paper 3), and no point-scale identification with the geometric saturation radius is made.

Two provenance details are bounded by the same diagnostic. First, GRG product-days are natively 30-s sampled, so on the configured 5-min-labelled frequency grid those station-days resolve a ~ 0.35 – 5 mHz band rather than the nominal 10–500 μHz band; substituting their correctly decimated contribution moves the pooled scale by $\sim 1\%$ ($1,271 \rightarrow 1,285$ km on the sampled day set; 9 of 486 days carry GRG data). The per-center consequence is larger — the GRG-only matched-day scale moves from 3,257 km to 1,933 km under the corrected pairwise epoch intersection (Step 3.1) — because the per-center path additionally truncated unequal-length series without timestamp alignment, a fault the shared-grid construction removes. The pairing step now thins finer-cadence products to the 300 s grid before spectral estimation. Second, single-window realisations remain epoch-dependent at the ~ 30 – 40% level, as reported in §4.1.

The fitted amplitude A decomposes along the same axes (Step 3.6; 46 COD days, 190,867 pair-days, all statistics evaluated on identical pair sets). The phase-alignment index is bounded in $[-1, +1]$ and saturates once the in-phase component dominates the band-integrated cross-spectrum, so A is not a linear estimator of the shared coherent power: on the same bins the canonical configuration returns $A = 0.789$ while the band-averaged real coherency — the linear in-phase coherence measure — returns $A = 0.250$. The same statistic-level inflation is visible inside the corpus on Paper 3's raw-SPP analysis, which reports both statistics on identical pair-days: normalised coherence amplitude 0.182 against phase-alignment 1.056, a $\sim 5.8\times$ spread on 61.8 million pairs. Estimator conditioning contributes the next factor: under the Papers-1–2 settings on the same MGEX station-days (single full-day Welch segment, no common-mode removal, no flat-line rejection) the fitted amplitude falls to $A = 0.466$, and the flat-line rejection alone moves $A = 0.436 \rightarrow 0.789$ — held-constant clocks constitute 11.5% of COD and 18.2% of GRG station-days and inject near-random pair phases. Per-product ordering inside MGEX confirms the remaining conditioning: on identical canonical settings GRG returns $A = 0.462$ against COD 0.789. A labelled transfer calibration (Step 3.6; synthetic red-noise pairs, estimator calibration only) inverts the published amplitudes to underlying band coherences of $\rho \approx 0.30$ (MGEX) and $\rho \approx 0.05$ (IGS finals) under each family's own spectral window — a product-level difference in the shared in-phase budget — not yet modelled from product construction, and not in contradiction with the measured structure. The fitted offset shows the same bookkeeping: all three product families return negative

C_0 , and the deeper MGEX asymptote is the zero-sum signature of its per-epoch median datum — toggling the common-mode subtraction on identical pair-days moves C_0 from -0.015 to -0.066 , the same distant-pair anticorrelation produced by the conditioned datum kernel (Paper 1, §4.6). The cross-product comparison is therefore carried by the signed, distance-ordered in-phase decay and its scale ordering, not by amplitude equality, and the $\sim 7\times$ headline ratio is a property of the statistic's saturation plus product and estimator conditioning.

4.6 Null Tests

Null tests were run using the same MGEX combined-clock phase-alignment pipeline. Three of the four nulls (temporal shuffle, spatial shuffle, and phase randomisation) collapsed to $R^2 \leq 0$, confirming that the recovered correlation length depends on the true temporal, spatial and phase information. The solar-rotation reassignment null, however, retains $R^2 = 0.50$ under the primary parameters. A sensitivity analysis attributes this to a small-sample bin artifact: short-baseline bins at 100–115 km (40 pairs) and 132–152 km (23 pairs) receive extreme coherence values (0.29 and 0.21) under the specific shuffle sequence, producing a spurious short-baseline spike that the exponential fit interprets as structure. Under a robust analysis (minimum baseline 500 km, minimum bin count 50) all four nulls give $R^2 \leq 0.122$, confirming that the distance-decay structure depends on the true temporal, spatial, and phase information and is not a solar-period artifact.

Null	R^2 (primary)	Passes?	R^2 (robust)	Passes?
Temporal shuffle	-0.343	Yes	-0.011	Yes
Spatial shuffle	-0.000	Yes	0.028	Yes
Phase randomisation	-0.001	Yes	0.122	Yes
Solar-rotation reassignment	0.499	No	0.010	Yes

The robust analysis (minimum baseline 500 km, minimum bin count 50) eliminates the small-sample bin artifact that caused the solar-rotation null to fail under the primary parameters. All four null controls pass under the robust analysis, confirming that the observed spatial covariance requires genuine spatial and temporal phase ordering.

5. Discussion

The combined MGEX receiver-clock product shows a distance-dependent structure with $\lambda = 1862$ km. The estimate remains within 1,819–2,000 km when the minimum baseline is raised from 50 to 2,000 km, while the binned-fit R^2 improves from 0.70 to about 0.92. It persists on quiet days and after distance matching, and it survives the two computed mundane channels rather than only label-permutation nulls: the forward-modelled shared satellite-error kernel is flat through the signal band and decays an order of magnitude more slowly than the measured profile; conditioning the same channel on the product's own ephemerides and satellite-clock content decays no faster than $\lambda = 6,650$ km ($\sim 3.6\times$ the measured scale; Paper 1, Step 5.2); and the weather-organized partitions return the opposite of the atmospheric signature — cross-hemisphere pairs, which share no synoptic weather, carry the cleanest decay, and same-latitude-band pairs beyond 8,000 km return negative coherence where a zonal atmospheric kernel requires positive coherence. The ± 112 km formal fit uncertainty treats 1.83 million pair-days as independent; cluster-robust resampling of whole epochs inflates the statistical error only modestly — ± 86 km under day-block bootstrap, ± 155 km under month-block bootstrap, and ± 94 km under leave-one-month-out jackknife — so the within-product statistical precision is about $\pm 8\%$. The dominant scale uncertainties are systematic rather than statistical: single-window realisations are epoch-dependent at the ~ 30 – 40% level, and the matched-day cross-center comparison contributes $\sim \pm 600$ km. A factorial decomposition on identical station-days (Step 3.5, §4.5b) shows that the estimator axes — Welch segment length and per-epoch common-mode removal — move the fitted scale by $\leq \pm 25\%$, so the residual offset to the IGS-final family (3,330–4,549 km, Papers 1–2) resides in the clock products rather than in the correlation estimator, and the same band-ordered profile (longest scale in the lowest resolved sub-band) is recovered in both product families. The amplitude is conditioned the same way and more strongly: the fitted A is a bounded, saturating phase-direction index (§4.5b, Step 3.6) whose $\sim 7\times$ offset from the IGS-final family decomposes into a $\sim 3\times$ statistic nonlinearity, the documented estimator axes, and a product-level difference in the shared in-phase budget — the linear coherence measured on identical pair-days is $A = 0.250$, and the deeper negative MGEX asymptote is the signature of its own zero-sum datum, not a sign discrepancy. The result is nonetheless a useful cross-product corroboration of a correlation scale in the thousands-of-kilometres range, not a direct measurement-equivalence claim with the earlier GPS PPP products.

The conventional directional measures do not replicate robustly. The full-range EW/NS ratio is 1.29 but is non-significant under station-clustered bootstrap ($p = 0.49$). The longitude-matched subset is suggestive under a pair bootstrap (mean ratio 1.71, $p = 0.058$) yet non-significant under spatially clustered resampling ($p = 0.214$). Its decline under more conservative baseline choices and station de-clustering makes a positive anisotropy conclusion unwarranted for this product.

The orbital finding is more informative but has a narrower evidential status. The primary monthly λ and EW/NS estimators are null after Bonferroni correction, unsurprising with only 16 complete chronological months and unstable monthly exponential fits. The model-free PA-difference statistic, monthly mean EW minus NS phase alignment, has $r = -0.716$ with orbital speed ($p = 0.0018$) and r

= -0.767 with velocity projected onto the CMB dipole ($p = 0.0005$). Because the nominal p -values assume independent monthly points, they are reported alongside autocorrelation-aware diagnostics: effective- degrees-of-freedom values $p = 0.072$ and $p = 0.018$, matched AR(1)-surrogate values $p = 0.019$ and $p = 0.003$, and circular-shift values $p = 0.125$ and $p = 0.063$ (the 1/16 floor for the projection channel). The association is therefore tempered but not erased, and it is the physically motivated velocity-projection channel that survives; leave-one-month-out correlations remain within $r = -0.66$ to -0.86 , excluding single-month dominance. These signs and magnitudes are consistent with the preceding TEP-GNSS analyses, which motivated the physical prediction before this MGEX analysis. That history makes the result corroborative across products. However, PA difference was chosen here after the primary estimators were unstable, so it is exploratory within this study and should not be counted as an independent confirmatory test or folded into confirmatory family-wise inference.

The annual-direction analysis keeps January 2025 through April 2026 as separate months. Its direct CMB-projection correlation is $r = -0.733$, consistent with the stronger weighted orbital result ($r = -0.767$, $p = 0.0005$). The full-sky scan has a nominal maximum at RA = 230° , Dec = $+60^\circ$ ($r = -0.745$), 82.7° from the CMB dipole. Earth's orbital velocity remains in the ecliptic plane, so dot-product regression identifies only an axis's in-plane projection; directions separated substantially in three dimensions can give nearly identical annual predictors. Here the CMB and nominal-best projected predictors correlate at $r = 0.984$, and 61 of 703 grid directions equal or exceed the direct-CMB $|r|$. The 30° three-dimensional rule is thus not an identifiable criterion for this design. The permutation result (plus-one $p = 0.005$ over 200 complete-grid shuffles) supports directional annual structure, while the direct CMB projection is only one non-unique member of that structure. It does not identify the CMB frame. The 60° sensitivity result and daily analysis are corroborative diagnostics, not independent confirmations. The 16-month series further limits precision; a preregistered multi-year analysis should lock the PA-difference definition and report an identifiable ecliptic-plane direction or add non-coplanar velocity information before claiming a unique sky-frame axis.

The satellite-clock analogue does not recover an exponential structure. GLONASS, BeiDou, and Galileo have negative fitted R^2 values, while GPS has $R^2 = +0.13$ with boundary convergence. That result is consistent with the single-reference-time architecture of the combined MGEX solution and limits what this product can say about satellite-clock effects; it is not evidence against a physical effect in independently estimated satellite clocks.

The central limitation is structural: CODE MGEX supplies one combined multi-GNSS receiver-clock solution, so the constellations cannot be separated in this analysis. The present study therefore contributes product-type and epoch diversity, rather than per-constellation independence. Longer, separately processed multi-GNSS or raw-observation analyses are needed to turn the corroborative orbital result into a prospective test and to evaluate sky-frame localization.

6. Conclusions

This product-type independence stress test uses the CODE COD0MGXFIN combined multi-GNSS receiver-clock product from 2025-01-01 to 2026-05-01. Its principal result is a distance-dependent correlation scale of $\lambda = 1862$ km ($R^2 = 0.699$; formal within-fit $\sigma = \pm 112$ km, cluster-robust $\sigma = \pm 155$ km from month-block epoch resampling), stable across the tested baseline cutoffs and present on quiet geomagnetic days, with epoch- and product-level scale systematics of ~ 30 – 40% reported alongside the statistical error. Under the primary short-baseline parameters the solar-rotation reassignment null retains $R^2 = 0.50$, traced to a small-sample bin artifact at 100 – 152 km; the robust analysis (minimum baseline 500 km, minimum bin 50) passes all four null controls and is reported as the clean result, with the fitted scale essentially unchanged ($1,819$ km). This is cross-product corroboration of a spatial structure at the thousands-of-kilometres scale reported in the earlier TEP-GNSS papers, while remaining subject to the different metric and combined-clock product construction. The fitted scale is a product-dependent phenomenological observable — the transfer map from the microscopic screening operator $\mathcal{S}_\Sigma(\mathcal{E})$ to λ remains to be derived — so the result is reported as corroboration of the TEP scale, not as a parameter-level prediction test.

The secondary results are deliberately qualified. Standard anisotropy and primary orbital metrics are not robust in this short MGEX interval. A model-free PA-difference statistic has strong associations with orbital speed ($r = -0.716$, $p = 0.0018$) and CMB-projected orbital velocity ($r = -0.767$, $p = 0.0005$) across 16 chronological months. Because those signs were anticipated by previous TEP-GNSS work, the result is corroborative across products; because this operationalisation was selected after primary MGEX fits were unstable, it remains exploratory within this study and needs a prospective, longer-baseline replication.

The CMB-labelled orbital template is correlated with the exploratory statistic: the direct monthly CMB projection has $r = -0.733$, while the weighted PA-difference test has $r = -0.767$ ($p = 0.0005$). The grid's nominal RA = 230° , Dec = $+60^\circ$ maximum is 82.7° from the CMB dipole. Because orbital velocity constrains only an axis's ecliptic-plane projection, the grid cannot uniquely localize a three-dimensional sky direction; its within-grid permutation statistic (plus-one $p = 0.005$ from 200 shuffles) measures annual directional structure; it does not detect a CMB rest frame. The satellite-clock analysis is limited by the combined product's shared reference time and does not recover an exponential signal; the MGEX product therefore cannot test the satellite-clock channel at all, and a per-constellation solution with independently estimated satellite clocks would be required for that test. The next decisive test is a preregistered chronological analysis over multiple annual cycles, reporting an identifiable plane-projected direction or using non-coplanar velocity information.

7. Data and Code Availability

All data are publicly available from NASA CDDIS (cddis.nasa.gov) and the IGS (igs.org) under their standard open-data policies. The analysis uses the MGEX combined multi-GNSS receiver-clock product COD0MGXFIN (CODE), with WUM (Wuhan) and GRG (CNES) MGEX clock products as day-level fallbacks, downloaded via authenticated HTTPS. No proprietary or restricted data are used, and no raw RINEX observation files are required.

Analysis code and pipeline outputs are available at github.com/matthewsmawfield/TEP-GNSS-MGEX.

8. References

8.1 TEP Series

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8.3 Cosmology and CMB

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9. Reproducibility Statement

This manuscript is accompanied by a complete, open-source reproduction pipeline available at github.com/matthewsmawfield/TEP-GNSS-MGEX. All figures, tables, and statistical values reported in this paper can be regenerated from publicly available data using the code and instructions provided in the repository.

Data: The analysis uses only publicly distributed MGEX combined multi-GNSS receiver-clock products (CODE COD0MGXFIN), with WUM and GRG products as day-level fallbacks. Data were downloaded from NASA CDDIS (cddis.nasa.gov) and the IGS (igs.org) via standard authenticated HTTPS endpoints. No proprietary, restricted, or raw RINEX observation files are required.

Software: The pipeline is written in Python (NumPy, SciPy) and standard geodetic libraries. All dependencies are listed in `requirements.txt` in the repository root. No commercial or closed-source software is required.

Execution: After installing dependencies and configuring access to the authenticated CDDIS endpoints, running `python scripts/run_all.py` from the repository root reproduces the analysis chain: data download, station-pair differencing, phase-coherent stacking, isotropic and anisotropic correlation fitting, orbital-velocity projection, and figure generation. Run `python`

`scripts/generate_figures.py` after an analysis-only rerun; then run `cd site && npm run build` to build the manuscript site. Execution time depends on hardware, cached downloads, and data availability.

Verification: All derived output files in `results/outputs/` are regenerated by the pipeline from raw data, permitting direct verification of reproduced results.

Environment: The pipeline has been tested on macOS 14 and Ubuntu 22.04 with Python 3.11. All dependencies are listed in `requirements.txt` for reproducible installation.