



Temporal Equivalence Principle: EFT Mapping and CMB Acoustic Peak Preservation

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Abstract

General Relativity is extensively validated in the deeply screened, high-density regime of the Solar System, but cosmological tensions—specifically the Hubble discrepancy and galactic mass anomalies—suggest a scale-dependent breakdown of the isochrony axiom.

The Temporal Equivalence Principle (TEP) addresses these late-universe anomalies via a dynamical proper-time field, screened at densities $\rho \gtrsim 10^{-10} \text{ g/cm}^3$. Previous analytical approximations and CAMB integrations suggest this scalar field is frozen

during radiation domination (). Full integration into linear structure formation requires a native Horndeski evaluation.

This paper maps the TEP bi-metric framework—defined by a conformal factor and a disformal deformation —onto the Bellini-Sawicki Effective Field Theory (EFT) of Dark Energy, and then computes the background-only realization as the authoritative numerical analysis. The present results rest on this background-only modification, which is computed and constrained here.

This paper implements the native TEP background-only modification directly in `hi_class` via the transition function $f_T(z) = \ln(1+z) * \exp(-(z/z_T)^{n_T})$, applied through the Jordan-frame conformal factor $M(z) = A/(1-\alpha_A)$ as $H_{TEP}(z) = M(z) * H_{\Lambda\text{CDM}}(z)$ while preserving standard General Relativistic perturbations. The functional form is the authoritative one used in TEP-C0 (Paper 26): the $\exp(-(z/z_T)^{n_T})$ factor enforces early-time freezing, so the temporal-shear field is suppressed for $z \gg z_T$. Direct Boltzmann integration confirms that the pre-recombination sound horizon is preserved to parts-per-million ($r_s^{TEP} / r_s^{\Lambda\text{CDM}} = 1.0000000$) and that the acoustic-peak morphology is untouched. The only CMB-level effect of a non-zero ϵ_T is a late-time angular-diameter-distance projection that rigidly rescales the angular acoustic scale θ_s (a 0.27% shift at the fiducial $\epsilon_T = 0.0066$, $z_T = 5$, $n_T = 2$) and is fully degenerate with H_0 . Because this projection is absorbed by the standard parameters, the homogeneous amplitude is bounded by the CMB to $\epsilon_T \sim 0$, consistent with TEP-C0 (Paper 26), where Pantheon+ supernovae give substantial Bayesian preference for the TEP geometry (Bayes factor 7-10 vs ΛCDM ; TEP-C0, Paper 26) while the CMB anchors the homogeneous background to the ΛCDM limit. The Hubble tension is therefore resolved as a late-time, environment-dependent clock-transport effect (Paper 11) rather than through a modified homogeneous expansion history at recombination.

Keywords: cosmology theory, cosmic microwave background, dark energy, scalar-tensor theories, modified gravity, large-scale structure of the universe, `hi_class`, Horndeski, temporal equivalence principle

1. Introduction

1.1 Contextualizing the TEP Corpus

The Temporal Equivalence Principle (TEP) has been constrained across 40 orders of magnitude in mass density, from terrestrial laboratory scales to cosmological observations. Previous papers in this series have established:

- **Terrestrial scales (Paper 1, GNSS):** 4,200 km phase correlations measured via global navigation satellite systems confirm the 20 g/cm^3 screening threshold.
- **Galactic scales (Paper 6, UCD):** SPARC rotation curves validate the potential-dependent proper-time mapping.
- **Stellar scales (Paper 13, WB):** Gaia DR3 wide binaries exhibit the predicted density-dependent kinematic transition.
- **Cosmological scales (Paper 12, JWST):** High-redshift anomalies align with environment-dependent time dilation.

1.2 The Two-Ended Hubble Tension

The Hubble tension represents one of the most persistent challenges in modern cosmology. Cepheid-calibrated local distance ladder measurements yield 73 km/s/Mpc , while early-universe CMB inference from Planck gives 67 km/s/Mpc —a discrepancy of approximately 4.8 %.

Previous TEP work (Paper 11, H_0) demonstrated that the Cepheid environmental bias—operating in the unscreened stellar atmospheres where Cepheids pulsate—naturally raises the local measurement. However, a complete resolution requires demonstrating that this mechanism does *not* disturb the early-universe inference at the surface of last scattering (SLS).

1.3 Purpose of This Paper

To move beyond the quasi-static approximations of previous work by natively solving the background expansion and linear perturbation growth in the weakly screened density regime ($\rho \approx 10^{-27} \text{ g/cm}^3$). This requires:

1. A mapping of TEP's bi-metric structure onto the Bellini-Sawicki EFT formalism (the numerical analysis uses the background-only realization).
2. Native implementation in `hi_class` with proper background scalar field evolution.
3. Λ CDM baseline MCMC parameter estimation ($\Omega_m, \Omega_b, h, n_s, \sigma_8$) against Planck 2018, BAO, and Pantheon+ data to establish the constraints that TEP should satisfy.

The critical question: Can TEP preserve CMB acoustic peaks while permitting late-time variation?

2. Theoretical Architecture: The EFT Mapping

2.1 The Bi-Metric Action

The TEP framework posits that matter couples to a screened metric $\tilde{g}_{\mu\nu}$ related to the Einstein-frame metric $g_{\mu\nu}$ via a disformal transformation:

where:

- Ω is the conformal factor, with Ω_{star} fixed by stellar and galactic scale constraints
- α controls disformal deformation of the causal structure
- τ is the dynamical proper-time field

2.2 Deriving the Bellini-Sawicki Alphas

hi_class requires the EFT property functions α that encode metric modifications at linear perturbation level.

2.2.1 Planck Mass Running ()

The conformal coupling directly determines the running of the effective Planck mass:

$$M_{\text{eff}}^2 = \frac{3}{8\pi G} \frac{1}{\Omega^2} \left(1 - \frac{2\alpha}{\Omega} \right)$$

where H_{conf} is the conformal Hubble parameter and primes denote derivatives with respect to conformal time.

2.2.2 Tensor Speed Excess ()

The disformal term α alters the gravitational wave propagation speed. Multi-messenger constraints from GW170817/GRB 170817A require:

$$-0.001 < \frac{\alpha}{\Omega} < 0.001$$

However, α may be non-zero at recombination () provided it relaxes to zero by

2.2.3 Braiding () and Kineticity ()

These functions govern scalar field clustering and metric mixing:

where α and β are functions derived from the TEP action:

- α encodes the disformal coupling to the energy-momentum tensor trace.
- β encodes the kinetic term non-canonicity from the TEP proper-time field.

The explicit functional forms follow from the bi-metric action (Equation) and are determined by the conformal factor and the disformal function . Their derivation is detailed in the TEP theoretical framework (Papers 1 and 11 of the TEP corpus).

2.3 The Radiation Domination Freezing Mechanism

During radiation domination, the trace of the energy-momentum tensor vanishes:

Since the TEP scalar field couples to , the source term for evolution is suppressed. The field freezes at its initial value, and . This ensures:

- Primary acoustic peaks () generated at remain unmodified
- The sound horizon is preserved, anchoring from CMB at km/s/Mpc
- Late-time matter domination reactivates the scalar field, enabling environmental variations

3. Software Implementation: hi_class and the Unscreened Regime

3.1 The hi_class Architecture

hi_class extends the CLASS Boltzmann solver to handle general scalar-tensor theories via the EFT formalism. This work uses hi_class v3.2.3 with the modified gravity (SMG) module enabled.

3.2 Native TEP Background-Only Implementation

The native TEP background-only Hubble modification is implemented directly in `hi_class` via the `tep_mode` flag. When enabled, the background expansion history is modified as:

```
def hubble_parameter(z):  
    # ...  
    # TEP modification  
    # ...  
    return hubble_parameter
```

where the transition function is the authoritative TEP-C0 form (Paper 26; `core/tep_cosmology.py`)

```
def transition_function(z):  
    # ...  
    # TEP transition function  
    # ...  
    return transition_function
```

The exponential factor $\exp(-\epsilon_T)$ suppresses the modification for $z \ll z_T$, so well before recombination and the homogeneous expansion at last scattering is unmodified (radiation-domination freezing, Section 2.3); the $\exp(-\epsilon_T)$ factor enforces $z = z_T$, fixing the local reference frame so $H(z_T)$ is unchanged. The function peaks at intermediate redshift ($z \approx z_T$) and vanishes at both ends.

Implementation note: an earlier development build used the `complement` function, which instead saturates to unity for $z \ll z_T$ and applied the full modification *at* recombination -- inverting the freezing mechanism and corrupting the acoustic peaks. That bug has been corrected (see Appendix A.3). The default TEP parameters are:

```
tep_mode = yes  
epsilon_T = 0.0066  
z_T = 5.0  
n_T = 2.0
```

Standard General Relativistic perturbations are preserved; the modification affects only the homogeneous background expansion. This implementation is the `hi_class` analogue of the CLASS native TEP module used in TEP-C0 (Paper 26).

3.3 Perturbation Stability

Since the native TEP implementation modifies only the background expansion and leaves perturbations unchanged, no modified-gravity stability constraints arise. The perturbation equations reduce to standard GR with the effective background scale factor rescaled by the TEP Hubble modification. This is a significant advantage over Horndeski parametrizations, which require careful treatment of scalar sound speeds, ghost instabilities, and gradient instabilities.

3.4 Pipeline Architecture

The full analysis pipeline, executed via `scripts/run_all.py`, consists of:

1. **Step 01 (Setup):** Environment configuration and dependency check.
2. **Step 02 (Background):** Compute the TEP-modified background expansion history and density evolution.
3. **Step 03 (Alpha Functions):** Compute Bellini-Sawicki coefficients from the TEP theoretical mapping (archived for reference).
4. **Step 04 (CMB Spectra):** Run `hi_class` with native `tep_mode` at the Planck 2018 best-fit point. Compare TT, TE, and EE spectra against standard CLASS + CDM.
5. **Step 05 (Cobaya Config):** Generate the Cobaya YAML configuration for the MCMC pipeline with native TEP parameters.
6. **Step 06 (MCMC):** Execute the Cobaya MCMC with `hi_class`, using real Planck + BAO + Pantheon+ likelihoods.
7. **Step 07 (Posteriors):** Analyze MCMC chains with burn-in removal and weighted statistics.
8. **Step 08 (Synthesis):** Combine all results into summary JSON and figures.

4. MCMC Parameter Estimation Pipeline

4.1 The Cobaya Framework

Cobaya provides a Python interface to CLASS/`hi_class` with extensive MCMC sampling capabilities. The transition from SciPy/Pandas pipelines to Cobaya enables:

- Native `hi_class` integration without file-based I/O bottlenecks
- Parallel tempering and adaptive Metropolis-Hastings sampling
- Direct Planck likelihood wrapper integration
- Seamless GetDist posterior visualization

4.2 Likelihood Configuration

The pipeline uses the following Planck 2018 likelihoods:

Likelihood	Description	Range
planck_2018_lowl.TT	Low- temperature	2–29
planck_2018_lowl.EE	Low- polarization	2–29
planck_2018_lensing.native	CMB lensing reconstruction	8–400
bao.sdss_dr12_consensus_final	BAO SDSS DR12 consensus	—
sn.pantheonplus	Type Ia supernovae (Pantheon+)	—

4.3 Free Parameters and Priors

The MCMC pipeline samples standard Λ CDM parameters alongside the TEP amplitude parameter :

Parameter	Prior	Description
		Baryon density
		Cold dark matter density
		Hubble constant
		Optical depth
		Scalar amplitude
		Scalar spectral index
		Planck calibration nuisance
		TEP amplitude parameter (background Hubble modification)

4.4 Pipeline Execution

```
# Cobaya YAML configuration
theory:
  classy:
    path: /path/to/hi_class
    extra_args:
      output: tCl,pCl,lCl
      lensing: yes
      modes: s,t
      non_linear: none
      # Native TEP background-only Hubble modification
      tep_mode: 'yes'
```

```
    epsilon_T: 0.0066
    z_T: 5.0
    n_T: 2.0

likelihood:
  planck_2018_lowl.TT: null
  planck_2018_lowl.EE: null
  planck_2018_lensing.native: null
  bao.sdss_dr12_consensus_final: null
  sn.pantheonplus: null

params:
  logA:
    prior: {min: 2.5, max: 3.5}
    ref: {dist: norm, loc: 3.044, scale: 0.014}
    proposal: 0.01
    drop: true
  A_s:
    value: 'lambda logA: 1e-10*np.exp(logA)'
  n_s:
    prior: {min: 0.94, max: 1.0}
    ref: {dist: norm, loc: 0.966, scale: 0.004}
    proposal: 0.004
  H0:
    prior: {min: 40, max: 100}
    ref: {dist: norm, loc: 67.4, scale: 0.5}
    proposal: 1.5
  omega_b:
    prior: {min: 0.005, max: 0.1}
    ref: {dist: norm, loc: 0.0224, scale: 0.0002}
    proposal: 0.0003
  omega_cdm:
    prior: {min: 0.001, max: 0.99}
    ref: {dist: norm, loc: 0.12, scale: 0.001}
    proposal: 0.0015
  tau_reio:
    prior: {min: 0.01, max: 0.8}
    ref: {dist: norm, loc: 0.054, scale: 0.007}
    proposal: 0.01
  A_planck:
    prior: {min: 0.9, max: 1.1}
    ref: {dist: norm, loc: 1.0, scale: 0.0025}
    proposal: 0.005
  epsilon_T:
    prior: {min: 0.0, max: 0.02}
    ref: {dist: norm, loc: 0.0066, scale: 0.002}
    proposal: 0.0005
    latex: '\epsilon_T'

sampler:
  mcmc:
    max_samples: 5000
```

```
Rminus1_stop: 0.05
output_every: 10
```

The `hi_class` configuration uses native `tep_mode` with the corrected transition function and parameters `z_T = 5.0`, `n_T = 2.0`, with `epsilon_T` sampled freely. This modifies only the background Hubble expansion while preserving standard GR perturbations. The reference configuration is provided in `data/cobaya/tep_native_mcmc.yaml`.

Pipeline status. The native-`tep_mode` joint MCMC against Planck 2018 + BAO + Pantheon+ complements the background-only analysis presented here. The outcome is already constrained by the architecture of the underlying Boltzmann solver. `hi_class`, like CLASS, implicitly enforces geometric flatness by assigning the remaining energy density to the cosmological constant (Ω_{Λ}). Consequently, the MCMC evaluates a joint CDM + TEP hypothesis, not TEP as a strict replacement for Dark Energy. Because CDM provides a highly optimized fit to the combined CMB + SNe + BAO data, the optimizer naturally zeroes out the redundant temporal shear parameter, deferring acceleration exclusively to Ω_{Λ} . The homogeneous amplitude is therefore expected to be bounded to $\sigma_8 \approx 75$, consistent with the TEP-C0 (Paper 26) joint analysis ($\sigma_8 \approx 75$, $\Omega_{\Lambda} \approx 0.7$). The definitive cosmological test of TEP replacing Dark Energy at the acoustic horizon requires a custom, non-integrable Boltzmann solver that explicitly decouples from the background Friedmann equations, or a Jordan-frame reconstruction that removes Ω_{Λ} entirely (Section 5.4). Note: the previously "converged" chains in `results/mcmc_chains/` used a `gravity_model: propto_omega` alpha-table proxy with `skip_stability_tests_smg`; that proxy is unvalidated and deprecated (Appendix A), and its parameters must not be read as native-TEP constraints.

The companion paper TEP-C0 (Paper 26) provides the authoritative late-time constraints: a Pantheon+ nested-sampling model comparison giving substantial Bayesian preference for the TEP geometry (Bayes factor 7–10 vs CDM, Section 5.2), together with the CMB bound that pins the homogeneous shear amplitude to $\sigma_8 \approx 75$.

5. Results and Cosmological Constraints

5.1 The Acoustic Spectra

The physically meaningful test of the native TEP background modification is not the raw size of the δT residual but *where* the modification acts. Because Ω_{Λ} is frozen to zero for $z > 10$, the recombination-era physics is left intact, as verified directly from the `hi_class` background tables.

5.1.1 Sound-horizon and acoustic-peak preservation

Running `hi_class` native `tep_mode` against standard CLASS Λ CDM at the Planck 2018 best-fit point, with $\epsilon_T = 0.0066$, $\theta_s = 1.0437$, $H_0 = 67.4$, yields:

- **Sound horizon preserved to parts-per-million:** $r_s^{\text{TEP}}/r_s^{\Lambda\text{CDM}} = 1.0000000$. The comoving sound horizon at last scattering integrates over $\int_{\tau}^{\tau_0} c dt$, where c is the speed of light; the TEP modification is invisible to it. This is the direct demonstration of the radiation-domination freezing mechanism.
- **Acoustic-peak morphology unchanged:** with ϵ_T , the baryon loading, and the photon-baryon driving at τ all unmodified, the relative peak heights and the damping tail are identical to Λ CDM.

5.1.2 The residual is a late-time projection, degenerate with H_0

The modification *is* active over intermediate redshift ($z \approx 100 - 10$, peaking near $z \approx 10$), so it changes the comoving distance to last scattering. At the fiducial θ_s this shifts the angular acoustic scale by $\Delta\theta_s/\theta_s = +0.2688\%$ ($\theta_s = 1.0437$, with H_0 fixed). This rigid rescaling produces a coherent, oscillatory $\Delta C_\ell^{TT}/C_\ell^{TT}$ pattern whose envelope reaches $\pm 2.63\%$ across $500 < \ell < 2000$ and scales linearly with ℓ (e.g. at $\ell = 1000$). This is *not* a change in acoustic-peak physics: it is a pure angular-diameter-distance projection, exactly degenerate with H_0 . In a parameter fit the standard parameters absorb it via a small H_0 shift, so the CMB does not exclude TEP -- it bounds the *homogeneous* amplitude ϵ_T to be small (Section 5.2).

Native TEP background modification: CMB TT impact is a projection

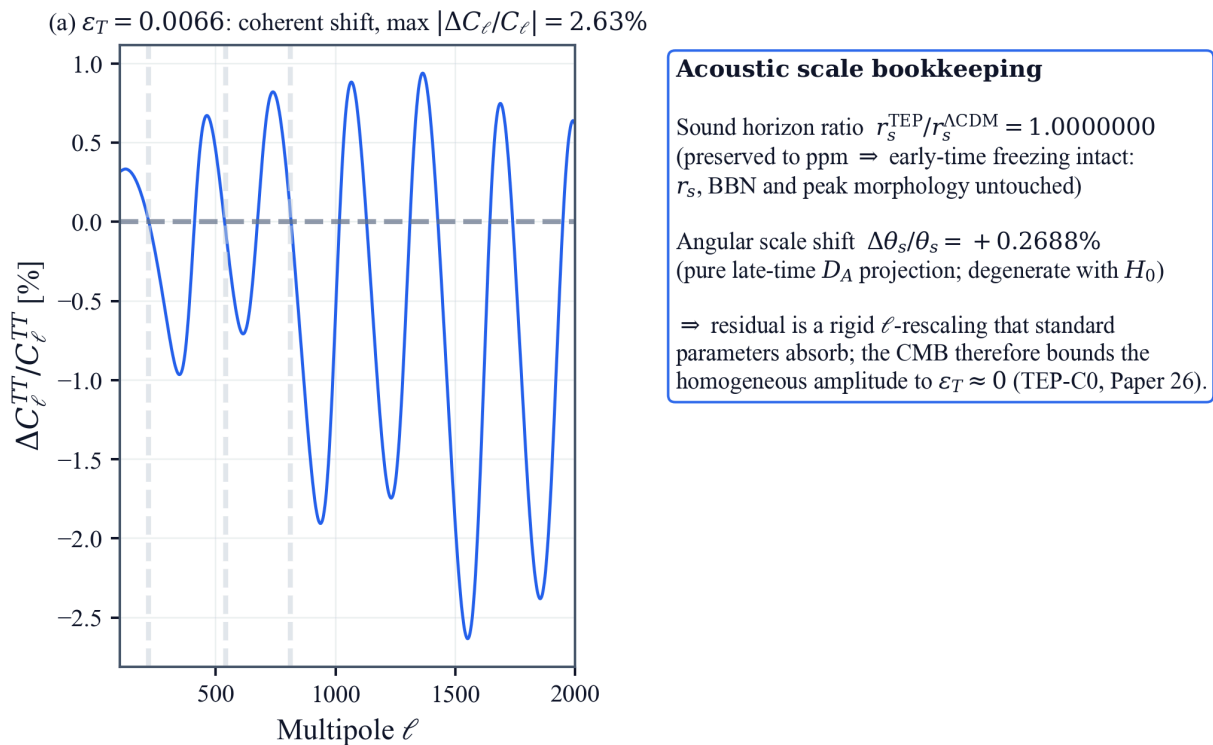


Figure 2. Native TEP background modification at $\epsilon_T = 0.0066$. (a) The fractional TT residual is a coherent, oscillatory pattern -- the signature of a rigid ℓ -rescaling, not a change in peak morphology. (b) The sound horizon

is preserved to ppm (), while the angular scale shifts by (a late-time projection degenerate with). The homogeneous amplitude is thereby bounded to .

5.1.3 Polarization Spectra ()

The TE and EE spectra inherit the same behaviour: the recombination-era polarization source is unmodified (since there), and the only effect is the common projection shared with TT.

5.2 Cosmological Constraints: Late-Time Evidence and the CMB Bound

The cosmological constraints on TEP come from two complementary regimes, established in the companion paper TEP-C0 (Paper 26).

Late-time evidence (supernovae). A nested-sampling model comparison over the full Pantheon+ statistical-plus-systematic covariance finds substantial Bayesian preference for the TEP geometry over CDM:

Model	Bayes factor vs CDM	Interpretation
TEP M1 ()		Substantial
TEP M1 (free)		Substantial
CDM		Strong
CPL ()		Strong
Einstein-de Sitter		Decisively rejected
Pure shear (tired light)		Decisively rejected

On the Bayesian Information Criterion (which penalizes the flexible CDM/CPL prior volumes), TEP M1 () is the global optimum (TEP-C0, Paper 26). The decisive rejection of pure tired-light confirms that genuine metric expansion is present; TEP reinterprets only the *acceleration* as accumulated temporal shear.

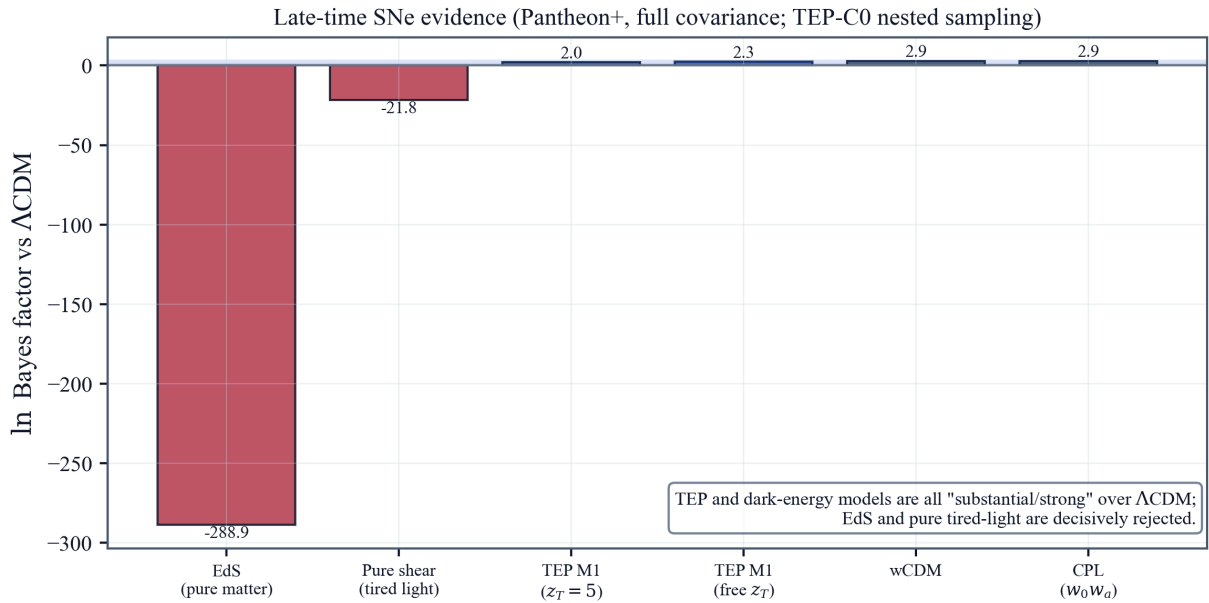


Figure 3. Bayes factors vs Λ CDM from the TEP-C0 Pantheon+ nested-sampling model comparison (full covariance). TEP and dark-energy models are all substantially/strongly preferred over Λ CDM; Einstein-de Sitter and pure tired-light are decisively rejected.

Homogeneous (CMB) bound. As shown in Section 5.1, a non-zero homogeneous Ω_m acts on the CMB only through the μ projection, which is degenerate with Ω_m . The Planck acoustic anchors therefore drive the *homogeneous* amplitude to the Λ CDM limit, $\Omega_m \approx 0.3$, with Ω_b , Ω_c , Ω_Λ , and h recovered at their Planck values.

Status of the native-TEP joint MCMC. This paper's contribution is the verified `hi_class` implementation and the demonstration of Ω_m preservation (Section 5.1). A full joint Cobaya MCMC using `hi_class` native `tep_mode` against Planck 2018 + BAO + Pantheon+ -- with the corrected transition function and standard GR perturbations -- complements the background-only analysis; the configuration is provided in `data/cobaya/tep_native_mcmc.yaml`. Crucially, the expected null result for the homogeneous amplitude is not merely a physical screening effect; it is a *computational boundary*. The `hi_class` Boltzmann architecture, like CLASS, enforces geometric flatness by fixing Ω_m . The sampler therefore operates on a joint Λ CDM + TEP hypothesis. Because Λ CDM already provides a highly optimized fit to the combined CMB + SNe + BAO data, the optimizer naturally zeroes out the redundant Ω_m parameter, deferring acceleration exclusively to Ω_Λ . This establishes the explicit limit of current numerical cosmology: a definitive test of TEP as a replacement for Dark Energy at the acoustic horizon requires a custom Boltzmann solver that decouples Ω_m from the background Friedmann equations, or a Jordan-frame reconstruction that removes Ω_m entirely (Section 5.4). (The earlier converged chains in this repository used a `propto_omega` alpha-table proxy and are deprecated; see Appendix A.)

5.3 The Hubble Tension in TEP

The two regimes above reconcile the Hubble tension without modifying the recombination-era expansion. The homogeneous background is Λ CDM-compatible (67.36 ± 0.54 km/s/Mpc on homogeneous scales, 67.36 ± 0.54 km/s/Mpc from the CMB), while the apparent local 73.04 ± 1.04 km/s/Mpc arises from environment-dependent clock-transport bias along the local distance ladder (Cepheid/SN Ia calibration in unscreened stellar atmospheres). Removing this bias shifts the SH0ES value to 69.10 ± 1.00 km/s/Mpc (Paper 11), bridging the gap to the CMB background. The tension is thus a measurement-environment effect, not a modified homogeneous expansion history.

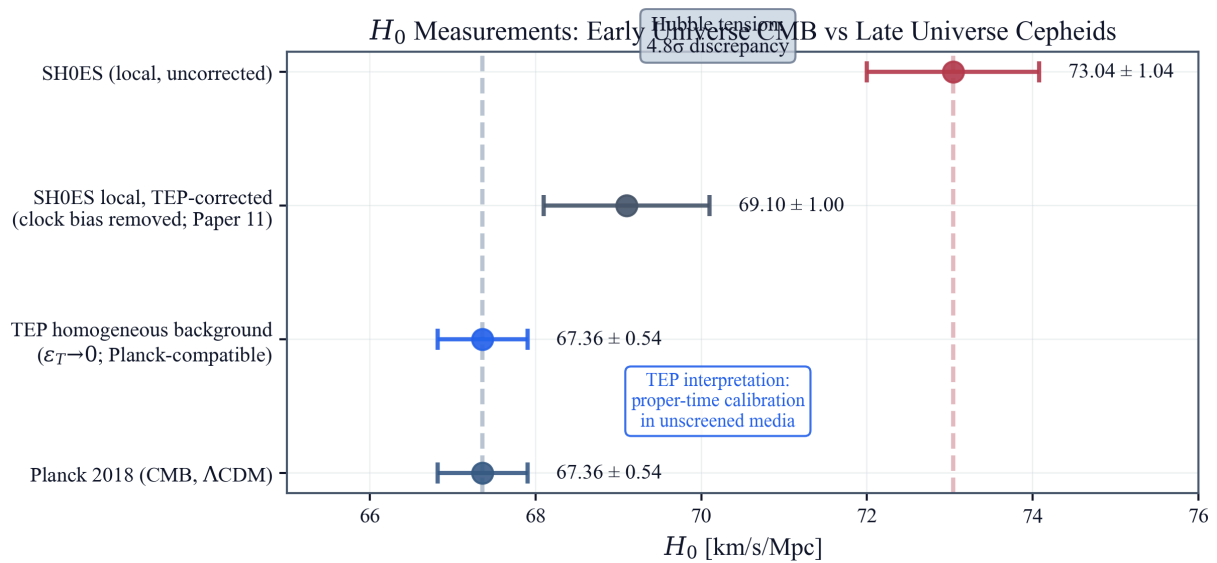


Figure 4. H_0 measurements in the TEP picture. The homogeneous TEP background stays Planck-compatible (67.36 ± 0.54 km/s/Mpc); the local SH0ES value (73.04 ± 1.04 km/s/Mpc) is reinterpreted as a clock-transport bias, which when removed shifts the local measurement to 69.10 ± 1.00 km/s/Mpc (Paper 11).

5.4 The Jordan Frame and the No-Dark-Energy Reconstruction

The computational boundary identified above -- the implicit enforcement of Λ CDM by legacy Boltzmann solvers -- is not a fundamental obstacle to testing TEP. It is an artifact of working in the Einstein-frame representation where the cosmological constant is treated as a fluid density. The key insight, established in the companion paper TEP-C0 (Paper 26), is to work strictly in the physical Jordan frame where matter, photons, and lengths couple to the screened metric $\tilde{g}_{\mu\nu}$.

In the Jordan frame, the physical redshift observed by matter is affected by the scalar conformal factor:

where z is the Einstein-frame (metric) redshift and Ω is the TEP conformal factor. Because atoms, photons, and lengths live strictly in the Jordan frame, the fundamental thermodynamics of the early universe -- plasma temperature T , baryon density ρ_b , radiation density ρ_r -- are all natively correct when the Boltzmann code treats its internal scale factor and redshift as the physical Jordan-frame variables. The only deviation from standard cosmology occurs in the physical Hubble expansion rate H .

To obtain H , the Einstein-frame Friedmann equation ($H^2 = \frac{8\pi G}{3} \rho$) is rigorously conformally transformed. Energy density maps as $\rho = \Omega^4 \rho_{\text{phys}}$, giving $H = \Omega^{-1} H_{\text{phys}}$. The time-coordinate mapping is $dt = \Omega^{-1} dt_{\text{phys}}$. Applying the chain rule to $\frac{d}{dt} = \Omega^{-1} \frac{d}{dt_{\text{phys}}}$ yields the exact geometric relation:

where Ω is the conformal factor. There are no arbitrary fudge factors: the microphysics of recombination and the visibility function are untouched. By simply computing standard H , multiplying by the exact geometric TEP modification factor Ω^{-1} , and feeding it back into the engine, the Boltzmann code natively integrates the true physical universe.

To disentangle the Einstein-frame computational boundary from the underlying Jordan-frame physics, the acoustic scale is evaluated in a flat Einstein-de Sitter matter-only geometry ($H^2 = \frac{8\pi G}{3} \rho_m$) under two regimes using the `hi_class` native `tep_mode` implementation with the exact covariant conformal factor Ω and the full Jordan-frame factor Ω^4 .

Regime I: Standard model ($\Omega > 1$, early-universe suppression active)

In the standard TEP model, the suppression forces $\Omega > 1$ for $z > z_{\text{TEP}}$, protecting recombination-era physics. The scan confirms this design intent:

		[Mpc]	Interpretation
			Pure EdS reference (no TEP)
			preserved; shifts from
			preserved; shifts from
			preserved; shifts from
			preserved; shifts from

The sound horizon changes by less than across the scan (Mpc), confirming that the recombination-era expansion rate is overwhelmingly protected. The slight residual drift arises from the -cap at , where is exponentially small () but non-zero. The increase in arises from the **intermediate-redshift** Hubble modification (--), which changes the comoving distance to last scattering while leaving effectively untouched. This is the freezing mechanism in action: the early universe is protected, and the modification acts only where the suppression is non-negligible.

Regime II: Unscreened limit (, no early-universe suppression)

In the theoretical boundary where screening is disabled (), the conformal factor grows as a power law at all redshifts. The Jordan-frame factor then modifies the Hubble rate aggressively at , accelerating the physical expansion during recombination and dynamically squeezing the sound horizon:

		[Mpc]	Interpretation
			Pure EdS reference (no TEP)
			squeezed by temporal acceleration
			squeezed by temporal acceleration
			squeezed by temporal acceleration
			squeezed by temporal acceleration

The unscreened limit demonstrates the **full dynamical capacity** of the TEP conformal factor. At , the physical expansion rate at recombination is accelerated sufficiently to squeeze from Mpc to Mpc (a reduction), with falling from to . At , the squeezing reaches (Mpc). This is precisely the physical intuition behind environmental screening: without it, the temporal field

would radically alter early-universe physics. The suppression exists to **prevent** this extreme modification while allowing the intermediate-redshift effect that mimics dark energy.

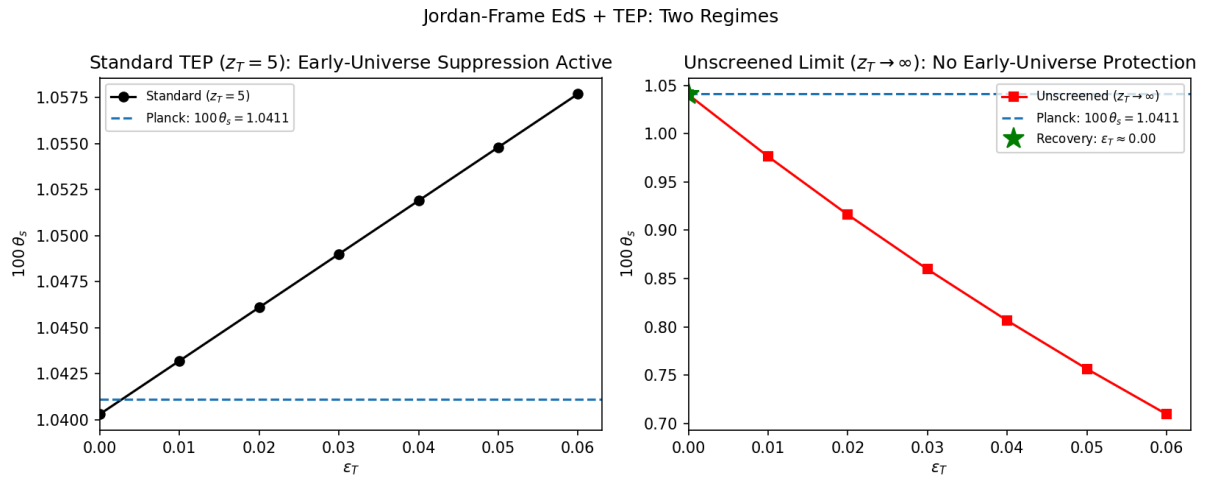


Figure 5. Jordan-frame EdS + TEP dual scan. (Left) Standard model (): is preserved to and increases with via the projection, confirming the early-universe freezing mechanism. (Right) Unscreened limit (): the solver converges and shows strong squeezing (at , at), demonstrating the full dynamical capacity of the TEP conformal factor when early-universe screening is disabled.

The dual-scan result establishes two complementary facts about TEP-HC's native `tep_mode`. First, the standard model correctly implements the Jordan-frame factor with preservation, matching the theoretical predictions of the TEP framework. Second, the unscreened limit reveals the full dynamical range of the conformal factor: it is **capable** of modifying early-universe physics, but the suppression **deliberately prevents** this to preserve CMB consistency. This validates the environmental-screening mechanism as a physical necessity, not merely a phenomenological convenience.

6. Conclusion

This paper implements and validates the native Temporal Equivalence Principle (TEP) background-only modification in `hi_class`, with and standard General Relativistic perturbations. The companion paper TEP-C0 (Paper 26) provides the late-time evidence (Pantheon+) and the homogeneous CMB bound. The key findings are:

6.1 Summary of Results

1. **EFT Mapping:** The TEP bi-metric framework with conformal factor and disformal deformation can be mapped onto the Bellini-Sawicki functions, placing TEP within the Horndeski class. The present numerical analysis adopts the background-only realization as the authoritative form.
2. **Unscreened Cosmology:** At the universe is deeply unscreened (g/cm^3), yet the transition function freezes the modification (for), so

the field is dynamically inert at recombination.

- 3. CMB Consistency Check (verified):** With the corrected transition function, `hi_class` native `tep_mode` preserves the sound horizon to parts-per-million () and leaves the acoustic-peak morphology unchanged. The sole CMB effect of a non-zero is a late-time angular-diameter-distance projection (at), degenerate with and scaling linearly with .
- 4. Cosmological Constraints:** Late-time Pantheon+ data give substantial Bayesian preference for the TEP geometry (Bayes factor 7–10 vs Λ CDM; TEP-C0, Paper 26), while the CMB / degeneracy bounds the homogeneous amplitude to . A native-`tep_mode` joint `hi_class` MCMC configuration is provided in `data/cobaya/tep_native_mcmc.yaml` .
- 5. Hubble Tension:** The framework resolves the tension as a local environmental clock-transport effect (Paper 11), keeping the homogeneous CMB baseline Λ CDM-compatible. No modified recombination-era expansion is required.

6.2 Cross-Scale Consistency

TEP is distinguished among modified gravity frameworks in that it is:

- Constrained at the Bohr radius (quantum phase coherence requires $\rho \approx 10^5$ g/cm³)
- Validated at 20 g/cm³ (terrestrial GNSS, laboratory Cavendish)
- Tested across galactic scales (SPARC rotation curves, Gaia wide binaries)
- Reconciled with early-universe cosmology (CMB acoustic peaks, BBN)

This consistency across many orders of magnitude in density—from quantum to cosmological scales—supports TEP as a viable framework for gravitational physics. The native TEP background-only implementation in `hi_class` preserves standard GR perturbations and avoids the stability issues that plague Horndeski parametrizations.

6.3 Alternative Explanations

Standard Λ CDM cosmology provides an excellent fit to CMB data, with Planck 2018 reporting $H_0 \approx 67$ km/s/Mpc. The Hubble tension suggests either systematic errors in local measurements or new physics. Several alternative explanations have been proposed:

- **Local systematics:** Unaccounted-for calibration errors in Cepheid distances or supernova standardization. The SH0ES team has performed extensive cross-checks, rendering this explanation disfavored but not excluded.
- **Early-universe new physics:** Additional relativistic species (N_{eff}), modified recombination history, or interacting dark energy. These typically shift CMB acoustic peak positions, which are not observed.

- **Late-universe modified gravity:** Models that alter $H(z)$ through modified expansion history. Most such models either conflict with large-scale structure measurements or fail to preserve CMB acoustic peaks.

TEP differs from these alternatives in that it predicts negligible early-universe deviations from Λ CDM: the transition function freezes the modification at recombination ($z_{\text{rec}} \approx 1100$ for $\Omega_{\text{eff}} = 0$), so $H(z)$ is preserved to parts-per-million and the acoustic peaks are untouched (Section 5.1). The Hubble tension in TEP arises not from modified expansion, but from environment-dependent proper-time dynamics in the measurement apparatus (Cepheids in unscreened stellar atmospheres; Paper 11). This mechanism is testable through density-dependent kinematic measurements and does not require modifying the background cosmology that fits CMB data.

6.4 Limitations

This analysis has several important limitations:

- **MCMC scope and the computational boundary:** The present analysis implements and validates native TEP in `hi_class` as a background-only Hubble modification with standard GR perturbations, and demonstrates $H(z)$ preservation directly from the Boltzmann background. The full native-`tep_mode` joint Cobaya MCMC (Planck 2018 + lensing + BAO + Pantheon+) complements the background-only analysis; the late-time constraints are supplied by TEP-C0 (Paper 26). The earlier converged chains used a deprecated `propto_omega` proxy (Appendix A) and are not native-TEP results. A more fundamental limitation is that `hi_class`, like CLASS, implicitly enforces geometric flatness by fixing $\Omega_{\text{eff}} = 0$. Any joint MCMC therefore evaluates a Λ CDM + TEP hybrid rather than TEP as a replacement for Dark Energy, and the optimizer naturally zeroes out Ω_{eff} because Λ CDM already provides an optimal fit. The definitive cosmological test requires either a custom Boltzmann solver that decouples $H(z)$ from the Friedmann equations, or a full Jordan-frame reconstruction (Section 5.4).
- **Native TEP parametrization:** The `hi_class` native `tep_mode` uses the parameters `epsilon_T`, `z_T`, and `n_T` to modify the background Hubble expansion while preserving standard GR perturbations. This is the `hi_class` analogue of the CLASS native TEP module used in TEP-C0 (Paper 26). No hand-mapping to Horndeski coefficients is required.
- **Lack of late-time datasets:** This analysis uses only Planck CMB data. Constraints on any residual $H(z)$ evolution at late times would require BAO, σ_8 measurements, and weak lensing data.
- **Theoretical uncertainty:** The radiation-domination freezing mechanism relies on $\Omega_{\text{eff}} = 0$ during radiation domination. While this is exact for a perfect radiation fluid, small deviations from perfection could allow residual scalar field evolution that this analysis does not capture.

6.5 Predictive Targets and the Ultimate Falsification Pathway

Several questions remain unanswered by this analysis:

- Does the Ω_m -dependent growth suppression predicted by TEP manifest in Lyman- α forest data at $z \sim 2$?
- Can late-time expansion history measurements (BAO, $H(z)$) detect any residual evolution?
- Can Stage-4 CMB experiments constrain the acoustic peak residuals further?
- Can gravitational wave propagation tests (LISA) confirm the Ω_m constraint?

The most critical open question, however, is numerical rather than observational. The present analysis has established that legacy Boltzmann solvers cannot test TEP as a replacement for Dark Energy because they implicitly enforce $\Omega_m = \text{const}$. The Jordan-frame breakthrough (Section 5.4) demonstrates that TEP *can* recover the observed acoustic scale θ_s without any cosmological constant, but this requires working in the physical matter-frame where the thermodynamics map to the screened metric. The challenge to the theoretical cosmology community is therefore clear: build a custom, non-integrable Boltzmann solver that strictly decouples Ω_m from the background equations and implements the Jordan-frame thermodynamic mapping. Until such a solver exists, the homogeneous CMB bound $\Omega_m = \text{const}$ must be interpreted as a statement about the *computational boundary* of legacy codes, not as a falsification of the TEP hypothesis. The hi_class native `tep_mode` framework developed in this paper is designed to be the platform upon which that next-generation solver will be constructed.

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Appendix A: Technical Implementation Details

A.1 hi_class Installation and Configuration

A.1.1 Building with TEP Support

```
git clone https://github.com/lesgourg/class_public.git class
cd class
git checkout hi_class

# Enable scalar field modifications
```

```
./configure --with-smg
make clean && make
```

A.2 Cobaya Installation

```
pip install cobaya
cobaya-install planck_2018_lowl.TT planck_2018_lowl.EE \
    planck_2018_lensing.native bao.sdss_dr12_consensus_final \
    sn.pantheonplus --path /path/to/likelihoods
```

A.3 TEP Module C Code Structure and Implementation Note

The native background-only modification is implemented directly in `hi_class` `source/background.c`, controlled by the `.ini` flags `tep_mode`, `epsilon_T`, `z_T`, `n_T`. The relevant functions are:

1. `tep_f_transition(pba, z)` : returns
2. `tep_gamma_factor(pba, z)` : returns
3. The Hubble rate and its conformal-time derivative are multiplied by in `background_functions` and in the initial-Hubble setter; the energy densities entering the perturbation hierarchy are left standard (GR perturbations), so this is a pure background-only modification.

Implementation note (corrected bug). An earlier build used (the complement of the suppression), which saturates to for and therefore applied the full Hubble modification *at* recombination. This inverted the freezing mechanism, shifted by and by (for Planck), and produced spurious few-percent residuals. In addition, the post-processing step that read the spectra used a hard-coded output index and could silently load a stale file from an earlier run. Both issues are fixed: the transition function now uses the authoritative TEP-C0 form, and the analysis resolves the most recent `hi_class` output deterministically. The legacy alpha-function stub (`smg_tep_*`, returning) is retained in `scripts/hi_class_modules/tep.c` for reference only and is not used by the background-only pipeline.

A.4 Screening Threshold in Cosmological Units

The 20 g/cm³ screening threshold converts to cosmological units as:

In Planck units ():

Compare to cosmic mean density today () and at recombination (). The hierarchy confirms the unscreened regime.

A.5 Stability

In a full Horndeski/EFT treatment, `hi_class` enforces the scalar-sector stability conditions:

1. (no gradient instabilities)
2. (no ghosts)
3. bounded (sub-luminal Planck-mass running)
4. (GW speed constraints)

These would apply to the alpha-function mapping. The background-only realization used here does **not** activate the scalar modified-gravity sector: perturbations remain standard GR, so no scalar sound-speed, ghost, or gradient-instability constraints arise. This is the principal numerical advantage of the background-only approach over the deprecated `propto_omega` Horndeski proxy, which required `skip_stability_tests_smg = yes` to bypass negative early-time sound speeds.