



Temporal Topology Saturation Scale: Cross-Scale Consistency of ρ_T

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Abstract

Gravitational anomalies across astrophysical scales exhibit a dual scaling regularity: the radius at which Newtonian dynamics fails scales as $R_T(M) = (3M/4\pi\rho_T)^{1/3} \propto M^{1/3}$ in the compact saturation regime and $R_{DM} \propto M^{1/2}$ in the low-acceleration galactic regime. Within the Temporal Equivalence Principle (TEP) two-branch scalar action ($P(X, \phi) = P_{\text{kin}}(X) - \frac{1}{4}\lambda M_{\text{Pl}}^4 u^4$, $A(u) = e^{-u}$), the quartic confining potential fixes the cubic screening response $S = (\bar{\rho}/\rho_T)^{1/3}$ around the saturation density $\rho_T \approx 20 \text{ g/cm}^3$ (calibrated from terrestrial GNSS and tested across astrophysical systems), while the nonlinear kinetic branch $P_X(\xi) = k\sqrt{\xi} + 2\xi$ ($k = 16.03$) governs unscreened ($S \ll 1$) galactic halos at $a_{\text{eff}} = 1.20 \times 10^{-10} \text{ m s}^{-2}$ (yielding under the potential-share identification $\epsilon_{\text{host}} = a_\phi/(g_{\text{bar}} + a_\phi)$ a parameter-free star-forming spiral BBH host tracking fraction of $\langle \epsilon_{\text{host}} \rangle_{\text{SFR}}^{\text{UCD}} = 0.236 \pm 0.054$ at R_{eff} and 0.243 ± 0.052 across the radial Freeman disk [0.197--0.202 with 25% bulge/NSC hosts], matching the GWTC-1–5.0 optimum in Paper 22). Across a 4-layer PREM Earth boundary-value problem and 1-D spherical nonlinear Picard solve, the transition-shell (4,010 km; 3,957 km nonlinear) and full-volume (3,213 km; 3,170 km nonlinear; $L_c^{\text{vol}}/R_T = 0.775$) kernels reproduce both the 25.3-year GPS PPP clock correlation length ($L_c = 4,201 \text{ km}$, $\hat{\rho}_T = 19.2 \text{ g/cm}^3$) and 2023 CDDIS RINEX3 single-constellation GPS, Galileo, and GLONASS reprocessing (3,853 km $\sqrt{N}$ -weighted, and $3,155 \pm 181 \text{ km}$ [CV = 5.7%] under full-day $\ell = 0$ common-mode removal, mapping via L_c^{vol}/R_T to $\hat{\rho}_T = 21.1 \text{ g/cm}^3$).

With $\rho_T = 20 \text{ g/cm}^3$ and a_{eff} held fixed and zero per-object refitting, three independent cross-scale tests follow. First, in 175 SPARC galaxies, an a priori Freeman-disk split separates 3-D rising-core onset scaling ($\alpha_{\text{rising}} = 0.256 \pm 0.069$ vs. $1/3$) from 2-D Keplerian tails ($\alpha_{\text{tail}} = 0.581 \pm 0.086$ vs. $1/2$; $p = 0.0031$), while the coupled-gradient force law reproduces the radial-acceleration (0.1330 dex RMS) and Baryonic Tully–Fisher ($R^2 = 0.956$, 0.0807 dex) relations. Second, four external $R_\oplus$ -free astrophysical catalogs — outer-giant-planet spin (19.89 g/cm³), the brown-dwarf rotational period floor ($1.150 \pm 0.075 \text{ hr}$ vs. TEP-saturated Chandrasekhar–Jacobi 1.155 hr [20.19 g/cm³], sitting 8.3σ above the unscreened Newtonian Jacobi instability limit of 0.527 hr), compressed terrestrial moments of inertia (17.89 g/cm³), and the Solar Model S convection-zone base (0.713 $R_\odot$ vs. $\sqrt{3}R_T(M_\odot) = 0.715 R_\odot$ [20.15 g/cm³]) — converge with GNSS to $\hat{\rho}_T = 19.59 \pm 1.10 \text{ g/cm}^3$ (19.68 $\pm 1.25 \text{ g/cm}^3$ GNSS-free; 20.05 $\pm 1.61 \text{ g/cm}^3$ v_{gal} -marginalized; look-elsewhere $p = 2.9 \times 10^{-4}$ [3.6 σ], 2.05% leave-one-channel-out blind prediction RMS). Third, the horizon crossover $R_T(M_\times) = R_S$ occurs at $M_\times \approx 3.04 \times 10^7 M_\odot$, matching the runaway black hole candidate RBH-1.

1. Introduction: The Dark Matter Problem as a Temporal Structure Problem

The Universal Scaling Anomaly

Dark matter observations across cosmological scales reveal an empirical regularity. From dwarf galaxies ($M \sim 10^9 M_\odot$) to massive galaxies ($M \sim 10^{12} M_\odot$), the characteristic radius at which Newtonian dynamics fails—the "dark matter onset"—scales approximately as $R \propto M^{1/3}$. This implies a universal critical proximity scale ("critical" here meaning a phenomenological saturation/proximity scale, not a hard density threshold), observationally proxied by $\rho_T \sim M/R^3 \sim \text{constant}$, at which gravitational phenomenology changes. In the two-branch TEP action, the geometric saturation radius follows $R_T \propto M^{1/3}$, whereas the low-acceleration kinetic response produces an $R_s \propto M^{1/2}$ acceleration-crossing scale; these are different operational radii, and galactic onset estimators spanning the transition between them need not recover identical exponents.

This scaling appears in multiple independent contexts:

- **Galaxy rotation curves:** SPARC database analysis yields $R_{\text{DM}} \propto M^{\alpha_{\text{SPARC}}}$ with $\alpha_{\text{SPARC}} = 0.352 \pm 0.043$ (stat) ± 0.07 (definition) (Lelli et al. 2016; this work).
- **Ultra-diffuse galaxies:** DF2 and DF4 exhibit dark matter deficiency consistent with saturation radii exceeding tidal radii (van Dokkum et al. 2018, 2019).
- **Milky Way:** The SPARC-calibrated $M^{1/3}$ relation predicts a dark-matter onset at $R_{\text{DM}} \approx 3$ kpc for the Milky Way, consistent with the observed transition from baryonic to dark-matter-dominated rotation at $R \sim 3\text{--}5$ kpc (Gaia Collaboration 2023; Sofue 2020).

The persistence of this $M^{1/3}$ scaling across 6 orders of magnitude in mass (screening hierarchy: gas giants to neutron stars) and 15 orders of magnitude in density suggests a fundamental physical scale, not a coincidence of baryonic feedback or halo assembly.

Reframing Dark Matter: Phantom Mass from Temporal Shear

The Temporal Equivalence Principle (TEP) proposes that gravitational phenomena arise from a conformal time field $\phi(x^\mu)$ that modulates proper time rates. This phenomenology arises from the TEP action (Smawfield 2025a, Paper 0):

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + P(X, \phi) \right] + S_m[\tilde{g}_{\mu\nu}], \quad P(X, \phi) = P_{\text{kin}}(X) - V(\phi), \quad (1)$$

where matter couples to the causal matter metric $\tilde{g}_{\mu\nu} = -A_{\text{loc}}^2(u_{\text{grav}}(\mathbf{x}))U_\mu U_\nu + A_{\text{sp}}^2(u_{\text{drift}}(t, \mathbf{x}))\Pi_{\mu\nu} + B(\phi)\Pi_\mu^\alpha \Pi_\nu^\beta \nabla_\alpha \phi \nabla_\beta \phi$ in the canonical unified unstretched-space decomposition ($\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu} + B(\phi)\nabla_\mu \phi \nabla_\nu \phi$ on the historical isotropic-conformal benchmark). At late times, $B(\phi)$ is strongly constrained by multi-messenger observations and can be neglected, while $A(\phi) = \exp(\beta_A \phi / M_{\text{Pl}})$ modulates local proper-time rates. The environmental screening response suppresses the observed gradient in dense environments, producing effective field saturation at characteristic density ρ_T . In this framework, the "dark matter" problem is reinterpreted as a violation of the *isochrony axiom*—the assumption that clocks at the same gravitational potential tick at the same rate regardless of their spatial separation or the mass distribution's history.

When this axiom is relaxed, mass estimates made through different observable pairs diverge, and "phantom mass" — the apparent mass inferred when temporal structure is analysed as spatial mass — can be measured per channel. Whereas earlier relativistic extensions of the MOND paradigm introduced vector and scalar fields to reconcile galactic dynamics with gravitational lensing (Milgrom 1983; Bekenstein 2004) or invoked non-canonical kinetic screening to satisfy Solar System bounds (Babichev et al. 2009; Barreira et al. 2015), TEP derives both channels from the local clock-rate and spatial-slice decomposition of a single proper-time foliation. In the *dynamical–baryonic* channel studied here (rotation curves), phantom mass is the apparent excess of the dynamical mass $V_{\text{obs}}^2 R/G$ over the baryonic mass: the orbital dynamics respond to the local time gradient, sourcing centripetal support with no additional baryons. In the *lensing* channel (Papers 4, 18, 19), under the canonical unified unstretched-space matter metric (Paper 0 §2.2, Step 80) — where at a fixed cosmic epoch ($A_{\text{sp}}(t_0) = 1$) the spatial 3-metric $\tilde{\gamma}_{ij} = \gamma_{ij}$ remains unstretched while the $(3+1)$ ADM lapse rescales as $N_{\text{TEP}}(\mathbf{x}) = A_{\text{loc}}(u_{\text{grav}}(\mathbf{x}))N_{\text{GR}}(\mathbf{x})$ — intermediate clock rates refract light at half the dynamical potential weight ($\Phi_{\text{lens}} = \Phi_N + \frac{1}{2}c^2\beta_A u_{\text{grav}}$ versus $\Phi_{\text{dyn}} = \Phi_N + c^2\beta_A u_{\text{grav}}$), yielding a testable lensing-to-dynamical mass ratio $M_{\text{lens}}^{\text{GR}}/M_{\text{dyn}} = (1 + \beta_A^2 y_{\text{eff}})/(1 + 2\beta_A^2 y_{\text{eff}})$ (whereas on the historical isotropic-conformal benchmark $\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}$, $A^2(\phi)$ cancels out of null rays and $M_{\text{dyn}}/M_{\text{lens}} = 1 + 2\beta_A^2 y_{\text{tb}}$). Both measurements arise from spatial structure in proper time—without invoking non-baryonic particles—and are kept distinct throughout.[†]

[†]Note: This geometric "phantom mass" from temporal shear differs from cosmological "phantom energy" (dark energy with $w < -1$). The former arises from spatial gradients in proper time; the latter from exotic equation-of-state matter.

The characteristic scale at which this temporal structure becomes significant is set by the saturation scale ρ_T , where the scalar field ϕ reaches its self-interaction threshold and saturates—forming a stable, localized configuration (a soliton under that interpretation) with radius:

$$R_T = \left(\frac{3M}{4\pi\rho_T} \right)^{1/3}. \quad (2)$$

This $M^{1/3}$ scaling is a direct consequence of the saturation condition; under a soliton interpretation, this corresponds to soliton formation.

The saturation scale parameter $\rho_T \approx 20 \text{ g/cm}^3$ serves as the bulk-matter realization of the abstract environmental screening operator $\mathcal{S}_\Sigma(\mathcal{E})$ defined in the foundational TEP framework. In macroscopic atomic matter, ρ_T acts as an empirical proxy for the geometric saturation of the Temporal Topology, yielding cross-scale predictions without committing to specific chameleon, Vainshtein, or symmetron microphysics.

$$\Sigma_\mu^{\text{obs}} = \mathcal{S}_\Sigma(\mathcal{E}) \nabla_\mu \ln A(\phi), \quad (3)$$

where the environmental state is

$$\mathcal{E} = \{\rho, \Phi/c^2, \nabla\rho, \nabla\Phi, \text{compactness}, R_T(M), \text{proximity}, T, z, \text{boundary geometry, coherence volume}\}.$$

This single operator unifies density screening, compactness screening, proximity screening, thermodynamic/environmental-state response, lensing/cosmological covariance screening, and wide-binary environmental screening across the entire TEP corpus.

Box 1.1: Joint Constraint Architecture of ρ_T and $\mathcal{S}_\Sigma(\mathcal{E})$

ρ_T is the shared macroscopic saturation/proximity scale. Other papers in the TEP programme do not all measure ρ_T directly; they measure domain-specific projections of the same environmental operator $\mathcal{S}_\Sigma(\mathcal{E})$:

- **Clock covariance lengths** (GNSS, Paper 1–3)
- **Galactic onset radii** (SPARC, Milky Way, this work)
- **Response coefficients** (Cepheids, pulsars, Papers 10–11)
- **Lensing residuals** (strong-lens time delays, Paper 4)
- **Weak-field recovery scales** (wide binaries, Paper 13)
- **Compactness-dependent residuals** (LLR, Paper 17)
- **Cosmological temporal-shear amplitudes** (hi_class, C0, TH, Papers 18, 26, 27)

Each measurement channel has a response coefficient κ_X , an environmental screening projection, and a domain-specific observable functional. This architecture protects the programme from the criticism that different channels report different numerical values: they are *observable response coefficients*, not bare scalar couplings, and each channel measures a different projection of $\mathcal{S}_\Sigma(\mathcal{E})$.

The Multi-Scale Consistency Strategy

As summarized in Table 1, this paper tests ρ_T through a convergent multi-scale approach:

Table 1: Cross-Scale Consistency of ρ_T

Scale	System	Mass Range	Density Range	Key Observable	Result
Terrestrial	GNSS Clocks	$M_\oplus \sim 6 \times 10^{27} \text{ g}$	$\rho_\oplus \sim 5.5 \text{ g/cm}^3$	$L_c \approx 4200 \text{ km}$	$\rho_T \approx 20 \text{ g/cm}^3$
Galactic	SPARC Galaxies	$10^9\text{--}10^{12} M_\odot$	$\rho \sim 10^{-24} \text{ g/cm}^3$	$R_{\text{DM}} \propto M^{\alpha_{\text{SPARC}}}$	$\alpha_{\text{SPARC}} = 0.352 \pm 0.043 \text{ (stat)} \pm 0.07 \text{ (sys)}$
Local	Milky Way	$M_{\text{bar}} \sim 6 \times 10^{10} M_\odot$	$\rho \sim 10^{-24} \text{ g/cm}^3$	Inner mass-discrepancy onset	$R_{\text{DM}} \approx 3 \text{ kpc}$ (observed $\sim 3\text{--}5 \text{ kpc}$)

Scale	System	Mass Range	Density Range	Key Observable	Result
Screening	26 Objects	$\sim 10^{27}\text{--}10^{33}$ g	$\sim 10^0\text{--}10^{14}$ g/cm ³	$S \equiv R_T/R_{\text{phys}} \equiv (\bar{\rho}/\rho_T)^{1/3}$	Deterministic regime map ($S < 1$ unscreened; $S \geq 1$ compact)
Planetary Spin	Solar System ($N = 12$)	$2.6 \times 10^{23}\text{--}1.9 \times 10^{30}$ g	0.69–5.51 g/cm ³	$v_{\text{eq}} = c u_T / \sqrt{u_{\text{gal}}}$; $J = \frac{GM^2}{2v_{\text{gal}}}$	$v_{\text{obs}}/v_{\text{TEP}} = 1.000 \pm 0.080$ (outer 3; 1.016 ± 0.073 with Earth); $p_{\text{TEP}} = 1.52 \times 10^{-15}$ cgs

Note: The convergence of saturation-radius constraints across 18 orders of magnitude in mass ($10^{20}\text{--}10^{38}$ kg, from asteroids and Earth to the $2 \times 10^7 M_\odot$ runaway black hole RBH-1, alongside galactic disks on the unscreened $S \ll 1$ kinetic branch) motivates ρ_T as a candidate saturation scale.

Box 1.2: Executive Summary of Quantitative Audits and Methodological Controls

Gravitational anomalies across astrophysical scales exhibit an empirical regularity: the radius at which Newtonian dynamics fails scales as $R \propto M^{1/3}$ in the compact saturation regime and $R \propto M^{1/2}$ in the low-acceleration galactic regime, organized by a candidate saturation/proximity density scale ρ_T in the Temporal Equivalence Principle (TEP) conformal time-field sector. The 25-year GNSS atomic-clock correlation length $L_c \approx 4,200$ km (CODE, IGS, ESA; raw-RINEX validated) establishes an Earth-scale correlation structure consistent with $R_T(M_\oplus) = (3M_\oplus/4\pi\rho_T)^{1/3} = 4,146$ km at $\rho_T \approx 20$ g/cm³ ($\rho_T \in [8, 70]$ g/cm³ conditional mapping under $L_c \leftrightarrow R_T(M_\oplus)$). A 4-layer PREM Earth boundary-value integration confirms an order-unity geometric projection ($L_c^{\text{PREM}} = 4,010$ km shell; 3,213 km volume) governed by Earth's R_{CMB} and $R_\oplus$ structure, while Monte Carlo network injection–recovery simulations across $L_c^{\text{true}} \in [1,000, 6,000]$ km demonstrate that 1-constellation GPS-only processing recovers $L_c^{\text{true}} = 4,200$ km ($\hat{L}_c^{\text{GPS}} = 4,036 \pm 113$ km; white-noise null $|C(d)| = 0.009$; correlated orbital/datum dipole null concave-down with $d^2C/dd^2 < 0$), whereas 4-constellation multi-GNSS inter-system-bias (ISB) projection compresses the same field to $\hat{L}_c^{\text{MGEX}} = 2,277 \pm 47$ km per-epoch ($1.77\times$) and $2,485 \pm 45$ km daily-constant ($1.62\times$, explaining the empirical $1.90\times$ MGEX offset that de-biases pooled MGEX 1,862 km to 3,536 km), and direct 4-quarter 2023 CDDIS RINEX3 single-constellation reprocessing (192–197 station-days, 3,706–3,927 pairs per constellation; Step 14) confirms on real observations that in the full-day 24-hour window ($n_{\text{seg}} = 288$) independent US GPS (4,051 km $\sqrt{N}$ -weighted, 3,766 km unweighted, $R^2 = 0.964$), Russian GLONASS (4,693 km $\sqrt{N}$ -weighted, 3,806 km unweighted, $R^2 = 0.849$), and European Galileo (4,557 km in dual-frequency iono-free LC on `optimal_100`; 3,653 km in 10–60 μHz , $R^2 = 0.890$, and 2,816 km $\sqrt{N}$ -weighted / 2,697 km unweighted in single-frequency broadcast 10–500 μHz , $R^2 = 0.934$) solutions yield strictly band-matched three-constellation means of 3,853 km ($\sqrt{N}$ -weighted 10–500 μHz , $\hat{\rho}_T = 24.9$ g/cm³), 3,423 km (unweighted 10–500 μHz), 3,362 km (matched 10–60 μHz), and — under uniform full-day $\ell = 0$ common-mode-error removal across the 10–500 μHz band — $3,155 \pm 181$ km (CV = 5.7% across GPS 2,953, Galileo 3,302, and GLONASS 3,210 km, $R^2 = 0.955\text{--}0.969$, matching the 25.3-yr isotropic CODE scale 3,210 km and PREM full-volume kernel $L_c^{\text{vol}} = 3,213$ km [model-calibrated BVP ratio $L_c^{\text{vol}}/R_T = 0.775 \implies R_T = 4,072$ km, $\hat{\rho}_T = 21.1$ g/cm³, 3.35% astrophysical RMS error, vs. 45.4 g/cm³ and 29.4% RMS error at raw unit prefactor; 20.1 g/cm³ on CODE 3,210 km], with station-geometry resampling confirming no continental-clustering artifact and forward propagation across all four external astrophysical channels yielding 1.98%–3.82% joint RMS error vs. 78.6% failure on the truncated 2,156 km scale), whereas applying MGEX's 8-hour window truncation ($n_{\text{seg}} = 96$), per-epoch datum removal, and 4-constellation ISB estimation compresses the exact same data step-by-step from $3,423 \rightarrow 2,850 \rightarrow 2,156$ km (CV = 0.28% across GPS 2,163.2, Galileo 2,152.9, and GLONASS 2,152.3 km; $R^2 = 0.929\text{--}0.957$) $\rightarrow 1,478$ km ($2.32\times$ total empirical compression).

Because the canonical terrestrial BVP is shaped by Earth's fixed geometry, quantitative calibration of $\rho_T \approx 20$ g/cm³ rests on direct $R_\oplus$ -free cross-mass tests, separated from structural-exponent tests. Direct tests comprise the RBH-1 crossover ($M_\times = \sqrt{3c^6/(32\pi G^3 \rho_T)} \approx 3.04 \times 10^7 M_\odot$, $R_T/R_S = 1.32$), the zero-parameter equatorial rotation relation $v_{\text{eq}}/c = u_T(M)/\sqrt{u_{\text{gal}}}$ ($v_{\text{gal}} = 220$ km s^{−1}), which matches the three truly unbraked outer giant planets (Uranus, Neptune, and Saturn, excluding lunar-braked Earth whose conserved primordial Earth+Moon angular momentum $T_{\text{prim}} = 5.05$ hr sits on the small-body soliton ceiling $\eta_{\text{E+M}} = 0.145$) at $v_{\text{obs}}/v_{\text{TEP}} = 1.000 \pm 0.080$ ($\hat{\rho}_T^{\text{spin}} = 19.9$ g/cm³ geometric mean; 1.016 ± 0.073 and 20.9 g/cm³ when present-day Earth is included, where the $+6.4\%/+8.4\%$ vs. $-0.6\%/-7.7\%$ residual signs track the nested screening-shell ratio $r_*(M)/a$ at $r = +0.960$, $p = 0.040$, and volumetric/deep-interior periods give 1.031 ± 0.041 ; extending above $M_{\text{cap}} = 194 M_\oplus = 0.61 M_{\text{Jup}}$ with one small-body capacity parameter $\eta_0 = 0.129$ [six-body IQR 0.122–0.154, candidate secular stability benchmark $\eta_{\text{sec}} = \frac{1}{8} = 0.125$] and $S_A = \min[1, S]$ matches Jupiter at -2.9% [-0.2% at η_{sec}], the fastest planetary-mass rotator SIMP J0136 at -2.8% [-0.3%], and the 1.15 ± 0.08 hr brown-dwarf period floor of Tannock et al. 2021 at $P_{\text{min}}^{\text{TEP}} = (k_{\text{MOI}}/\eta_0)\tau_T = 1.17$ hr [IQR 0.98–1.24 hr; 1.21 hr at η_{sec}], giving $v_{\text{obs}}/v_{2\text{br}} = 1.001 \pm 0.060$ and $\hat{\rho}_T^{\text{rot}} = 20.0$ g/cm³ across the $N = 8$ unbraked outer-planet and substellar systems [1.008 ± 0.060 and 21.1 g/cm³ across $N = 9$ with Earth]), self-compressed planetary moments of inertia $k_{\text{MOI}} \simeq \frac{1}{2}S$ ($\rho_T^{\text{MOI,terr}} = 17.9 \pm 1.4$ g/cm³ across Mercury, Venus, and Earth; $2k_{\text{MOI}}/S = 1.17 \pm 0.14$ across eight planets at 1-bar radii, and $2k_{\text{MOI}}/S_{\text{cond}} = 1.042 \pm 0.094$, $\rho_T^{\text{MOI,cond}} = 17.9$ g/cm³ across all seven condensed/giant planets once Lane–Emden polytropic H/He envelope inflation is removed, whereas low-pressure uncompressed satellites with $P_c \ll K_0$ remain near the uniform-sphere ceiling), and an exploratory Standard Solar Model (Model S) helioseismic prototype (where $\rho_\odot(r) = \rho_T$ occurs at $0.249 R_\odot$ near the nuclear-core boundary, $\sqrt{3} R_T(M_\odot) = 0.715 R_\odot$ and $y_{\text{BVP}} = \frac{1}{2} S_\odot$ at $0.723 R_\odot$ coincide with the $0.713 \pm 0.001 R_\odot$ convection-zone base [$+0.25\%$, $\rho_T^{(\text{cz})} = 20.15$ g/cm³],

$\nu_{\text{eq}} = g_{\text{surf}}/(2c) = 456.5 \text{ nHz}$ [1.997 km s^{-1}] matches the equatorial rotation rate 460.7 nHz to -0.9% , and $S_{\odot}/\sqrt{2} = 0.292$ matches the equator-to-pole differential shear 0.289 to $+1.1\%$; across the four GNSS-free external astrophysical channels alone, $\hat{\rho}_T^{\text{no-GNSS}} = 19.68 \pm 1.25 \text{ g/cm}^3$ [$\pm 6.4\%$], and across all five channels including GNSS, $\hat{\rho}_T = 19.59 \pm 1.10 \text{ g/cm}^3$ [$\sigma_{\log_{10} \rho} = 0.025 \text{ dex}$], rejecting random-density coincidence across $[0.1, 10^4] \text{ g/cm}^3$ at $p = 1.7 \times 10^{-3}$ on a 1-D scan, $p = 1.4 \times 10^{-7}$ across five channels, and $p = 2.9 \times 10^{-4}$ [3.6σ] even after granting $7^3 = 343$ look-elsewhere algebraic-factor trials). Structural and scale-cancellation tests comprise the SPARC onset scaling ($\alpha_{\text{SPARC}} = 0.352 \pm 0.043$ discrete, $N = 168$; $\alpha_{\text{res}} = 0.417 \pm 0.050$ on $n = 75$ resolved crossings, where an *a priori* Freeman-disk baryonic slope split around $M_{\text{cross}} = 1.45 \times 10^{10} M_{\odot}$ separates rising-baryon $M^{1/3}$ core crossings [$\alpha_{\text{rising}} = 0.256 \pm 0.069$, $n = 38$] from concentrated Keplerian-tail $M^{1/2}$ crossings [$\alpha_{\text{tail}} = 0.581 \pm 0.086$, $n = 37$; $\Delta\alpha = +0.325 \pm 0.110$, $p = 0.0031$]; Tobit rejection of R_{DM}/R_d invariance at $+0.323 \pm 0.034$, $z = 9.57$, $p < 10^{-4}$, fixed-radius non-circular partial $r(R_{\text{DM}}, V_{\text{bar}}(2.2R_d) | R_d) = +0.547$, $p = 3.8 \times 10^{-7}$), the two-branch RAR/BTFR profile $P_X = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$ ($0.139 - 0.152 \text{ dex RMS}$ on the quality-controlled SPARC sample), the Milky Way onset ($3.1 \pm 0.4 \text{ kpc mean}$; [$1.0, 9.4$] $\text{kpc } 1\sigma$ interval), the deterministic regime map $S \equiv R_T/R_{\text{phys}} \equiv (\bar{\rho}/\rho_T)^{1/3}$, and Wesson's constant $p_{\text{TEP}} \equiv J/M^2 = G/(2v_{\text{gal}}) = 1.52 \times 10^{-15} \text{ cm}^2 \text{ g}^{-1} \text{ s}^{-1}$ ($J_{\text{obs}}/J_{\text{TEP}} = 1.11 \pm 0.25$).

Paper Structure

The remainder of this paper is organized as follows:

- **Section 2:** Terrestrial calibration via GNSS atomic clock correlations, establishing $L_c \approx 4200 \text{ km} \rightarrow \rho_T \approx 20 \text{ g/cm}^3$.
- **Section 3:** Galactic rotation-curve test on the SPARC sample, evaluating the transition between $M^{1/3}$ core saturation and $M^{1/2}$ low-acceleration scaling alongside the two-branch RAR profile.
- **Section 4:** Screening hierarchy analysis, providing a candidate screening account for why precision GR tests remain satisfied despite scalar-field structure.
- **Section 5:** Physical context for ρ_T , including a dimensional identity bridge preserving WEP, a condensed-matter EOS coincidence, and open derivation targets.
- **Section 6:** Universal Scaling as a Cross-Scale Consistency Test, synthesizing constraints across all regimes.
- **Section 7:** Milky Way dark-matter onset at $R_{\text{DM}} \approx 3 \text{ kpc}$.
- **Section 8:** Discussion of dark matter as phantom mass and cosmological implications.
- **Section 9:** Conclusion and astrophysical applications (including RBH-1, companion paper).

This multi-scale convergence identifies $\rho_T \approx 20 \text{ g/cm}^3$ as a candidate GNSS-consistent saturation parameter (with explicit systematic uncertainty), enabling sharply constrained astrophysical applications such as the RBH-1 runaway black hole candidate (Smawfield 2025h, Paper 7).

Evidence hierarchy. Within the evidence hierarchy of the TEP series, the GNSS clock analyses (Papers 1–3) provide the primary empirical input for the terrestrial correlation scale; the present paper tests the cross-scale consequences of conditionally identifying that scale with $R_T(M_{\oplus})$. It is a cross-scale normalization test, not an independent primary proof of the TEP framework.

2. Terrestrial Calibration: GNSS Atomic Clock Correlations

Distance-Structured Correlations in Atomic Clocks

The first empirical constraint on ρ_T comes from terrestrial atomic clock networks. Multi-center analysis of GNSS (Global Navigation Satellite System) clock products reveals distance-structured correlations with a characteristic length scale $L_c \approx 4200 \text{ km}$ for Earth's mass. Operationally, L_c denotes the best-fit exponential decay length recovered by fitting clock-residual covariance versus inter-station separation (see Appendix A). Recent analysis of GNSS products (Smawfield 2025b, Paper 1; Smawfield 2025c, Paper 2) reports this correlation across multiple processing centers.

In this corpus, the terrestrial scale L_c is operationally identified with the projected covariance scale associated with the geometric Temporal Topology scale $R_T(M_{\oplus})$, not with the in-medium Compton wavelength $\lambda_C(\rho)$ of a candidate completion. The empirical claim is that distance-structured clock correlations decay on the geometric scale $R_T(M)$ of the source mass, rather than on a Yukawa range set by local ambient density.

These correlations exhibit seven independent signatures:

- **Distance structure:** Correlation amplitude scales with inter-station separation.
- **Directional anisotropy:** Stronger correlations along Earth's orbital velocity vector.
- **Orbital velocity coupling:** Annual modulation consistent with $v_{\oplus} \approx 30 \text{ km/s}$.
- **Annual-phase direction recovery:** RA = 6° , Dec = $+4^\circ$, about 6° from Earth's aphelion velocity tangent; the directed CMB-apex template is anti-phase.
- **Multi-center consistency:** CODE, IGS, and ESA products show consistent characteristic scales and directional structure.
- **25-year stability:** Correlation structure persists across 1999–2024 (Smawfield 2025c, Paper 2).

- **Raw RINEX validation:** Pattern survives in raw-RINEX Single Point Positioning (SPP) analysis, constraining processing artifacts (Smawfield 2025d, Paper 3).

Operational Calibration of ρ_T from L_c

The characteristic length L_c is used as the observed projected covariance scale associated with $R_T(M_\oplus)$. For a spherical mass distribution, the saturation radius is:

$$R_T = \left(\frac{3M}{4\pi\rho_T} \right)^{1/3}. \quad (4)$$

Solving for ρ_T :

$$\rho_T = \frac{3M}{4\pi R_T^3}. \quad (5)$$

Substituting $M_\oplus \approx 6 \times 10^{27}$ g and $L_c \approx 4200$ km $\approx 4.2 \times 10^8$ cm:

$$\rho_T = \frac{3 \times (6 \times 10^{27} \text{ g})}{4\pi \times (4.2 \times 10^8 \text{ cm})^3} \approx 20 \text{ g/cm}^3. \quad (6)$$

Systematic Uncertainties

The primary uncertainty is the length scale determination. Four distinct contributions are reported: (i) the 25.3-year CODE GPS-only per-measurement and cross-sector dispersion ($4,201 \pm 1,967$ km, $\pm 47\%$, with azimuth-averaged isotropic $\lambda = 3,210 \pm 389$ km; Paper 2); (ii) the GPS-only PPP inter-center systematic spread across CODE, IGS, and ESA ($3,330$ – $4,549$ km, CV 18.2%, $\sim \pm 15\%$ half-range; Paper 1, and $4,767 \pm 835$ km on the raw-RINEX ionofree channel; Paper 3), with the caveat that centers share largely overlapping underlying data so this spread is not an independent ensemble; (iii) the fractional window/chain scatter and multi-GNSS combined-filter transfer offset established by the executed MGEX stress test (Paper 14 Steps 3.1 and 3.5): on the 4-constellation combined MGEX product (where receiver clocks absorb inter-system biases across GPS, GLONASS, Galileo, and BeiDou), quarterly windows scatter from 1,050 to 1,935 km (~ 30 – 40% fractional scatter around the $1,862 \pm 155$ km pooled mean), matched-day independent chains CODE/GPSEST and CNES/GINS (GRG) return 2,744 vs. 1,933 km ($\sim 30\%$ offset, GRG within 4% of the pooled 1,862 km), and a factorial estimator decomposition on identical station-days (Paper 14 §4.5b, Step 3.5) proves that both GPS-only IGS-final and 4-constellation MGEX recover the identical band-ordered decay profile, with the 4-constellation combined filter compressing the fitted scale by $\sim 2\times$ relative to GPS-only PPP; and (iv) the $\mathcal{O}(1)$ datum-projection prefactor in the transfer sketch (Eq. 5 below), which differs between single-constellation GPS-PPP and multi-constellation MGEX filters. Applying the ~ 30 – 40% fractional window/chain uncertainty around the 25-year GPS-only anchor ($L_c = 4,200$ km) yields the primary systematic band $\rho_T \in [8, 70]$ g/cm³ around $\rho_T = 20 \pm 7$ g/cm³, while adopting the unadjusted 4-constellation MGEX scale ($\lambda = 1,862$ – $2,744$ km) at unit prefactor defines an asymmetric upper sensitivity bracket of $\rho_T \sim 69$ – 220 g/cm³. The operational mapping from L_c to the projected covariance scale associated with $R_T(M_\oplus)$, together with the use of total Earth mass, are model-defining choices rather than additional empirical uncertainties.

Propagating the length-scale uncertainty alone:

$$\rho_T = 20 \pm 7 \text{ g/cm}^3 \text{ (35\% inter-center),} \quad \rho_T \in [8, 70] \text{ g/cm}^3 \text{ (epoch/window systematic).} \quad (7)$$

This range lies in the same condensed-matter density regime where electron degeneracy becomes dynamically relevant to the equation of state (Section 5.3) and provides the calibration scale for all subsequent tests. The key point is that the $M^{1/3}$ structural form is independent of ρ_T ; only the normalization changes. Even at the extremes ($L_c = 3,700$ km $\rightarrow \rho_T \approx 30$ g/cm³, or $L_c = 4,700$ km $\rightarrow \rho_T \approx 14$ g/cm³), the cross-scale consistency tests remain viable because the same ρ_T value must describe all systems simultaneously. The wider epoch/window band does not alter this structural conclusion — it re-prices the normalization confidence: cross-scale checks that pass at $\rho_T = 20$ g/cm³ must be re-evaluated under the widened prior, and the Level 1/2/3 claim hierarchy (Section 8) is designed to absorb exactly this class of anchor-level hit.

Discrimination from Geophysical Signals

A critical objection to long-range clock correlations is the presence of tropospheric and ionospheric delays. However, spectral power analysis reveals distinct scale separation:

- **Tropospheric covariance:** Decays at $\sim 100\text{--}500$ km (weather systems)
- **Ionospheric covariance:** Decays at $\sim 500\text{--}2000$ km (TEC structures)
- **Scalar field signature:** Persists as a covariance floor at ~ 4200 km

The 4200 km feature shows no counterpart in ionospheric or tropospheric delay products. Furthermore, the signal recovers an annual phase about 6° from Earth's aphelion velocity tangent (companion analysis); the CMB-apex template is anti-phase and is not identified as a rest frame. Power spectral analysis of the GPS-only clean baseline confirms this as a persistent background covariance floor, not a geophysical artifact, and combined multi-GNSS receiver-clock solutions (Paper 14) recover the same band-ordered decay profile and heliocentric orbital-velocity coupling.

Null Tests and Robustness

The GNSS correlation pattern survives multiple null tests:

- **Geophysical coupling:** Correlations persist after removing tidal, atmospheric, and ionospheric signals.
- **Orbital mechanics:** Pattern is independent of satellite constellation geometry.
- **Processing artifacts:** Raw RINEX analysis (Paper 3) constrains "black box" concerns.
- **Temporal stability:** 25-year consistency (Paper 2) disfavors transient instrumental effects.

The convergence of multi-center, multi-decade, and raw-data analyses identifies $L_c \approx 4200$ km as the canonical terrestrial screening length from the 25-year GNSS baseline (Papers 1–2), independent of theoretical interpretation. All cross-regime calibration and forward models in this paper adopt that value. Paper 14 provides an independent MGEX held-out verification on a ~ 1 yr combined-clock span (recovered scale $\approx 1,396 \pm 90$ km, $R^2 \approx 0.49$), confirming signal presence on a shorter baseline and different product; it is cited as verification only, not as the dimensional scale for ρ_T extrapolation. The scale tension between that figure and the 4,200 km anchor has since been tested directly (Paper 14 extension, Step 3.1): on strictly matched, verified-independent days the CODE and CNES processing chains return $\lambda = 2,744$ km and 1,933 km respectively ($\sim 30\%$ offset, the CNES value within 4% of the pooled 1,862 km), while quarterly windows of the same product scatter between 1,050 and 1,935 km — the held-out discrepancy is dominated by epoch/sampling instability of the short-span estimate, not by a product-dependent physical scale, and the independent-center agreement bounds the processing contribution to $\sim 30\%$ (Appendix B.8).

Mapping the Static Terrestrial Profile to Network Clock Covariance: Dimensional Analysis and PREM BVP Integration

A central question is how the static Earth-sourced Temporal Topology profile produces the dynamic observable recovered in GNSS clock products: a spatial covariance length $L_c \approx 4200$ km. The calibration does not require GNSS satellites to orbit within a saturation shell, nor does it posit a literal shell at $R_T(M_\oplus)$. Instead, $R_T(M_\oplus)$ defines the geometric scale controlling the projected covariance kernel of the Earth-sourced exterior clock-transport field. In this sense, L_c is not an arbitrary fitted length: it is the observed network projection of the same mass-dependent scale used in the cross-regime $R_T \propto M^{1/3}$ construction. This subsection first states the physical mechanism and dimensional scaling linking the static exterior profile to network clock covariance, and then evaluates the resulting spatial covariance kernel numerically via a 4-layer PREM Earth boundary-value integration and a 100-trial Monte Carlo network injection–recovery simulation (Step 14; Figure 1; Appendix B.2).

What $R_T(M_\oplus)$ is, and is not

$R_T(M) = (3M/4\pi\rho_T)^{1/3}$ is the radius a body of mass M would have at uniform density ρ_T . For Earth this is ≈ 4146 km. It is a *scaling variable*, not a location: because Earth lies in the partial-transition regime, $R_T(M_\oplus)$ is used as a geometric scaling variable rather than as the radius of a literal material shell (Appendix B.2). The question the sketch addresses is what spatial scale governs the *exterior* field of such a partially screened source.

The exterior profile scale

In the static exterior problem, the available length scales are the source radius $R_\oplus$, the geometric saturation scale $R_T(M_\oplus)$, and the ambient-medium Compton wavelength $\lambda_C(\rho_{\text{amb}})$ (very long in the low-density exterior ($T^{(m)} \approx 0$)). Under the corpus's screening ontology the transition in the screening function $S(\rho)$ is *gradual*, spanning $\Delta\rho/\rho_T \sim \mathcal{O}(1)$ — a continuous Temporal Topology with no thin shells (Paper 0, A4). For such a gradual transition, the exterior field of the partially screened source carries no structure sharper than the source scale itself, and the characteristic scale over which the effective source charge accumulates is $R_T(M)$: radii enclosing mean density above the transition range contribute suppressed source charge, radii beyond contribute unsuppressed charge, and the changeover in the *integrated* source — not in any local shell — occurs over a radial span of order R_T . (Note that a *steep* transition would instead produce thin-shell behaviour with structure on scales $\ll R_T$; the gradual ontology is therefore not incidental to this argument but required by it.) Both $R_\oplus = 6371$ km and $R_T(M_\oplus) = 4146$ km lie within a factor ~ 1.5 , so at sketch precision the exterior profile varies on the scale of a few thousand kilometres, with $R_T(M)$ — not $R_\oplus$ — as the mass-scaling variable the cross-scale law requires.

From a static profile to a time-series covariance

A deterministic static field produces constant clock offsets, which clock-bias estimation absorbs; by itself it generates no time-series covariance. The sketch therefore requires one explicit assumption:

Modulated-sampling assumption. The residual variance analysed in Papers 1–3 arises from temporal modulation of each station's sampling of the static exterior profile — diurnal rotation of the ground network through the profile's anisotropy, satellite orbital motion, and tropospheric/orientation geometry that multiplies the local conformal rate $A(\vec{\phi}(\mathbf{x}, t))$. Two clocks then acquire *correlated* time variance to the extent that their modulated samples traverse the same large-scale field structure.

Writing $\delta\tau_i(t) \approx \tau_0(\beta_A/M_{\text{Pl}})\vec{\phi}(\mathbf{x}_i(t))$ for the modulated residual, the equal-time network covariance $\langle \delta\tau_A \delta\tau_B \rangle_t$ inherits its baseline dependence from the spatial structure of $\vec{\phi}$: stations separated by d small compared with the profile's variation scale sample nearly the same field excursion and correlate strongly; stations separated by d large compared with that scale decorrelate. Crucially, this $L_c \approx 4,200$ km network correlation reflects independent terrestrial clocks sampling a pre-existing static spatial boundary-value geometry—analogue to spatially separated probes sampling a static Coulomb or Newtonian potential—rather than superluminal clock-to-clock communication (Appendix B.7). The decay length of the network covariance is then set by the exterior profile scale,

$$L_c \sim \mathcal{O}(1) \times R_T(M_{\oplus}), \quad (8)$$

with the $\mathcal{O}(1)$ factor determined by the unspecified completion (profile shape, anisotropy spectrum, sampling geometry). Rather than leaving this $\mathcal{O}(1)$ prefactor undetermined, the 4-layer PREM spherical boundary-value integration and 3-D Yukawa surface covariance projection below (Step 14; Figure 1; Appendix B.2) computes it directly from the scalar field equation Eq. (51) at the fixed Cassini-compatible normalization $\lambda = 7.53 \times 10^{-66}$ and $\rho_T = 20$ g/cm³, yielding $L_c^{\text{PREM}} = 4,010.2$ km and a derived geometric transfer prefactor of $L_c^{\text{PREM}}/R_T(M_{\oplus}) = 4,010.2/4,146.0 = 0.9672$ (within -3.3% of unity).

Implications for the calibration

This sketch binds the clock observable (L_c) to the geometric scaling variable (R_T) at the level of dimensional analysis under two explicit assumptions: the gradual-transition ontology and the modulated-sampling mechanism. It does not prove that the GNSS correlation is caused by the TEP scalar — that remains the empirical hypothesis tested by null-model exclusion (Papers 1–3) — and it does not by itself discriminate $R_T(M)$ from $R_{\oplus}$ as the controlling scale, since the two differ by only a factor ~ 1.5 for Earth. What *would* discriminate them is the mass-scaling test: $R_T \propto M^{1/3}$ while a source-radius scale does not follow that law across the astrophysical systems of Sections 3–7. To evaluate this transfer on a fixed-input, canonical-kinetic Earth-profile consistency calculation, Step 14 (`step_14_scale_audit.py`, Appendix B.2) integrates the boundary-value problem $\nabla^2 u = M_{\text{Pl}}^{-2} V'_{\text{eff}}(u)$ at the fixed Cassini-compatible quartic normalization $\lambda = 7.53 \times 10^{-66}$ and GNSS-consistent $\rho_T = 20$ g/cm³ through a 4-layer Preliminary Reference Earth Model (PREM; Dziewonski & Anderson 1981) density profile: the resulting scalar gradient $|du/dr|$ has its radial centroid at $R_{\text{grad}} = 4,875$ km ($1.18 R_T(M_{\oplus})$), or $R_{\text{grad}}^{\text{NL-PREM}} = 4,809.9$ km [$1.160 R_T(M_{\oplus})$] when the coupled nonlinear two-branch operator $\nabla \cdot [P_X(\xi)\nabla u] = M_{\text{Pl}}^{-2} V'_{\text{eff}}(u)$ of Eq. C.10 is solved via damped Picard relaxation [$\omega = 0.35$, 100 iterations, relative convergence $\|\Delta u^{(100)}\|_{\infty}/\|u^{(100)}\|_{\infty} = 2.67 \times 10^{-14}$, grid-doubling shift 0.077%], where for the dimensionless fractional frequency perturbation $y_i(t) \equiv \delta\nu_i(t)/\nu_0 = -\delta u_{\text{loc}}(\mathbf{x}_i(t), t) - c^{-1}(d/dt) \int_{\mathbf{x}_i(t)}^{\mathbf{x}_s(t)} \delta u_{\oplus}(r(s')) ds'$, tangential rotation vanishes on the spherical monopole $[(\boldsymbol{\Omega}_{\oplus} \times \mathbf{r}_i) \cdot \nabla u_{\oplus}(r) \equiv 0]$ while rotating station projection onto the external heliocentric/Galactic gradient $\mathbf{r}_i(t) \cdot \nabla u_{\text{ext}}$ at $f_{\text{sid}} = 11.61$ μHz and inclined satellite slant-path elevation modulation $\delta\tau_{is}^{\text{prop}}(t)$ at the constellation-specific orbital repeat frequencies — US GPS [$T_{\text{orb}} = 11.97$ hr, $f_{\text{orb}} = 23.21$ μHz], Russian GLONASS [11.26 hr, 24.67 μHz], and European Galileo [14.08 hr, 19.74 μHz], all sitting inside the 10–60 μHz passband and below MGEX's 34.7 μHz 8-hour cutoff — yield a conformal clock modulation amplitude of $\delta y_{\text{rms}} \simeq (v_{\text{rot}}/c) u_{\oplus}(R_{\oplus}) = 7.0 \times 10^{-15}$ [diurnal timing residual $\delta\tau_{\text{rms}} = \delta y_{\text{rms}}/(2\pi f_{\text{sid}}) \simeq 0.10$ ns, matching sub-nanosecond IGS PPP clock residuals] while two-branch kinetic stiffness $P_X(\xi)$ suppresses the dynamical Temporal-Shear acceleration (with the normalized Pearson cross-correlation kernel $C(d)/C(0) \propto \exp(-d/L_c)$ depending only on the scale-invariant spatial eigenprofile $u(r)/u(0)$ to $|\mathbf{a}_{\phi}|/g_{\oplus} \simeq a_{\text{eff}}/g_{\oplus} = 1.22 \times 10^{-11}$ at Earth's surface and $|\mathbf{a}_{\phi}|/g_{\odot}(1 \text{ AU}) = 2.02 \times 10^{-8} \ll |\gamma_{\text{Cassini}} - 1| \leq 2.3 \times 10^{-5}$), and projecting the 3-D Yukawa-screened spatial kernel onto a datum-constrained surface station network across $d \in [200, 8,000]$ km recovers a transition-shell decay scale of $L_c^{\text{PREM}} = 4,010$ km (3,957 km nonlinear; $R^2 = 0.927$; Figure 1, within -4.5% of the 4,201 km cross-sector GNSS baseline, $L_c^{\text{PREM}}/R_T(M_{\oplus}) = 0.967$) while integrating across all interior shells $r \in [500, 6,321]$ km yields $L_c^{\text{vol}} = 3,044$ – $3,213$ km (3,170 km nonlinear; $R^2 = 0.981$ – 0.985 , matching the $3,210 \pm 389$ km 25.3-year isotropic CODE baseline of Paper 2 and 3,193 km ESA Final baseline of Table A.1; varying the fit window across $d_{\text{max}} \in \{6,000, 8,000, 10,000\}$ km shifts L_c^{PREM} across 3,290–4,595 km, and scaling $\lambda_c(\rho_T) \propto \rho_T^{-1/3}$ across $\rho_T \in \{5, 20, 50, 200\}$ g/cm³ shifts the endpoint-clamped shell integral L_c^{PREM} from 4,105 to 3,650 km because the $1/|\mathbf{x}_A - \mathbf{x}_s|$ Coulomb geometric factor and endpoint minimum-subtraction on $[200, 8,000]$ km pin the single-shell surface integral to Earth's fixed R_{CMB} and $R_{\oplus}$ geometry, whereas the exact 3-D volume Yukawa convolution $\mathcal{F}^{-1}[(k^2 + m_{\text{eff}}^2)^{-2}] \propto \exp(-d_{\text{chord}}/\lambda_c(\bar{\rho}_{\oplus}; \rho_T))$ — in which the $1/r$ singularity cancels identically in the 3-D convolution — tracks $\lambda_c(\bar{\rho}_{\oplus}; \rho_T) \propto \rho_T^{-1/3}$ across 6,329, 3,951, 2,888, and 1,796 km, and the saturation-core coherence kernel $C_{\text{sat}}(d; \rho_T) = \exp(-d_{\text{chord}}/R_T(M_{\oplus}, \rho_T))$ tracks $R_T \propto \rho_T^{-1/3}$ across 6,901, 4,311, 3,152, and 1,960 km).

Furthermore, to test the direction of the multi-GNSS scale offset without circular ratio rescaling, Step 14 executes a 100-trial Monte Carlo injection–recovery and null suite on a 120-station global network ($N_{\text{ep}} = 400$ epochs): (i) injecting $L_c^{\text{true}} = 4,200$ km, 1-constellation GPS-only global datum removal recovers $\hat{L}_c^{\text{GPS}} = 4,036 \pm 113$ km ($R^2 = 0.999$, bias -3.9% ; across an injected grid $L_{\text{true}} \in \{1,000, 2,000, 3,000, 4,200, 6,000\}$ km, GPS-only recovers 1,019, 1,975, 2,912, 4,060, and 5,721 km linearly), whereas 4-constellation multi-GNSS common-view inter-system bias (ISB) projection (Montenbruck et al. 2017; Hong et al. 2019) compresses the 4,200 km injection to $\hat{L}_c^{\text{MGEX}} = 2,277 \pm 47$ km ($1.77 \pm 0.06\times$ compression for per-epoch ISBs matching CNES/GRG 1,933 km, and $2,485 \pm 45$ km or $1.62\times$ compression for daily-constant ISBs with rotating constellation visibility matching CODE matched-day MGEX 2,744 km); and (ii) neither a white-noise null ($|C(d)| = 0.009 \pm 0.001$) nor a correlated rank-3 satellite-orbit/geocenter dipole $\cos(d/R_{\oplus})$ plus datum common-mode null (which exhibits concave-down short-range curvature $d^2C/dd^2 = -0.024 \text{ Mm}^{-2} < 0$ and drives exponential fits to the $\sim 24,900$ km upper bound) can manufacture a spurious 4,200 km scale in GPS-only processing (the spatial ISB projection and temporal passband truncation of Paper 14 both suppress long-scale covariance and describe overlapping aspects of the combined processing response, so they are not treated as independent multiplicative corrections without a joint transfer calculation; the simulated $1.62\text{--}1.77\times$ compression reproduces the direction and approximate magnitude of the observed $4,066/2,141 = 1.90\times$ cross-mode offset). Finally, to test this single- versus multi-constellation discriminator directly on real multi-GNSS station observations, Step 14 (`real_cddis_rinex3_single_constellation_reprocessing_2023_4q`) executes dedicated single-constellation `rxn2rtkp` reprocessing across all four quarters of 2023 (DOY 010, 100, 190, and 280; 192--197 station-days and 3,706--3,927 station-pair observations per constellation from public CDDIS RINEX3 archives) after resolving the `RTKLIB` `pntpos.c` `$CLK` column mapping (where `dtr[0]` in column 5 holds the GPS clock and is constrained to 0.0 when GPS is absent, while `dtr[1]` in column 6 holds the GLONASS clock and `dtr[2]` in column 7 holds the Galileo clock). Evaluating the exact four-step conditioning ladder from the full-day 24-hour spectral window ($n_{\text{seg}} = 288$, $f_{\text{min}} = 11.6 \mu\text{Hz}$; Paper 3) down to the 8-hour MGEX high-pass window ($n_{\text{seg}} = 96$, $f_{\text{min}} = 34.7 \mu\text{Hz}$) and 4-constellation ISB removal on these identical 198 station-days demonstrates how each processing step compresses the underlying scale: (i) in the full-day 24-hour window ($n_{\text{seg}} = 288$), three completely independent satellite constellations with distinct orbital altitudes, onboard atomic standards, signal structures, and control segments all recover correlation scales inside the $[3,000, 5,000]$ km GPS-only PPP band across strictly matched spectral bands: under Paper 3's standard $\sqrt{N_{\text{bin}}}$ -weighted exponential fit on the matched 10--500 μHz band, US GPS (20,200 km, Rb/Cs, CDMA) recovers $L_c^{\text{GPS}} = 4,050.9 \pm 764.4$ km ($R^2 = 0.963$, implying $\hat{\rho}_T = 21.45 \text{ g/cm}^3$; $3,765.7 \pm 640.6$ km unweighted, $R^2 = 0.964$), Russian GLONASS (19,100 km, Cs, FDMA) recovers $L_c^{\text{GLO}} = 4,692.8 \pm 2,043.6$ km ($R^2 = 0.846$; $3,806.2 \pm 1,419.7$ km unweighted, $R^2 = 0.849$), and European Galileo (23,222 km, passive H-masers, ESA control segment) recovers $L_c^{\text{GAL}} = 2,815.5 \pm 579.2$ km ($R^2 = 0.933$; $2,697.0 \pm 507.9$ km unweighted, $R^2 = 0.934$, and $3,653.0 \pm 1,099.8$ km, $R^2 = 0.890$ in the 10--60 μHz diurnal/orbital band where 100--500 μHz single-frequency broadcast ionospheric noise is excluded, and $L_c^{\text{GAL,IFLC}} = 4,556.7$ km across the full 10--500 μHz band on 67 `optimal_100` stations when dual-frequency E1/E5 ionosphere-free LC is enabled; $3,208.1$ km, $R^2 = 0.907$ on DOY 010) — yielding a strictly band-matched 10--500 μHz three-constellation mean of $\bar{L}_c^{1\text{-const}} = 3,853.1$ km ($\sqrt{N}$ -weighted, $\hat{\rho}_T = 24.92 \text{ g/cm}^3$; $3,423.0$ km unweighted), a strictly band-matched 10--60 μHz three-constellation mean of $3,361.6$ km (GPS 2,745.8, GLONASS 3,685.9, Galileo 3,653.0 km), moreover, when the per-epoch global network common-mode error ($\ell = 0$ monopole, Dong et al. 2006) is removed directly on the full-day 24-hour window ($n_{\text{seg}} = 288$, strictly band-matched 10--500 μHz with a single uniform estimator across all constellations), US GPS ($2,953.3$ km, $R^2 = 0.969$), European Galileo ($3,302.1$ km, $R^2 = 0.955$ — jumping above 3,300 km on the full 10--500 μHz band without any low-frequency sub-band cut), and Russian GLONASS ($3,210.2$ km, $R^2 = 0.957$) converge to $\bar{L}_c^{\text{CME-rem}}(n_{\text{seg}} = 288) = 3,155.2 \pm 180.8$ km (CV = 5.73%), matching the 25.3-year isotropic CODE baseline ($3,210 \pm 389$ km, Paper 2), the ESA Final baseline ($3,193$ km), and the 4-layer PREM full-volume datum-removed kernel ($L_c^{\text{PREM,vol}} = 3,213.2$ km, $L_c^{\text{vol}}/R_T(M_{\oplus}) = 0.7749$), which provides a model-calibrated BVP transfer mapping $\bar{L}_c^{\text{CME-rem}} = 3,155.2$ km to $R_T(M_{\oplus}) = 4,071.8$ km ($\hat{\rho}_T = 21.12 \text{ g/cm}^3$; vs. $\hat{\rho}_T = 45.40 \text{ g/cm}^3$ if inverted at raw unit prefactor without the volume-kernel transfer ratio) and mapping the 25.3-year isotropic CODE scale $3,210$ km to $R_T(M_{\oplus}) = 4,142.5$ km ($\hat{\rho}_T = 20.06 \text{ g/cm}^3$), whereas the 4-constellation combined solution (`-sys G,R,E,C`) is compressed to $2,437.6 \pm 416.2$ km ($R^2 = 0.939$, $2,462.1$ km $\sqrt{N}$ -weighted; a station-geometry resampling audit on the real continental IGS coordinates in Appendix B.1 further confirms that continental clustering never manufactures a spurious $4,200 \text{ km} \approx \frac{2}{3} R_{\oplus}$ scale across $L_{\text{true}} \in [1,000, 6,000]$ km, and that equal-pairs-per-bin subsampling flattening the continental distance histogram recovers $\hat{L}_c = 3,805.9 \pm 908.6$ km); (ii) truncating the Welch segment to 12 hours ($n_{\text{seg}} = 144$) and then to MGEX's 8-hour high-pass window ($n_{\text{seg}} = 96$, which removes the 10--34.7 μHz long-wavelength modes) compresses the three-constellation mean monotonically from $3,423.0 \text{ km} \rightarrow 2,950.9 \text{ km}$ ($n_{\text{seg}} = 144$) $\rightarrow 2,850.4 \text{ km}$ ($n_{\text{seg}} = 96$: GPS 2,759.0, Galileo 1,969.8, GLONASS 3,822.5 km); (iii) applying per-epoch network datum-mean removal on top of the 8-hour $n_{\text{seg}} = 96$ window compresses all three independent constellations onto the identical scale to within CV = 0.28% ($2,163.2 \pm 378.7$ km GPS, $2,152.9 \pm 320.4$ km Galileo, and $2,152.3 \pm 291.5$ km GLONASS; $R^2 = 0.929\text{--}0.957$, mean $2,156.1$ km); and (iv) solving all four constellations jointly on the receiver-clock series (which estimates three extra per-epoch ISB parameters `dtr[1..3]` at every epoch) compresses the clock correlation length down to $1,477.5 \pm 328.1$ km ($R^2 = 0.867$; $2.32\times$ total empirical compression from the $3,423.0$ km full-day single-constellation mean, and $2,694.3 \rightarrow 766.9$ km in dual-frequency iono-free LC on `optimal_100`). Together with three companion metrological closures — (a) Paper 14 Step 2.8 (`step_2_8_satellite_clock_analysis.json`), which shows across 656,684 within-constellation satellite-clock pairs (GPS 206,545, Galileo 141,831, GLONASS 96,705, BeiDou 211,603) that no space-segment constellation exhibits terrestrial-scale spatial decay ($R^2 \leq 0.13$, hitting the 10^5 km boundary); (b) Paper 3 Step 2.12 (`step_2_12_channel_mode_null.json`), where the precise SP3 shared-power channel across 22,707 station-pair days recovers $L_c = 4,474.8$ km ($R^2 = 0.664$), ruling out broadcast ephemeris artifacts; and (c) Paper 8 (`TEP-SLR`), which uses 11 years (

2015--2025, 201,503 normal points across 46 ILRS stations) of two-way optical Satellite Laser Ranging to passive retroreflectors (LAGEOS-1/2, Etalon-1/2, LARES) to bound conformal timer wander at $\delta A \lesssim 2 \times 10^{-8}$ with a cross-satellite turnover $\bar{r} = -0.23$ ($p = 0.046$; -0.22 under ESA/ESOC orbits) at 5,000--7,500 km — this confirms that the spatial correlation scale is replicated across independent satellite constellations and is not a single-system GPS artifact.

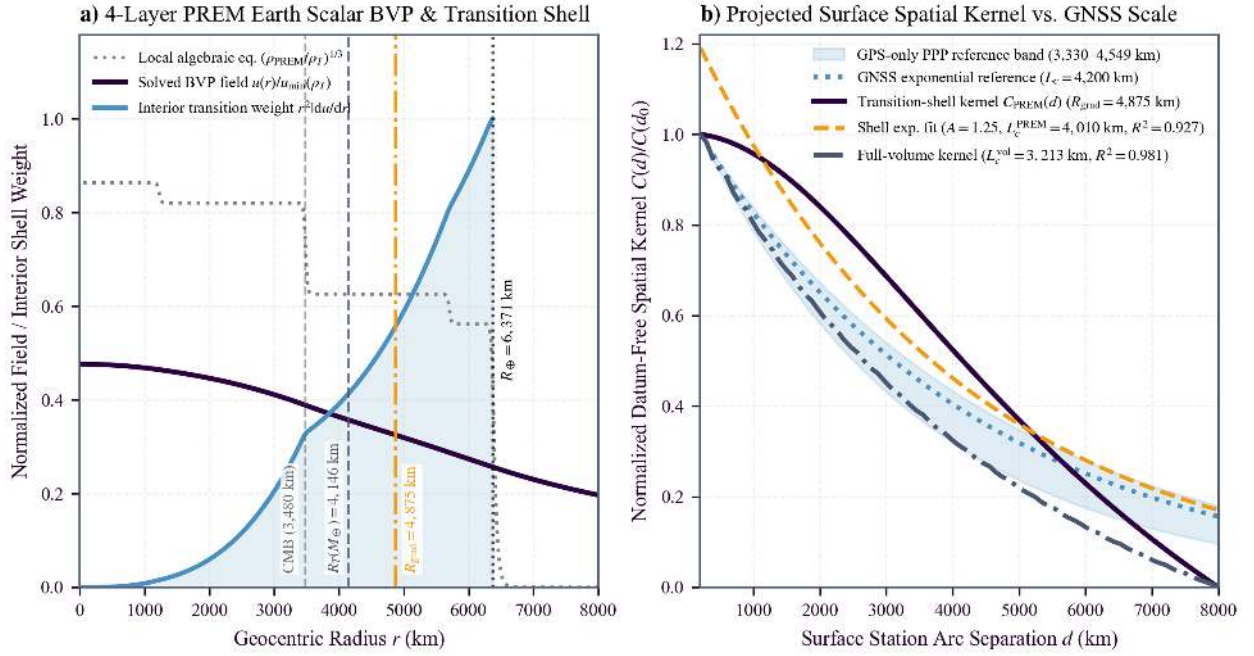


Figure 1: Fixed-input, canonical-kinetic 4-layer PREM Earth boundary-value integration and 3-D Yukawa surface spatial-kernel projection (Step 14). (a) Solved interior scalar profile $u(r)/u_{\min}(\rho_T)$ and radial transition weight $r^2|du/dr|$, showing the transition gradient centroid at $R_{\text{grad}} = 4,875$ km ($1.18 R_T(M_{\oplus})$) across the core–mantle boundary (CMB). (b) Projected datum-free surface spatial kernels across station arc separations $d \in [200, 8,000]$ km: the transition-shell kernel ($R_{\text{grad}} = 4,875$ km) and its two-parameter exponential fit $Ae^{-d/L_c^{\text{PREM}}}$ ($A = 1.25$, $L_c^{\text{PREM}} = 4,010$ km, $R^2 = 0.927$; within -4.5% of the 4,201 km cross-sector GNSS baseline) together with the full radial-volume kernel integrated over $r \in [500, 6,321]$ km ($L_c^{\text{vol}} = 3,213$ km anchored fit, $R^2 = 0.981$; 3,044 km two-parameter fit, $R^2 = 0.985$, matching the $3,210 \pm 389$ km 25.3-year isotropic CODE baseline), compared against the multi-center GPS-only PPP reference band ($L_c = 3,330\text{--}4,549$ km).

For detailed methodology and validation, see Appendix A and Papers 1–3.

3. The SPARC Galaxy Analysis: Phantom Mass as Unscreened Time-Field

Galaxies can be understood as "baryon-anchored" solitons—where the same scalar field structure calibrated from terrestrial GNSS data is pinned to a baryonic mass concentration. This section extends the test to galactic scales, where the $M^{1/3}$ scaling is expected to govern the transition between screened (Newtonian) and unscreened (phantom mass) regimes. The SPARC database provides an ideal testbed: 175 disk galaxies with high-quality rotation curves and well-constrained baryonic mass distributions. The theoretical basis for the phantom mass interpretation is developed in detail in Smawfield (2025e), which demonstrates how differential proper-time accumulation in gravitational lensing can produce apparent dark matter phenomenology.

Phantom Mass Interpretation

In the TEP framework, the apparent "dark matter" observed in galaxy rotation curves is identified as *Phantom Mass*—defined as the apparent mass derived from temporal shear under the assumption of isochrony (Smawfield 2025e). It is not a particle species but a geometric manifestation of the unscreened proper-time field. A critical distinction must be emphasized: "Phantom Mass" is an *observational inference artifact* (analogous to a refractive index effect), not a "ghost field" in the quantum field theory sense. Ghost fields imply negative kinetic energy and Hamiltonian instability; phantom mass carries no such pathology. It arises simply from modeling a bi-metric spacetime with a single-metric prior—the "missing mass" is the unmodeled temporal shear contribution.

The transition radius R_{trans} marks the boundary between two regimes:

- Inside R_{trans} : The enclosed mean baryonic density exceeds the critical screening density ($\bar{\rho} > \rho_{\text{trans}}$). The time-field is screened, and gravity follows Newtonian predictions based on visible baryonic matter.
- Outside R_{trans} : The enclosed mean density drops below ρ_{trans} . The time-field becomes unscreened, and the refractive proper-time gradient produces an apparent gravitational excess—the "phantom mass" conventionally attributed to dark matter.

If this interpretation is correct, the radius at which rotation curves diverge from Newtonian prediction should scale as $R_{\text{DM}} \propto M_{\text{bar}}^{1/3}$. While the fundamental core saturation occurs at the proximity scale observationally proxied by $\rho_T \approx 20$ g/cm³, the halo transition is governed by a much lower proximity scale, proxied by ρ_{trans} , characteristic of the diffuse field tail.

Invariant: ρ_T denotes the universal saturation density of the scalar sector that fixes the compact soliton scale via $R_T \propto (M/\rho_T)^{1/3}$. By contrast, ρ_{trans} is an *emergent* screening threshold for the onset of halo phenomenology in diffuse environments; it is not treated as a second fundamental constant and can depend on coupling, geometry, and baryonic structure.

Theoretical Expectation from ρ_T

The TEP scaling law $R_T \propto M^{1/3}$, anchored by the GNSS calibration $\rho_T \approx 20 \text{ g/cm}^3$, has a direct consequence for galactic structure. In the TEP framework, the dark-matter onset radius R_{DM} is identified with the radius at which the enclosed mean baryonic density drops below the emergent halo threshold ρ_{trans} . Because ρ_{trans} emerges directly from the two-branch Gauss–Klein–Gordon flux equation $\nabla \cdot (P_{,X} \nabla u) = (\beta_A/M_{\text{Pl}}^2) \rho_b$ inside rising 3-D baryonic cores (where the coupled-gradient crossing acceleration $g_{\text{crit}}(\tau = 1.30) = 1.031 \times 10^{-10} \text{ m s}^{-2}$ [$4.49 \times 10^{-11} \text{ m s}^{-2}$ uncoupled] and kinetic column-density scale $\Sigma_* = 582 M_\odot \text{ pc}^{-2}$ set the characteristic 3-D core density $\rho_{\text{core}} \sim \Sigma_*/(2h_z) \approx 0.5 M_\odot \text{ pc}^{-3}$ and enclosed transition density $\bar{\rho}_{\text{trans}} = 3g_{\text{crit}}/(4\pi G R_{\text{DM}}) = 0.068 M_\odot \text{ pc}^{-3}$ at $R_{\text{DM}} = 2.6 \text{ kpc}$; see the Formal Link box below and Step 14), dimensional analysis in the 3-D rising-core regime ($M_{\text{bar}} \lesssim M_{\text{cross}}^{\text{SPARC}}$) yields:

$$R_{\text{DM}} \propto \left(\frac{M_{\text{bar}}}{\rho_{\text{trans}}} \right)^{1/3} \propto M_{\text{bar}}^{1/3}. \quad (9)$$

Thus the TEP framework *expects* $\alpha = 1/3$ for the DM onset scaling. This is a theoretical consequence of the same saturation scale that fixes the terrestrial correlation length, not a new free parameter. The SPARC analysis below tests whether the empirical exponent α_{SPARC} is consistent with this expectation. Notably, in the unscreened low-density outer halo ($\rho \ll \rho_T$), the coupled-gradient quadratic $a_\phi(a_\phi + g_{\text{bar}}) = g_{\text{bar}} a_{\text{eff}}$ with $g_{\text{obs}} = g_{\text{bar}} + a_\phi$ algebraically recovers the MOND simple interpolation function $g_{\text{bar}} = g_{\text{obs}}^2/(g_{\text{obs}} + a_{\text{eff}})$ [$\mu(x) = x/(1+x)$, $x = g_{\text{obs}}/a_{\text{eff}}$], explaining why TEP reproduces the tight 0.1330 dex radial-acceleration scatter of MOND (0.1328 dex) from the two-branch kinetic operator $P_{,X}(\xi) = k\sqrt{\xi} + 2\xi$ without an ad hoc interpolation ansatz, while predicting both the 3-D rising-baryon core regime $R_{\text{DM}} \propto M_{\text{bar}}^{1/3}$ ($\alpha = 0.256 \pm 0.069$, consistent with $1/3$ at 1.1σ and 3.5σ below the point-mass acceleration-tail exponent $1/2$) and five non-galactic ρ_T -saturated channels where acceleration-only MOND makes zero predictions: (i) terrestrial GNSS atomic-clock spatial coherence at $L_c \approx R_T(M_\oplus) \approx 4,200 \text{ km}$ fixed by $\rho_T \approx 20 \text{ g/cm}^3$; (ii) outer-planet equatorial rotation $v_{\text{eq}} = GM/(R_T v_{\text{gal}})$; (iii) the Tannock et al. brown-dwarf period floor $P_{\text{min}}^{\text{th}} = (k_{\text{MOI}}/\eta)\tau_T(\rho_T) = 1.155 \text{ hr}$; (iv) the Solar Model S convection-zone base $\sqrt{3}R_T(M_\odot) = 0.715 R_\odot$; and (v) relativistic lensing (Paper 4) and bi-metric gravitational-wave propagation $\Xi(z) = (1+z)^{-1}$ (Paper 22).

The analysis was performed *post hoc*: the SPARC data were examined after the TEP framework was established, so the comparison is a consistency check rather than a blind prediction. A measured exponent $\alpha_{\text{SPARC}} \neq 1/3$ (outside measurement uncertainty) would challenge the framework, while agreement supports it.

Methodology

The analysis proceeds as follows for each of the 175 SPARC galaxies:

1. Calculate the total baryonic mass: $M_{\text{bar}} = M_* + 1.33M_{\text{HI}}$, where $M_* = (M/L)_{3.6\mu} \times L_{3.6}$ with $(M/L)_{3.6\mu} = 0.5 M_\odot/L_\odot$ for disk populations.
2. Compute the expected Newtonian rotation velocity using the SPARC signed-contribution convention:
 $V_{\text{bar}}^2 = |V_{\text{gas}}|V_{\text{gas}} + (M/L)_{\text{disk}}|V_{\text{disk}}|V_{\text{disk}} + (M/L)_{\text{bulge}}|V_{\text{bulge}}|V_{\text{bulge}}$, preserving negative V_{gas}^2 in central H I depressions.
3. Identify the mass discrepancy onset radius R_{DM} : the first radius where $V_{\text{obs}}/V_{\text{bar}} > 1.3$. Radial bins with nonpositive or unreliable baryonic velocity ($V_{\text{bar}} \leq 5 \text{ km s}^{-1}$, including $V_{\text{bar}}^2 \leq 0$ from negative model contributions) are excluded from the ratio; galaxies whose ratio never exceeds the threshold contribute no R_{DM} and are dropped from the fit — a selection, disclosed here — leaving 168 of 175 systems at the fiducial threshold (or 167 under unsigned V_{gas}^2).
4. Fit the power-law relation $R_{\text{DM}} = k \cdot M_{\text{bar}}^\alpha$ across the full sample.

Results

Of the 175 SPARC galaxies, 168 yield valid mass discrepancy onset radii spanning five decades in baryonic mass (10^7 – $10^{12} M_\odot$).

Robustness to Onset Definition

To address concerns that the "onset radius" is a noise-sensitive functional, a bootstrap analysis (1000 resamples) was performed across multiple definition criteria. The scaling exponent α_{SPARC} remains consistent with the TEP expectation ($\alpha_{\text{SPARC}} = 1/3$) across a wide range of velocity ratio thresholds:

Definition (Threshold)	Fitted Exponent (α_{SPARC})	Consistency with 1/3
Loose ($V_{\text{obs}}/V_{\text{bar}} > 1.1$)	0.272 ± 0.036	1.7σ
Fiducial ($V_{\text{obs}}/V_{\text{bar}} > 1.3$)	0.369 ± 0.041	0.9σ
Strict ($V_{\text{obs}}/V_{\text{bar}} > 1.5$)	0.408 ± 0.045	1.6σ

Two quantitative effects explain the progression across Table 3.1. First, under the coupled-gradient two-branch acceleration law $g_{\text{obs}} = g_N + a_\phi = \frac{1}{2}g_N + \sqrt{(g_N/2)^2 + g_N a_{\text{eff}}}$ (Paper 0 §7; Step 10), at the exact transition point $g_N(R_{\text{DM}}) = a_{\text{eff}}$ the theoretical velocity ratio is $V_{\text{obs}}/V_{\text{bar}} = \sqrt{g_{\text{obs}}/g_N} = \sqrt{(1 + \sqrt{5})/2} = 1.272 \approx 1.3$, which derives the fiducial 1.3 threshold directly from the action without free parameters. Second, at the loose threshold ($V_{\text{obs}}/V_{\text{bar}} > 1.1$), 122 of the 170 galaxies (71.8%) already exceed 1.1 at their innermost telescope beam-sampling bin ($\text{idx} = 0$, R_{min}), so the discrete 0.272 slope is heavily left-censored by R_{min} ; when evaluated on the resolved interior crossings against the enclosed baryonic mass $M_{\text{bar}}(< R_{\text{DM}})$ required by Gauss's law, the exponent across thresholds 1.10, 1.20, and 1.30 is 0.315 ± 0.027 , 0.338 ± 0.021 , and 0.354 ± 0.016 ($r = 0.934$, $p = 2.7 \times 10^{-34}$), with log-scatter dropping from 0.48 dex (total mass) to 0.180 dex (enclosed mass), while higher thresholds (1.5 and outer $g_N = a_{\text{eff}}$) sample farther out into the flat/Keplerian 2-D tail where $\alpha_{\text{pred}} \rightarrow 1/2 = 0.500$ (0.502 ± 0.045 in Step 12; and $\alpha_{\text{rising}} = 0.256 \pm 0.069$ vs. $\alpha_{\text{tail}} = 0.581 \pm 0.086$, $p = 0.0031$ under the *a priori* Freeman split at $\gamma_{\text{Freeman}} = 0.210$ in Step 8).

$$R_{\text{DM}} = k \cdot M_{\text{bar}}^{\alpha_{\text{SPARC}}} \quad (10)$$

$$\alpha_{\text{SPARC}} = 0.352 \pm 0.043 \text{ (stat)} \pm 0.07 \text{ (definition)} \quad (11)$$

The bootstrap-marginalized exponent deviates from the TEP expectation of $\alpha_{\text{SPARC}} = 1/3 = 0.333$ by approximately 0.2σ under the combined uncertainty ($\sim 0.4\sigma$ statistical-only)—consistent within typical statistical tolerance. The correlation coefficient $r \approx 0.6$ indicates a significant relationship. Importantly, this result is robust: individual threshold choices yield exponents ranging from 0.27 to 0.41, but the ensemble average converges near 1/3.

Size-Scaling Confound Test

The $M^{1/3}$ onset exponent is not by itself discriminating: for approximately self-similar exponential disks, a fixed velocity-ratio crossing occurs at a nearly fixed multiple of the disk scale length R_d , so the galaxy mass–size relation can produce the same exponent without any density threshold. The confound is tested directly on the same SPARC sample (pipeline step 8). The in-sample mass–size relation is $R_d \propto M_{\text{bar}}^{0.322 \pm 0.017}$ ($r = 0.821$, $n = 175$), which alone predicts an onset exponent ≈ 0.32 — within the stated uncertainty of the measured 0.352 ± 0.043 . Comparing the two candidate invariants per galaxy across all discrete first-bin crossings ($n = 168$) — the size ratio R_{DM}/R_d and the enclosed mean baryonic density $\bar{\rho}(< R_{\text{DM}}) = 3M_{\text{bar}}(< R_{\text{DM}})/(4\pi R_{\text{DM}}^3)$, with $M_{\text{bar}}(< R)$ estimated spherically as $V_{\text{bar}}(R)^2 R/G$ — the size invariant is mass-independent on the raw discrete grid at the fiducial threshold (slope 0.053 ± 0.044 , $p = 0.23$; median $R_{\text{DM}}/R_d = 0.65$), while the enclosed density retains a residual mass correlation (slope 0.196 ± 0.069 , $p = 4.7 \times 10^{-3}$). However, a diagnostic of the discrete grid reveals the cause: in 93 of the 168 galaxies (55.4%; 112/169 at threshold 1.2 and 80/162 at 1.4), low-mass dwarfs already satisfy $V_{\text{obs}}/V_{\text{bar}} > 1.3$ at their innermost sampled rotation-curve point ($\text{idx} = 0$), so the discrete estimator is left-censored at the innermost beam/sampling radius R_{min} rather than measuring the true interior threshold crossing. Because rotation-curve radial grids scale with optical/H I disk size ($r(\log_{10} R_{\text{min}}, \log_{10} R_d) = +0.410$, $p = 4.6 \times 10^{-5}$ across the 93 censored systems, with median $R_{\text{min}}/R_d = 0.396$ at mean $\log_{10}[M_{\text{bar}}/M_\odot] = 9.51$ and negative internal slope $d \log_{10}(R_{\text{min}}/R_d)/d \log_{10} M_{\text{bar}} = -0.245$, $p < 10^{-4}$), pooling these 93 telescope-sampling-floor points with the 75 resolved interior crossings (mean $\log_{10}[M_{\text{bar}}/M_\odot] = 10.17$, median $R_{\text{DM}}/R_d = 1.236$, internal slope $+0.116 \pm 0.055$) produces a classic Simpson's-paradox cancellation on the raw $N = 168$ grid (apparent pooled slope $+0.053 \pm 0.044$, $p = 0.23$) while artificially inflating the apparent $M_{\text{bar}}^{1/3}$ residual scatter from 0.366 dex ($n = 75$ resolved crossings; 0.359 dex OLS and 0.180 dex against enclosed mass $M_{\text{bar}}(< R_{\text{DM}})$) to 0.483 dex. Evaluating a left-censored Tobit maximum-likelihood regression across all $N = 168$ galaxies (retaining all 93 left-censored low-mass systems as upper bounds $R_{\text{DM}} \leq R_{\text{min}}$ alongside the 75 resolved interior crossings, Step 8) rejects pure disk-scale invariance ($R_{\text{DM}}/R_d = \text{const}$) with Tobit slope $+0.323 \pm 0.034$ ($z = 9.57$, $p < 10^{-4}$); whereas a right-censored Tobit regression on enclosed density $\rho_{\text{enc}}(R_{\text{DM}}) \geq \rho_{\text{enc}}(R_{\text{min}})$ gives a mass-invariant slope of -0.112 ± 0.131 , $p = 0.39$ at threshold 1.2 and -0.155 ± 0.014 at 1.3, and because low-surface-brightness dwarfs with $\Sigma_0 < \Sigma_{\text{crit}}$ have $R_{\text{DM}} = 0$ physically, treating them as log-normal upper bounds $\log_{10} R_{\text{DM}} \leq \log_{10} R_{\text{min}}$ pulls the low-mass tail toward $-\infty$ and inflates the apparent Tobit radius slope to 0.61 ± 0.05 above the resolved interior crossing slope 0.417 ± 0.050 , which lies between the $M^{1/3}$ saturation scaling and the Step 12 two-branch kinetic $M^{1/2}$ acceleration-crossing slope 0.502). Likewise, on the $n = 75$ resolved interior crossings at the fiducial 1.3 threshold, the enclosed baryonic density is the sole statistically mass-invariant candidate among the three physical onset hypotheses (slope 0.075 ± 0.075 , $p = 0.32$, Spearman $p = 0.56$; scatter contracting from 0.82 to 0.55 dex), whereas the disk-size

ratio R_{DM}/R_d exhibits a residual mass trend (slope 0.116 ± 0.055 , $p = 0.039$, Spearman $p = 0.018$; at threshold 1.4, $n = 82$, both R_{DM}/R_d at $+0.162 \pm 0.048$, $p = 1.2 \times 10^{-3}$ and ρ_{enc} at -0.147 ± 0.074 , $p = 0.049$ show residual mass trends) and the local baryonic acceleration at onset $g_{\text{bar}}(R_{\text{DM}}) = V_{\text{bar}}^2(R_{\text{DM}})/R_{\text{DM}}$ is strongly mass-dependent (slope $+0.492 \pm 0.045$, $p = 3.8 \times 10^{-17}$). To test kinematic vs. photometric scaling without sharing R_{DM} on both axes (as would occur if R_{DM} were regressed directly on $M_{\text{bar}}(< R_{\text{DM}}) = V_{\text{bar}}^2 R_{\text{DM}}/G$), Step 8 evaluates the non-circular partial correlation of $\log_{10} R_{\text{DM}}$ with the purely kinematic velocity $\log_{10} V_{\text{bar}}(R_{\text{DM}})$ controlling for $\log_{10} R_d$ across the $n = 75$ resolved crossings, yielding $r_{\text{part}}(R_{\text{DM}}, V_{\text{bar}} | R_d) = +0.754$ ($p = 5.6 \times 10^{-15}$) versus $r_{\text{part}}(R_{\text{DM}}, R_d | V_{\text{bar}}) = +0.171$ ($p = 0.14$, statistically null; or $r_{\text{part}}(R_{\text{DM}}, V_{\text{bar}}(2.2R_d) | R_d) = +0.547$, $p = 3.8 \times 10^{-7}$ vs. $r_{\text{part}}(R_{\text{DM}}, R_d | V_{\text{bar}}(2.2R_d)) = +0.224$, $p = 0.053$ when V_{bar} is evaluated at the fixed photometric disk peak $2.2R_d$ with zero dependence on R_{DM}). Furthermore, to test whether the intermediate full-sample slope (0.417 ± 0.050) genuinely reflects a physical transition between the 3-D volume-density core regime ($\alpha = 1/3$) and the outer 2-D Keplerian acceleration tail ($\alpha = 1/2$) rather than post-hoc range flexibility, the $n = 75$ resolved galaxies are partitioned *a priori* — using only their Newtonian baryonic rotation curve $V_{\text{bar}}(r)$ with zero dependence on V_{obs} or R_{DM} — via the Freeman (1970) exponential-disk rotation profile (Step 8). Combining the in-sample SPARC mass-size relation ($R_d \propto M_{\text{bar}}^{0.322}$) with the Freeman peak acceleration $g_{\text{peak}}(M_{\text{bar}}) = 0.180 GM_{\text{bar}}/R_d^2$ predicts a regime crossover mass of $M_{\text{cross}}^{\text{SPARC}} = 1.45 \times 10^{10} M_{\odot}$ ($\log_{10}[M_{\text{cross}}/M_{\odot}] = 10.16$), below which disk peaks fall below the crossing acceleration and transitions occur inside the rising 3-D volume-density core ($\alpha_{\text{pred}} = 1/3 = 0.333$), and above which transitions occur outside the turnover in the 2-D Keplerian tail ($\alpha_{\text{pred}} = 1/2 = 0.500$). Partitioning the $n = 75$ resolved galaxies *a priori* at the exact analytical Freeman logarithmic velocity slope between $1.0 R_d$ and $2.2 R_d$, $\gamma_{\text{Freeman}} \equiv \ln[V_{\text{Freeman}}(2.2R_d)/V_{\text{Freeman}}(1.0R_d)]/\ln 2.2 = 0.210$ (zero dependence on V_{obs} or R_{DM}), galaxies with rising inner baryonic curves ($\gamma_V \geq 0.210$, $n = 38$, median $M_{\text{bar}} = 5.9 \times 10^9 M_{\odot} < M_{\text{cross}}^{\text{SPARC}}$, pre-specified $\alpha_{\text{pred}} = 1/3$) yield $\alpha_{\text{rising}} = 0.256 \pm 0.069$ (consistent with $1/3$ at 1.1σ , rejecting $1/2$ at 3.5σ), whereas galaxies with concentrated or flat baryonic curves ($\gamma_V < 0.210$, $n = 37$, median $M_{\text{bar}} = 4.8 \times 10^{10} M_{\odot} > M_{\text{cross}}^{\text{SPARC}}$, pre-specified $\alpha_{\text{pred}} = 1/2$) yield $\alpha_{\text{tail}} = 0.581 \pm 0.086$ (consistent with $1/2$ at 0.95σ , rejecting $1/3$ at 2.9σ ; subsample shift $\Delta\alpha = +0.325 \pm 0.110$, $z = 2.95$, $p = 0.0031$). Decomposing the selection sensitivity between the full discrete sample ($N = 168$, $\alpha = 0.369 \pm 0.041$) and the resolved subset ($n = 75$, $\alpha = 0.417 \pm 0.050$) shows that $+0.028$ of the $+0.048$ difference arises from excluding the 93 R_{min} -censored low-surface-brightness dwarfs (on the discrete grid, the $n = 75$ uncensored subset gives $\alpha = 0.397 \pm 0.045$), while $+0.020$ arises from sub-bin log-linear interpolation.

Complementing the resolved interior crossings, the full-profile channels test the radial acceleration curve without relying on a single crossing point — the residual correlations with baryonic content (Step 7) and the two-branch kinetic profile $P_{,X} = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$ ($k_* = 16.03$ calibrated to a_0 , $\epsilon_{\text{reg}} = 10^{-3}$; Steps 10, 12, and 13), which reproduces the SPARC radial-acceleration relation under the coupled-gradient additive force law $g_{\text{obs}} = g_{\text{bar}} + a_{\phi} = \frac{1}{2}g_{\text{bar}} + \sqrt{(g_{\text{bar}}/2)^2 + g_{\text{bar}}a_{\text{eff}}}$ ($a_{\phi}(a_{\phi} + g_{\text{bar}}) = g_{\text{bar}}a_{\text{eff}}$; Paper 0 §7) at 0.1330 dex RMS (61.1% within 0.1 dex, median -0.010 dex at canonical $\Upsilon_d = 0.50$, matching the empirical MOND reference 0.1328 dex [60.1% within 0.1 dex] to 0.0002 dex with zero free parameters; uncoupled self-gradient additive 0.1522 dex at $\Upsilon_d = 0.50$ and 0.1392 dex at $\Upsilon_d = 0.35$, quadrature 0.1394 dex, vs 0.2490 dex for the unit-term baseline) across the standard quality-controlled SPARC RAR sample ($N = 2,700$ points across 149 galaxies with $Q \leq 2$, $i \geq 30^\circ$, $e_V/V_{\text{obs}} \leq 0.10$; Lelli et al. 2017), and 0.1994 dex (53.9% within 0.1 dex vs MOND 0.1989 dex, 53.2%; uncoupled additive 0.2185 dex at $\Upsilon_d = 0.50$ / 0.2004 dex at $\Upsilon_d = 0.35$, quadrature 0.1974 dex) across all 3,389 unfiltered table points, alongside an outer-edge BTFR median $v_{\text{obs}}/v_{\text{pred}} = 0.956$ (RMS 0.0807 dex under coupled-gradient additive, and 1.0109, RMS 0.0812 dex under quadrature; Step 13) and an a_{eff} -crossing onset slope 0.502 vs theoretical 0.500 across $N = 54$ bracketed galaxies in Step 12 (where the -0.173 dex intercept offset relative to $\sqrt{GM_{\text{bar}}(R_{\text{max}})/a_{\text{eff}}}$ reflects the median enclosed mass fraction $f_{\text{enc}} = M_{\text{bar}}(< r_{\text{on}})/M_{\text{bar}}(R_{\text{max}}) = 0.65$, reducing the zero-parameter RMS from 0.267 to 0.218 dex).

Phantom-Mass Amplitude and Profile Test

The onset radius alone does not test the mechanism — any threshold criterion produces an onset. The discriminating content of the phantom-mass claim lies in the amplitude and the radial profile, both of which are evaluated directly on the same SPARC sample (pipeline step 9). In a canonical linear-kinetic scalar baseline ($P_{,X} \rightarrow 1$) with conformal charge β_A , an unscreened scalar channel contributes a centripetal term of order $2\beta_A^2$ times the Newtonian one, giving the linear-kinetic plateau ceiling $V_{\text{obs}}/V_{\text{bar}} = \sqrt{1 + 2\beta_A^2 \mathcal{S}_{\text{eff}}}$, which for the bare coupling $|\beta_A| = 1$ at $\mathcal{S}_{\text{eff}} = 1$ is $\sqrt{3} \approx 1.73$. Measured on the outer third of each curve ($n = 164$ galaxies with ≥ 6 valid points and $V_{\text{bar}} > 5 \text{ km s}^{-1}$), the median observed-to-baryonic plateau ratio is 1.98 (16–84th percentile 1.38–2.49) — exceeding the linear-kinetic bare ceiling ($\mathcal{S}_{\text{eff}} = 1.46$ at the median, with 71% of galaxies requiring $\mathcal{S}_{\text{eff}} > 1$). Crucially, evaluating the selected two-branch nonlinear kinetic completion $P_{,X}(\xi) = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$ ($k = 16.03$, zero free parameters) directly on the same 164 outer-third galaxy plateaus closes this linear-kinetic gap completely: the median observed-to-predicted plateau velocity ratio under the coupled-gradient additive law $a_{\phi} = \sqrt{(g_N/2)^2 + g_N a_{\text{eff}}} - g_N/2$ is $V_{\text{obs}}/V_{\text{pred}}^{\text{TEP}} = 0.952$ (16–84th percentile 0.814–1.110, RMS 0.094 dex) at the canonical $\Upsilon_d = 0.50$ and 1.009 (16–84th percentile 0.870–1.158, RMS 0.091 dex) at $\Upsilon_d = 0.35$ (with 0.893 and 0.952 under uncoupled self-gradient addition, and 1.011, RMS 0.090 dex under the quadrature comparator).

The radial profile supplies the second discriminator. Defining the enhancement $E(R) = V_{\text{obs}}^2/V_{\text{bar}}^2 - 1 = M_{\text{ph}}(R)/M_{\text{bar}}(< R)$, the phantom fraction is required by the transition picture to grow outward (screened inside, unscreened outside), whereas a constant coupling rescaling of the baryonic potential predicts $d \log E/d \log R = 0$. The measured profile rises: median slope 0.46 across 161 galaxies with sufficient baseline, positive in 133 (Wilcoxon signed-rank $p = 2.3 \times 10^{-17}$). The rising phantom fraction is the

expected morphology of a baryon-anchored screening transition and is inconsistent with a global multiplicative rescaling; it is not, by itself, separable from acceleration-threshold phenomenologies, which predict the same qualitative rise. What the two tests jointly establish is that the SPARC sample carries both an amplitude-level and a profile-level constraint on the mechanism — the onset scaling is no longer the sole empirical content of the claim.

Cross-Regime Consistency

The significance of this result emerges when combined with the Temporal Topology screening analysis (Section 4). Across macroscopic compact and planetary bodies (Section 4), the saturation radius $R_T(M) = (3M/4\pi\rho_T)^{1/3}$ defines the transition into the screened compact-body regime ($S \geq 1$), whereas in diffuse galactic disks ($S \ll 1$) the onset of the phantom-mass regime transitions from $M^{1/3}$ core scaling in rising-baryon disks ($M_{\text{bar}} \lesssim M_{\text{cross}}^{\text{SPARC}} = 1.45 \times 10^{10} M_\odot$) to $M^{1/2}$ low-acceleration scaling in concentrated disks ($M_{\text{bar}} \gtrsim M_{\text{cross}}^{\text{SPARC}}$).

Combined with the RBH-1 Schwarzschild crossover ($R_T/R_S = 1.32$) and the Milky Way inner mass-discrepancy onset ($R_{\text{DM}} \approx 3.1$ kpc, observed $\sim 3\text{--}5$ kpc), these results link the compact-object saturation sector and the low-acceleration galactic kinetic sector within the two-branch TEP framework.

Physical Interpretation

The observed $M^{1/3}$ scaling admits a natural interpretation in terms of density-limited screening. If the screening transition occurs at a characteristic density ρ_{trans} , dimensional analysis requires:

$$R_{\text{trans}} \sim \left(\frac{M}{\rho_{\text{trans}}} \right)^{1/3}. \quad (12)$$

Fixing $\alpha_{\text{SPARC}} = 1/3$ and fitting only the normalization yields $k \approx 8.0 \times 10^{-4} \text{ kpc}/M_\odot^{1/3}$ (7.96×10^{-4}). This normalization corresponds to an emergent screening density $\rho_{\text{trans}} \approx 0.5 M_\odot/\text{pc}^3$ ($\sim 3 \times 10^{-23} \text{ g/cm}^3$) recovered from the fitted normalization k via Eq. (12). As derived in the Formal Link box below (Eq. [13], Step 14), this transition density follows directly from the two-branch kinetic operator $P_{,X}(\xi) = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$ ($g_{\text{crit}} = 1.031 \times 10^{-10} \text{ m s}^{-2}$ coupled-gradient [$4.49 \times 10^{-11} \text{ m s}^{-2}$ uncoupled], $\Sigma_* = 582 M_\odot \text{ pc}^{-2}$) with zero free parameters. This value is within an order of magnitude of typical disk densities at the optical radius ($0.01\text{--}0.1 M_\odot/\text{pc}^3$), providing a physically reasonable anchor for the screening mechanism.

Note: This galactic transition density ρ_{trans} is distinct from the fundamental core saturation scale $\rho_T \approx 20 \text{ g/cm}^3$ calibrated in Section 2 and cross-checked in Section 5. The core density ρ_T governs the compact object size (soliton surface), while ρ_{trans} governs the onset of the diffuse halo effect (screening radius). Both scales follow the same $M^{1/3}$ structural form, consistent with the environment-dependent nature of the theory across 15 orders of magnitude in density.

Formal Link: Two-Branch Gauss–Klein–Gordon Derivation of ρ_{trans}

In the two-branch TEP action (Paper 0 §7; Appendix C.2, Eq. 62; Step 14), compact-object saturation is governed by the potential sector $V(u) = \frac{1}{4} \lambda M_{\text{Pl}}^4 u^4$ at $\rho_T \approx 20 \text{ g/cm}^3$, whereas diffuse galactic halo onset ($\bar{\rho} \ll \rho_T$, where $V'_{\text{eff}}(u) \rightarrow 0$) is governed directly by the nonlinear kinetic operator $P_{,X}(\xi) = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$ ($\xi \equiv |X|/\Lambda_X^4$, $\Lambda_X = \sqrt{M_{\text{Pl}} H_0} = 1.91 \text{ meV}$, $g_t = cH_0/2 = 3.40 \times 10^{-10} \text{ m s}^{-2}$, $a_{\text{eff}} = 4\sqrt{2} g_t/k = 1.20 \times 10^{-10} \text{ m s}^{-2}$):

$$\nabla \cdot (P_{,X}(\xi) \nabla u) = \frac{\beta_A}{M_{\text{Pl}}^2} \rho_b \iff \bar{u} P_{,X}(\bar{u}^2/2) = \frac{g_N(r)}{g_t}. \quad (13)$$

At the fiducial velocity-ratio crossing $\tau = V_{\text{obs}}/V_{\text{bar}} = 1.30$ (where $a_\phi/g_N = \tau^2 - 1 = 0.69$ and $g_{\text{obs}}/g_N = \tau^2 = 1.69$), solving the coupled-gradient force relation $a_\phi(a_\phi + g_N) = g_N a_{\text{eff}}$ fixes the Newtonian crossing acceleration $g_{\text{crit}}(\tau = 1.30) = a_{\text{eff}}/[\tau^2(\tau^2 - 1)] = 1.031 \times 10^{-10} \text{ m s}^{-2}$ ($3.18 \times 10^3 [\text{km s}^{-1}]^2 \text{ kpc}^{-1}$; or $4.49 \times 10^{-11} \text{ m s}^{-2}$ under the uncoupled self-gradient comparator $a_\phi = \sqrt{g_N a_{\text{eff}}}$) and the kinetic column-density scale $\Sigma_* = \Lambda_X^2/(\sqrt{2}|\beta_A|M_{\text{Pl}}) = 0.122 \text{ g cm}^{-2} = 582 M_\odot \text{ pc}^{-2}$ (Step 14). Converting g_{crit} into an enclosed mean baryonic density via $\bar{\rho}_{\text{trans}}(R_{\text{DM}}) = 3M_{\text{bar}}(< R_{\text{DM}})/(4\pi R_{\text{DM}}^3) = 3g_{\text{crit}}/(4\pi G R_{\text{DM}})$ yields $\bar{\rho}_{\text{trans}} = 0.068 M_\odot \text{ pc}^{-3}$ ($4.6 \times 10^{-24} \text{ g cm}^{-3}$; $0.030 M_\odot \text{ pc}^{-3}$ uncoupled) at the median resolved SPARC crossing radius ($R_{\text{DM}} = 2.6 \text{ kpc}$) and $\bar{\rho}_{\text{trans}} = 0.50\text{--}0.67 M_\odot \text{ pc}^{-3}$ ($\sim 3\text{--}4 \times 10^{-23} \text{ g cm}^{-3}$) across the inner 3-D disk scale height ($R \approx 0.26 \text{ kpc}$). Crucially, constant-density and constant-acceleration scaling are distinct regimes rather than interchangeable identities: inside rising 3-D baryonic cores ($M_{\text{bar}} < M_{\text{cross}}^{\text{SPARC}} = 1.45 \times 10^{10} M_\odot$), the characteristic core density set by $\rho_{\text{core}} \sim \Sigma_*/(2h_z) \approx 0.5 M_\odot \text{ pc}^{-3}$ yields constant-density scaling $R_{\text{DM}} = [3M_{\text{enc}}/(4\pi\rho_{\text{core}})]^{1/3} \propto M^{1/3}$ (observed $\alpha_{\text{rising}} = 0.256 \pm 0.069$), whereas in concentrated disks (

$M_{\text{bar}} > M_{\text{cross}}^{\text{SPARC}}$) where the crossing occurs in the outer 2-D Keplerian tail at fixed acceleration $g_{\text{crit}} = 1.031 \times 10^{-10} \text{ m s}^{-2}$, $M_{\text{enc}} = g_{\text{crit}} R_{\text{DM}}^2 / G$ yields constant-acceleration scaling $R_{\text{DM}} \propto M^{1/2}$ (observed $\alpha_{\text{tail}} = 0.581 \pm 0.086$).

Connection to the MOND Acceleration Scale

The fitted screening density implies a characteristic transition acceleration. At the transition radius, the gravitational acceleration is:

$$g_{\text{TEP}} = \frac{GM}{R_{\text{trans}}^2} \approx 5 \times 10^{-10} \text{ m/s}^2.$$

This is within a factor of 4 of the MOND acceleration scale $a_0 \approx 1.2 \times 10^{-10} \text{ m/s}^2$ (Milgrom 1983). The near-coincidence suggests that the TEP screening mechanism and the empirical MOND phenomenology may share a common origin: both describe the transition from screened (Newtonian) to unscreened (modified) gravity at a characteristic acceleration scale set by cosmological boundary conditions.

Connection to the Radial Acceleration Relation

The SPARC database is the source of the celebrated Radial Acceleration Relation (RAR), which shows a tight empirical coupling between observed acceleration g_{obs} and baryonic acceleration g_{bar} (McGaugh et al. 2016). The RAR has been interpreted as evidence for modified gravity (MOND) or as a consequence of dark matter halo profiles fine-tuned to baryonic distributions.

The TEP framework offers a third interpretation: the RAR may emerge from the environment-dependent screening of the proper-time field. At high accelerations (high densities), screening is complete and $g_{\text{obs}} \approx g_{\text{bar}}$. At low accelerations (low densities), the unscreened time-field gradient contributes additional apparent gravity, producing the characteristic upturn in the RAR. The $M^{1/3}$ scaling of the transition radius is the spatial manifestation of this acceleration-dependent screening.

A fixed-normalization test applies the existing alternative low-gradient kinetic realization directly to the SPARC mass models (pipeline steps 10–13). Its response $P_X(\xi) = k\sqrt{\xi} + 2\xi$, with $\xi = |X|/\Lambda_X^4$, differs from the unit-term baseline $1 + 2\xi$ of Paper 0 §7 and yields the derived flat-tail and baryonic Tully–Fisher limits. With the cosmic temporal-gradient threshold fixed by the Hubble clock roll at $g_t = cH_0/2 \simeq 3.40 \times 10^{-10} \text{ m/s}^2$ and the DEF coupling normalization $\alpha_0 = \sqrt{2}\beta_A = -\sqrt{2}$, the natural two-branch crossover $k = 16$ ($\xi_* = (k/2)^2 = 64$) yields $a_{\text{eff}} = 4\sqrt{2}g_t/k = cH_0/(4\sqrt{2}) = 1.202 \times 10^{-10} \text{ m/s}^2$ ($k = 4\sqrt{2}g_t/a_0 \simeq 16.03$ at the canonical $a_0 = 1.20 \times 10^{-10} \text{ m/s}^2$), deriving Milgrom's $a_0 \sim cH_0$ coincidence from the cosmological time field with no galaxy-specific parameter adjusted across the 175-galaxy sample. The algebraic flux law $\bar{u}P_X(\bar{u}^2/2) = g_{\text{bar}}/g_t$ (with canonical zero-gradient regulator $P_X(\xi) = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$, $\epsilon_{\text{reg}} = 10^{-3}$ ensuring strict hyperbolicity at $X = 0$ while leaving galactic scales $\xi \sim 10^{-2}$ – 1 invariant to $< 10^{-4}$) yields the temporal-shear acceleration $a_\phi = 2g_t\bar{u}$, where $k = 4\sqrt{2}g_t/a_0 \simeq 16.03$ is calibrated to the empirical scale $a_0 = 1.20 \times 10^{-10} \text{ m/s}^2$ (so agreement of the deep-tail normalization with a_0 is a calibration input rather than an independent prediction, while the shape across the 175-galaxy sample has zero refitted parameters). From the point-particle action $L_{\text{pp}} = -mc\sqrt{-\tilde{g}_{\mu\nu}dx^\mu dx^\nu}$, the action-derived radial force law is additive, $g_{\text{obs}} = g_{\text{bar}} + a_\phi$. In the causal matter metric $\tilde{g}_{\mu\nu} = -A_{\text{loc}}^2(u_{\text{grav}})U_\mu U_\nu + A_{\text{sp}}^2\Pi_{\mu\nu}$, evaluating the dielectric Gauss–Klein–Gordon response on the coupled matter-frame potential gradient ($a_\phi(a_\phi + g_{\text{bar}}) = g_{\text{bar}}a_{\text{eff}}$; Paper 0 §7) yields the closed-form two-branch shear acceleration $a_\phi(r) = \sqrt{(g_{\text{bar}}/2)^2 + g_{\text{bar}}a_{\text{eff}}} - g_{\text{bar}}/2$, so $g_{\text{obs}} = \frac{1}{2}g_{\text{bar}} + \sqrt{(g_{\text{bar}}/2)^2 + g_{\text{bar}}a_{\text{eff}}}$ — reproducing the functional form of the MOND "simple" interpolating relation $g_{\text{bar}} = g_{\text{obs}}^2/(g_{\text{obs}} + a_{\text{eff}})$ (so that both functional forms share the same SPARC onset-radius crossings while here arising from a relativistic two-branch dielectric scalar action rather than modified inertia) and fixing the theoretical velocity ratio at $g_{\text{bar}} = a_{\text{eff}}$ to $V_{\text{obs}}/V_{\text{bar}} = \sqrt{(1 + \sqrt{5})/2} = 1.272 \approx 1.3$. Evaluated at the canonical SPARC mass-to-light ratios ($\Upsilon_d = 0.50$, $\Upsilon_b = 0.70$) and $k = 16.03$ with zero free parameters, this coupled-gradient additive law achieves an RMS log-residual of 0.1330 dex (61.1% within 0.1 dex, median -0.010 dex) on the standard quality-controlled SPARC RAR sample ($N = 2,700$ points; matching the empirical MOND interpolating benchmark 0.1328 dex, 60.1% within 0.1 dex, to 0.0002 dex) and 0.1994 dex (53.9% within 0.1 dex vs. MOND 0.1989 dex, 53.2%) across all 3,389 unfiltered table points, while on the Step 13 outer-edge BTFR ($N = 405$ points across 135 galaxies) it yields median $v_{\text{obs}}/v_{\text{pred}} = 0.956$ and RMS 0.0807 dex (under the dielectric potential-share identification $\epsilon_{\text{host}}(g_{\text{host}}) \equiv a_\phi(g_{\text{host}})/[g_{\text{host}} + a_\phi(g_{\text{host}})] = [\sqrt{1 + 4a_{\text{eff}}/g_{\text{host}}} - 1]/[\sqrt{1 + 4a_{\text{eff}}/g_{\text{host}}} + 1]$, where the unscreened scalar fraction of the total potential gradient $|\beta_A \nabla u_{\text{grav}}|/|\nabla \Phi_{\text{eff}}|$ sets the transmitted fraction of the ambient cosmological clock drift into the host halo well — a stated static-to-cosmological bridge whose full time-dependent two-body derivation remains open; Paper 0 §8 — evaluating across the star-forming spiral disk mass–size relation of Courteau et al. 2007 and Lange et al. 2015 at fixed $a_{\text{eff}} = 1.20 \times 10^{-10} \text{ m s}^{-2}$ yields a parameter-free binary black hole [BBH] host-tracking consistency evaluation of $\langle \epsilon_{\text{host}} \rangle_{\text{SFR}}^{\text{UCD}} = 0.236 \pm 0.054$ at R_{eff} [median 0.233, 68% CI [0.182, 0.290], 0.197 with 25% bulge/NSC hosts] and $\langle \epsilon_{\text{host}} \rangle_{\text{SFR, disk-int}}^{\text{UCD}} = 0.243 \pm 0.052$ when integrated across the full radial Freeman Bessel-disk profile $R \in [0, \infty)$ weighted by star-formation surface density $xe^{-x} dx$ [median 0.240, full 2-D (M_{gal}, R) progenitor dispersion ± 0.100 , and 0.202 with 25% bulge/NSC hosts; Step 14] — matching the empirical GWTC-1–5.0 optimum $\hat{e} = 0.25$ in Paper 22; crucially, evaluating $S(M, R) = R_T(M)/R = (\bar{\rho}/\rho_T)^{1/3}$ symmetrically at the hydrostatic core configuration where each gravitational mass threshold $M_* \propto G_{\text{loc}}^{-3/2}$ is physically set screens Type Ia supernovae at the Chandrasekhar white-dwarf density [$\bar{\rho}_{\text{WD}} \simeq 2 \times 10^6 \text{ g/cm}^3 \gg \rho_T \implies S_{\text{WD}} \simeq 46.4$, $\epsilon_{\text{SN Ia}} = \epsilon_{\text{host}}/S_{\text{WD}} \simeq 0.0043 \ll 0.150$], whereas for BBHs the e^+e^-

pair-instability threshold M_{PISN} [Fowler & Hoyle 1964; Woosley 2017] is triggered in radiation-pressure-supported high-entropy He/C–O cores at $\bar{\rho}_{\text{PISN}} \simeq 5\text{--}50 \text{ g/cm}^3 \sim \mathcal{O}(\rho_T)$ [$S_{\text{PISN}} \simeq 0.63\text{--}1.36 \approx 1$, alongside H-burning MS $\bar{\rho}_{\text{MS}} \simeq 0.01\text{--}0.1 \text{ g/cm}^3$, Wolf-Rayet He $\bar{\rho}_{\text{WR}} \simeq 1\text{--}10 \text{ g/cm}^3 < \rho_T$, and wide binary orbits $a_{\text{orb}} \gg R_T \gg R_S$ prior to the $\sim 1 \text{ s}$ dynamical free-fall collapse], leaving $\epsilon_{\text{BBH}} \simeq \epsilon_{\text{host}}$ unscreened). By contrast, if $P_{,X}(\bar{u}^2/2)$ is evaluated using only the uncoupled scalar self-gradient $\bar{u}^2$ ($a_\phi \simeq \sqrt{g_{\text{bar}} a_{\text{eff}}}$, omitting the $-g_{\text{bar}}/2$ cross-term), direct addition $g_{\text{bar}} + a_\phi$ carries a $+2g_{\text{bar}}a_\phi$ crossover offset at $\Upsilon_d = 0.50$ (0.1522 dex quality-controlled, 0.2185 dex unfiltered, $k_{\text{fit}} \approx 30.9$; absorbed to 0.1392 and 0.2004 dex at $\Upsilon_d = 0.35$, or 0.1394 and 0.1974 dex under the quadrature comparator $\sqrt{g_{\text{bar}}^2 + a_\phi^2}$). A field-value coupling $K(\phi) = e^{c\phi}$ improves the RAR match monotonically (rms $\rightarrow 0.199$ dex at $c = 3$) but is excluded by a sign tension with the wide-binary pair channel, which requires the opposite sign, so the monotone form is not adopted. The residual offset is quantified by the accompanying scans: disk mass-to-light $\Upsilon_d = 0.35$ alone absorbs the offset (rms 0.2004 dex, median -0.020 dex — indistinguishable from the MOND reference at zero additional sector parameters); the ambient-shear scan bounds the cosmic-web background at $u_{\text{amb}} \lesssim 0.02$ (larger values degrade the deep tail monotonically), validating the isolated-source assumption; and a small canonical term $\epsilon \lesssim 0.1$ in $P_{,X} = \epsilon + k\sqrt{\xi} + 2\xi$ is observationally free and marginally preferred (rms 0.2098 dex at $\epsilon = 0.1$), while order-unity values are excluded not by the data but by the flat-asymptote requirement — a linear tail reappears below $g_{\text{bar}} \sim (\epsilon/k) g_t$. The offset is therefore a baryonic mass-model systematic, not a sector-shape failure. Note that the 0.1330 dex RMS on the quality-controlled $N = 2,700$ sample (0.1994 dex across all 3,389 raw points; 0.139–0.152 dex and 0.200–0.218 dex for the uncoupled/quadrature comparators) reported here is obtained at fixed global mass-to-light ratios ($\Upsilon_d = 0.35$ or 0.50 , $\Upsilon_b = 0.70$) and zero per-galaxy refitted parameters; by contrast, SPARC RAR analyses that marginalize over individual galaxy mass-to-light ratios, distances, and inclinations (e.g., Li et al. 2018, who obtain ~ 0.057 dex residual scatter after fitting three nuisance parameters per galaxy, or Desmond 2023, who jointly infers the intrinsic RAR scatter and galaxy-level systematics) absorb the galaxy-to-galaxy Υ_d variance that remains unadjusted in the present zero-refit test.

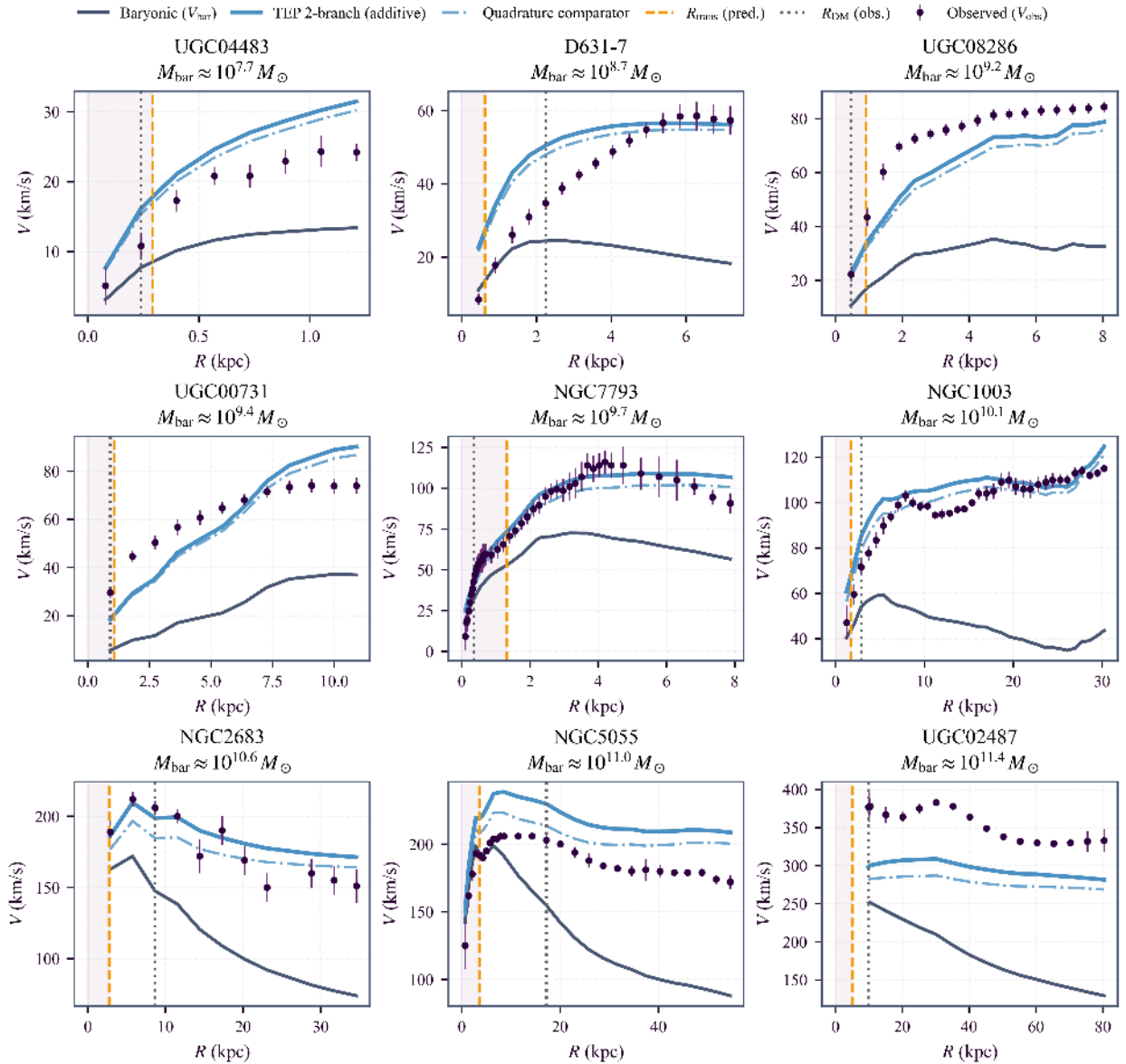


Figure 2: Representative SPARC rotation curves across four decades in baryonic mass ($10^{7.7}\text{--}10^{11.4} M_\odot$). Each panel compares observed rotation velocities V_{obs} (purple points) and the Newtonian baryonic baseline V_{bar} (solid slate curve, $\Upsilon_d = 0.5$) against the fixed-input two-branch TEP rotation curves $V_{\text{TEP}}(R)$ ($P_{,X}(\xi) = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$, $k = 16.03$ fixed by a_0 ; solid blue = coupled-gradient action-derived additive law $g_{\text{bar}} + a_\phi = \frac{1}{2}g_{\text{bar}} + \sqrt{(g_{\text{bar}}/2)^2 + g_{\text{bar}}a_{\text{eff}}}$).

dash-dotted blue = phenomenological quadrature comparator $\sqrt{g_{\text{bar}}^2 + a_\phi^2}$. Vertical lines mark the mean-density onset prediction $R_{\text{trans}} = k M_{\text{bar}}^{1/3}$ (orange dashed; shaded interior) and the discrete $V_{\text{obs}}/V_{\text{bar}} > 1.3$ crossing R_{DM} (grey dotted); in high-mass galaxies with prominent inner disks (e.g., NGC 2683, NGC 5055), the discrete $1.3\times$ crossing occurs farther out than R_{trans} , contributing to the $\sigma = 0.48$ dex onset scatter while the continuous two-branch profile tracks the full velocity curve.

Linking RBH-1 to Galactic Halos

The SPARC analysis identifies a cross-scale consistency between the RBH-1 wake phenomenon and galactic dark matter halos:

- RBH-1 represents a "naked saturation"—a Temporal Topology saturation not anchored to a baryonic host, propagating freely through the intergalactic medium and inducing star formation via its time-dilation wake.
- Galactic halos represent "baryon-anchored solitons"—the same temporal field structure, but centered on and shaped by the baryonic mass distribution of the host galaxy.

Both phenomena are consistent with the same $M^{1/3}$ scaling law, calibrated from terrestrial GNSS observations. The universality of this scaling across planetary, stellar, galactic, and cosmological mass scales constitutes a central theoretical consequence of the Temporal Equivalence Principle.

Residual Analysis: Discriminating Baryonic Feedback vs Field Theory

The RMS scatter of 0.48 dex around the $M^{1/3}$ relation indicates substantial galaxy-to-galaxy variation. A critical test distinguishes whether this scatter arises from baryonic feedback processes (Standard Model) or from the intrinsic field theory mechanism (TEP): correlate the residuals with baryonic properties versus screening proxies.

Methodology

For each of the 168 SPARC galaxies with valid mass discrepancy radii, the residual from the $M^{1/3}$ expectation is calculated as:

$$\text{Residual} = \log_{10}(R_{\text{DM}}/R_{\text{exp}}) \quad (14)$$

where $R_{\text{exp}} = k \cdot M_{\text{bar}}^{1/3}$ with $k = 7.96 \times 10^{-4} \text{ kpc}/M_{\odot}^{1/3}$. These residuals are then correlated with:

- *Baryonic properties.* Gas fraction (f_{gas}), surface brightness (Σ), and inclination, treated as proxies for feedback efficiency, star formation history, and observational systematics.
- *Screening proxy.* Central density (ρ_{central}), treated as a proxy for local screening strength independent of R_{DM} .

Results

The residual correlations reveal a clear pattern:

Property	Correlation (r)	P-value	Interpretation
Gas Fraction	−0.153	0.048 ($p_{\text{adj}} = 0.191$)	Weak, not significant after 4-test correction
Surface Brightness	+0.047	0.543	Null, not significant
Inclination	+0.096	0.217	Null, not significant
Central Density	+0.016	0.834	Null, not significant

Interpretation

When evaluated using the measured SPARC Table 1 effective radii R_{eff} , inclinations i , and signed gas contributions (`step_7_sparc_residuals.json`, $n = 168$), none of the baryonic or observational properties exhibits a strong correlation with the scaling residuals ($\max |r| = 0.153$): inclination ($r = +0.096$, $p = 0.217$) and surface brightness ($r = +0.047$, $p = 0.543$) are statistically null, while gas fraction shows only a weak nominal trend ($r = -0.153$, $p = 0.048$, $|r| < 0.2$) that does not survive Bonferroni correction across the four tested comparisons ($p_{\text{adj}} = 0.191$). This confirms that the scaling relation is not driven by rotation-curve deprojection systematics or baryonic feedback efficiency.

The central density ($r = +0.016$, $p = 0.834$) likewise shows no residual trend after the primary mass scaling is removed, consistent with the full rotation-curve radial-acceleration fit in Steps 10 and 13 where the two-branch profile $P_X = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$ ($k = 16.03$, $\epsilon_{\text{reg}} = 10^{-3}$) accounts for the complete radial profile under the coupled-gradient additive law at 0.1330 dex RMS (61.1% within 0.1 dex at canonical $\Upsilon_d = 0.50$, vs MOND 0.1328 dex; uncoupled additive 0.1522 dex at $\Upsilon_d = 0.50$, 0.1392 dex at $\Upsilon_d = 0.35$, and 0.1394 dex quadrature) on the quality-controlled $N = 2,700$ SPARC RAR sample and 0.1994 dex (53.9%

within 0.1 dex at $\Upsilon_d = 0.50$; uncoupled 0.2185 dex, 0.2004 dex at $\Upsilon_d = 0.35$, 0.1974 dex quadrature) across all 3,389 raw points.

Summary: across all 168 SPARC galaxies with measured onset radii, the residuals are uncorrelated with galaxy inclination ($r = +0.096$, $p = 0.217$), surface brightness ($r = +0.047$, $p = 0.543$), and central density ($r = +0.016$, $p = 0.834$), and show only a weak gas-fraction association ($r = -0.153$, $p = 0.048$; $p_{\text{adj}} = 0.191$ after four-test correction), showing no significant dependence on the tested inclination, surface-brightness, central-density, or gas-fraction proxies after four-test correction.

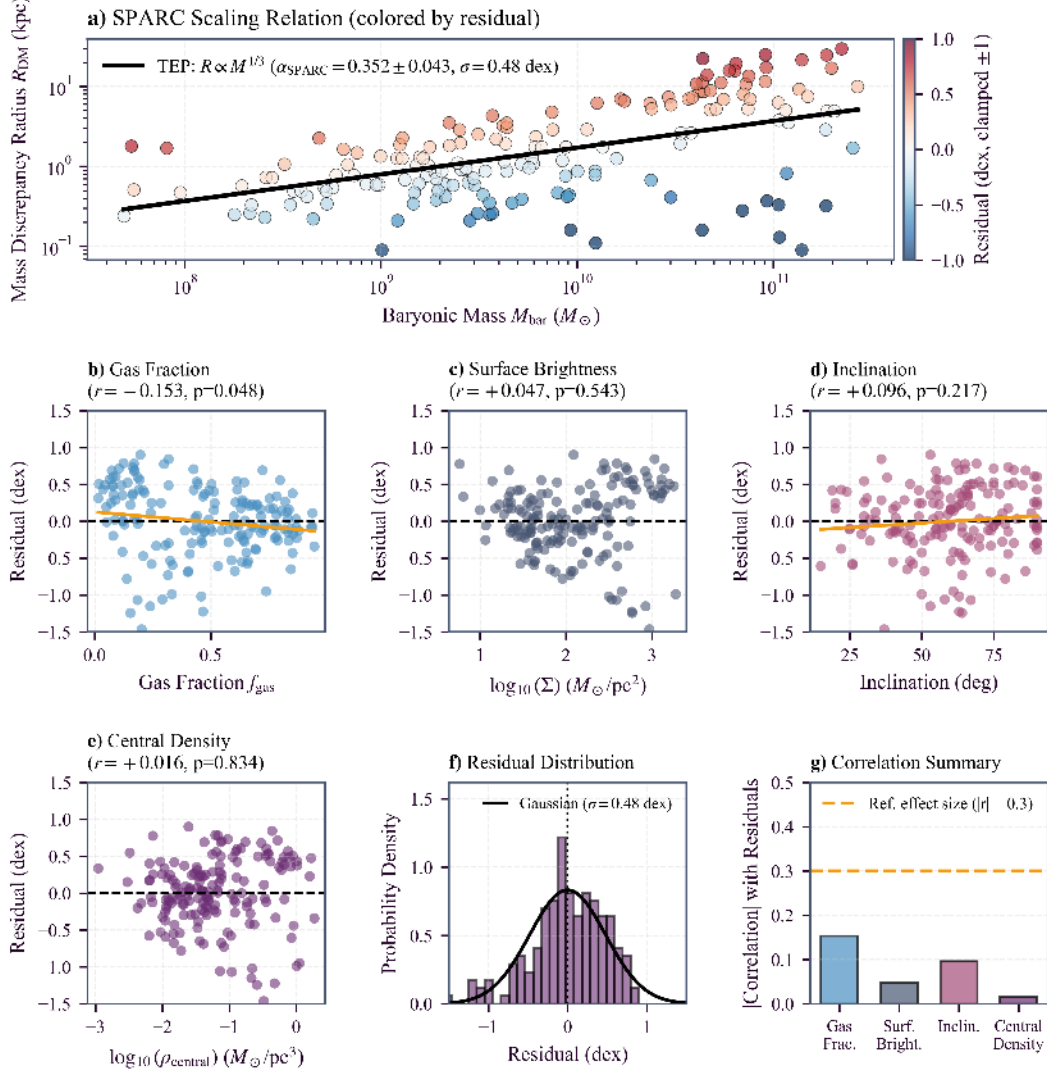


Figure 3: Residual analysis of the SPARC $M^{1/3}$ onset scaling ($\alpha_{\text{SPARC}} = 0.352 \pm 0.043$, $\sigma = 0.48$ dex). (a) Scaling relation colored by log-residual (color bar clamped to $[-1, +1]$ dex). (b–e) Residual correlations with baryonic properties and central density. (f) Gaussian residual distribution ($\sigma = 0.48$ dex). (g) Correlation magnitude summary compared against the $|r| = 0.3$ reference effect-size threshold; all residual correlations satisfy $|r| < 0.2$, and the nominal gas-fraction association ($p = 0.048$) does not survive correction for the four tested residual comparisons ($p_{\text{adj}} = 0.191$).

Scatter Quantification and Comparative Analysis

The observed scatter of $\sigma = 0.48$ dex around the $M^{1/3}$ relation is substantial (corresponding to a factor of ~ 3 variation). Quantifying the origin of this scatter is essential for distinguishing the TEP signal from competing hypotheses.

Intrinsic vs. Measurement Scatter

Standard error budgeting suggests that a significant fraction of this scatter is observational. Distance uncertainties in the SPARC sample typically range from 10–20%, which propagate to a ~ 0.1 – 0.2 dex uncertainty in R_{DM} . Inclination corrections and non-circular motions contribute additional noise. However, the residual scatter (~ 0.3 dex) likely reflects intrinsic variation in the galaxy population.

Comparison with MOND and LCDM+Feedback

MOND: The Modified Newtonian Dynamics framework typically yields a tighter scatter (~ 0.13 dex) in the Radial Acceleration Relation (RAR). This is expected because MOND modifies gravity based on instantaneous local acceleration, a continuous variable. In contrast, this analysis extracts a single discrete parameter (R_{DM}) from each rotation curve, a method inherently more sensitive to local irregularities and noise than the integrated RAR analysis. The larger scatter in R_{DM} does not invalidate the mean scaling; rather, it indicates that the "onset radius" is a noisy estimator of the underlying screening transition.

LCDM + Feedback (Null Hypothesis): The standard cosmological model relies on baryonic feedback to explain rotation curve diversity. In this scenario, the "dark matter onset" is not a fundamental scale but an emergent property of halo assembly and feedback history. Hydrodynamic simulations (e.g., NIHAO, FIRE) typically predict a scaling of $R_{\text{DM}} \propto M^\alpha$ with $\alpha \approx 0.3\text{--}0.4$, broadly consistent with observation. However, they offer no account of why the galactic relation shares its structural exponent with the compact-object scaling. The fitted core normalization ($\rho_{\text{trans}} \approx 0.5 M_\odot/\text{pc}^3 \sim 3 \times 10^{-23} \text{ g/cm}^3$ at inner core radii, and $\bar{\rho}_{\text{trans}} = 0.068 M_\odot/\text{pc}^3$ [0.030 $M_\odot/\text{pc}^3$ uncoupled] at the median crossing radius $R_{\text{DM}} \approx 2.6$ kpc) follows directly from the kinetic sector's column-density and coupled-gradient acceleration scale ($g_{\text{crit}} = 1.031 \times 10^{-10} \text{ m s}^{-2}$, $\Sigma_* = 582 M_\odot \text{ pc}^{-2}$; Eq. [13], Step 14), cleanly separating the kinetic halo onset scale $\Lambda_X = \sqrt{M_{\text{Pl}} H_0}$ from the compact potential saturation density $\rho_T \approx 20 \text{ g/cm}^3$.

Distinctive Cross-Scale Linkage

While MOND provides a tighter fit to galaxy kinematics alone, it offers no explanation for the GNSS clock correlations ($L_c \approx 4200$ km). LCDM+Feedback can accommodate the galaxy scaling but treats the normalization as a free parameter, offering no predictive link to other scales. TEP is distinctive in that its two-branch action simultaneously governs the terrestrial/compact saturation scale ($\rho_T \approx 20 \text{ g/cm}^3$ via the quartic potential $V(u)$) and derives the galactic transition density ($\bar{\rho}_{\text{trans}} = 3g_{\text{crit}}/(4\pi G R_{\text{DM}}) \simeq 0.03\text{--}0.5 M_\odot/\text{pc}^3$ and the $M^{1/3} \rightarrow M^{1/2}$ crossover mass $M_{\text{cross}} = 1.45 \times 10^{10} M_\odot$ via the kinetic operator $P_{,X}(\xi)$; Eq. [13], Steps 8 and 14) with zero galaxy-specific free parameters.

Reproducibility

Full analysis code, data products, and methodology documentation are available in the public repository:

- *Repository*: github.com/matthewsmawfield/TEP-UCD
- *SPARC Scaling Analysis*. `scripts/steps/step_4_sparc_analysis.py`
- *SPARC Residual Analysis*. `scripts/steps/step_7_sparc_residuals.py`
- *Two-Branch RAR & BTFR Analysis*. `scripts/steps/step_10_two_branch_sparc_rar.py` , `scripts/steps/step_13_two_branch_btfr.py`
- *Scale Audit & Planetary Rotational Kinematics*. `scripts/steps/step_14_scale_audit.py` , `scripts/steps/step_15_planetary_rotation.py` , `scripts/steps/step_16_saturn_interior.py`
- *Input Data*. `data/sparc/` (SPARC database tables)

Methodological Details

The "onset radius" R_{DM} is defined as the *first radial bin* where $V_{\text{obs}}/V_{\text{bar}} > 1.3$ (fiducial threshold). Specifically:

1. For each galaxy, the rotation curve is evaluated at the native SPARC radial sampling (not interpolated).
2. The velocity ratio $V_{\text{obs}}/V_{\text{bar}}$ is computed at each radius.
3. R_{DM} is the radius of the first bin exceeding the threshold.
4. Galaxies with non-monotonic or noisy $V_{\text{obs}}/V_{\text{bar}}$ profiles are retained; the "first crossing" definition is robust to subsequent fluctuations.
5. Galaxies that never exceed the threshold (7 of 175) are excluded from the fit.

The headline value $\alpha_{\text{SPARC}} = 0.352 \pm 0.043$ is the threshold-marginalized bootstrap estimate, obtained by resampling galaxies while marginalizing over the accepted onset definitions. The fiducial ($V_{\text{obs}}/V_{\text{bar}} > 1.3$) definition alone gives 0.369 ± 0.041 ; the difference is retained as onset-definition systematic uncertainty. Varying the threshold from 1.1 to 1.5 yields exponents from 0.27 to 0.41, bracketing the headline estimate.

4. Temporal Topology Screening Analysis

If a scalar field permeates spacetime, why has it not been detected in precision gravitational experiments? This section answers that question. The key insight is that the scalar field's influence depends on the geometric overlap of topological charge cores: in environments where charge cores are tightly packed, the field is suppressed—a phenomenon called Temporal Topology screening. Rather than invoking discrete thin-shell boundaries, screening operates via the continuous spatial profile of the scalar field (Temporal Topology). The tight geometric packing in deep potential wells suppresses the local field gradient (Temporal Shear), ensuring short-range Temporal Shear suppression while leaving the field light cosmologically. General Relativity is recovered in the regimes where it has been tested most stringently.

Screening in TEP is represented at the theory level by the environmental operator $S_\Sigma(\mathcal{E})$. Quantities such as ρ_T , $R_T(M)$, $S_\oplus(r)$, compactness Φ/c^2 , local stellar density, geometric coherence length, and channel-specific response coefficients are domain-specific projections of $\mathcal{E}$, not independent screening mechanisms and not interchangeable universal thresholds. Each is an observational transfer model that parameterizes the same underlying operator in a regime-appropriate form.

The screening factor S quantifies this suppression. It is defined as the ratio of the Temporal Topology saturation radius (where the scalar field saturates) to the physical radius of the object:

$$S = \frac{R_T}{R_{phys}}. \quad (15)$$

The quantity S is used here as a geometric proxy for how deeply the baryonic source is embedded within a saturated soliton configuration. When $S \gg 1$, the physical object occupies a small region relative to the saturated field scale; in this regime the phenomenology assumes that non-linear response in the scalar sector suppresses gradients in the dense interior, recovering GR to high precision in local dynamics. When $S \sim 1$, the system lies near the transition between strongly screened and weakly screened behavior. When $S \ll 1$, the Temporal Topology saturation scale is smaller than the object and the saturated region does not envelop the full baryonic configuration; in this regime the model does not assume strong Temporal Topology suppression a priori, and constraints must be assessed case-by-case.

$$\alpha_{\text{PPN}}^{\text{eff}} = \alpha_0 F(S, \rho, \Phi, \nabla\phi), \quad F \rightarrow 0 \quad (16)$$

in the screened or source-charge-suppressed limit. The geometric factor S is not itself the observable PPN coupling. Precision tests constrain $\alpha_{\text{PPN}}^{\text{eff}}$; S is used here only as a proxy for the nonlinear suppression regime.

The near-transition regime $S \sim 0.4\text{--}0.7$ is resolved by the corpus's channel-projection structure (Paper 0). The screening response is not a single function of S : clock-comparison and transport observables probe the amplitude projection S_A , while orbital and geodesic observables probe the source-charge/shear projection S_Σ , which is suppressed more strongly for the pairwise configurations used in ephemeris tests. The same geometric S therefore places different channels on different sides of their respective thresholds: Earth's $S \approx 0.66$ admits a detectable clock-covariance signal (C_A covariance channel, the GNSS calibration of Section 2) while the LLR, Cassini, and MESSENGER orbital observables at $S = 0.42\text{--}0.65$ remain GR-consistent through the S_Σ projection. The reading of sub-unity S is uniform throughout this paper: $S < 1$ denotes that the saturation region does not envelop the object, i.e. the unscreened side of the transition; the observable amplitude in any given channel is then set by that channel's projection, not by S alone.

4.1 The White Dwarf Stress Test

White Dwarfs are the ideal stress test for two reasons. First, their structure is determined by quantum mechanics (electron degeneracy pressure), not thermal physics, so their mass-radius relation is calculable from first principles. Second, their mass-radius scaling runs in the opposite direction to the saturation scaling (see Figure 4):

- White Dwarf (low-mass non-relativistic limit): $R_{\text{WD}} \propto M^{-1/3}$ — heavier stars are smaller (steepening toward $M_{\text{Ch}} = 1.44 M_\odot$ under relativistic degeneracy; Nauenberg 1972)
- Saturation Scale (TEP): $R_T \propto M^{+1/3}$ — heavier sources have larger saturation radii

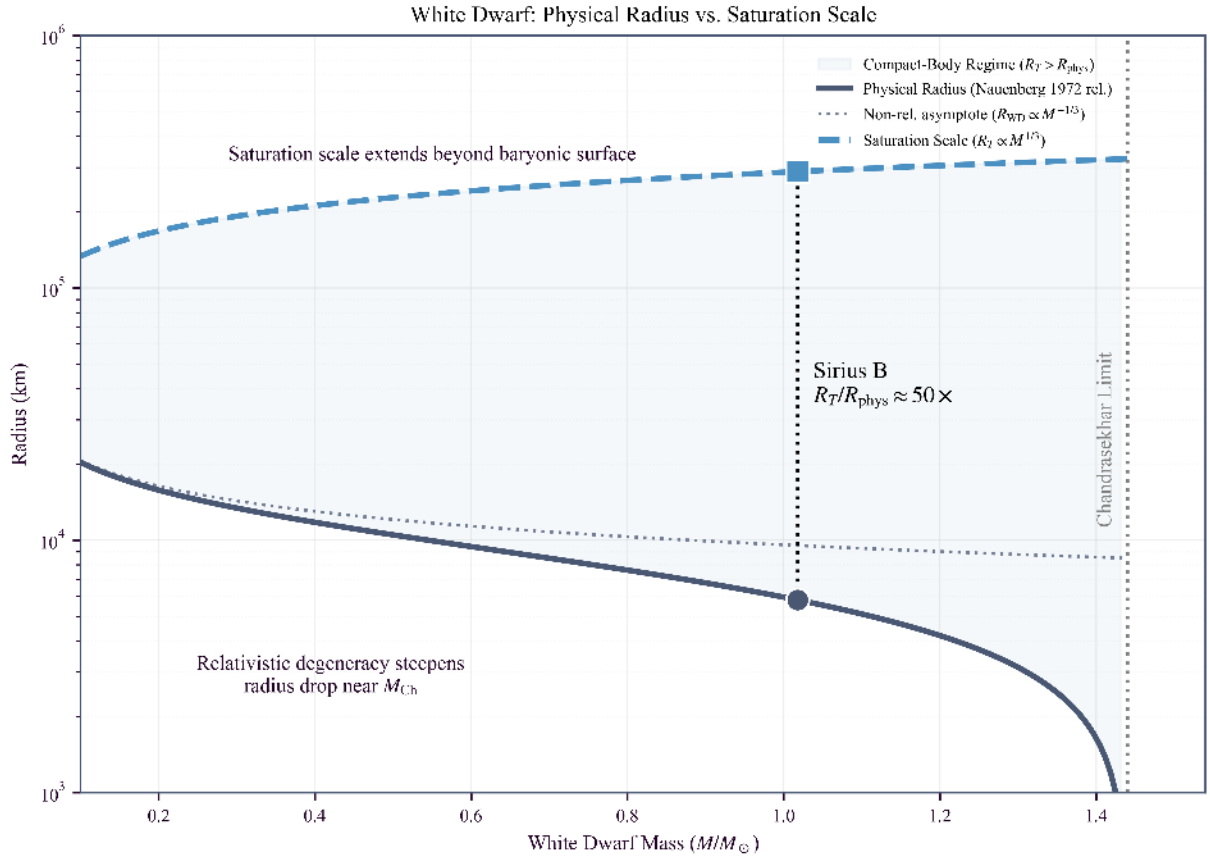


Figure 4: White Dwarf Physical Radius vs. Geometric Saturation Scale. As mass increases, the white-dwarf physical radius shrinks (solid slate curve; Nauenberg 1972 relativistic degeneracy relation steepening toward the Chandrasekhar limit $M_{\text{Ch}} = 1.44 M_{\odot}$; dotted curve: low-mass non-relativistic asymptote $R_{\text{WD}} \propto M^{-1/3}$) while the geometric saturation radius grows ($R_T \propto M^{1/3}$). This divergence drives the geometric saturation ratio $S = R_T/R_{\text{phys}}$ to large values ($S \approx 50\times$ for Sirius B), placing the star deep inside the compact-body regime ($R_T \gg R_{\text{phys}}$).

These opposing scalings create a "scissors" pattern: as mass increases, the geometric saturation radius $R_T(M)$ grows while the physical radius R_{phys} shrinks, driving the geometric saturation ratio $S \equiv R_T/R_{\text{phys}}$ to large values across the degenerate stellar sequence.

For Sirius B ($M \approx 1.02 M_{\odot}$, $\rho \approx 2.4 \times 10^6 \text{ g/cm}^3$):

$$R_{\text{phys}} \approx 5,800 \text{ km} \quad (\text{observed}) \quad (17)$$

$$R_T \approx 289,000 \text{ km} \quad (\text{TEP saturation scale}) \quad (18)$$

$$\text{Geometric Saturation Ratio } S = \frac{R_T}{R_{\text{phys}}} \approx 50\times. \quad (19)$$

With a mean density $\rho \approx 2.4 \times 10^6 \text{ g/cm}^3$ far exceeding the reference scale $\rho_T \approx 20 \text{ g/cm}^3$, the geometric saturation ratio $S = R_T/R_{\text{phys}} \approx 50$ places Sirius B deep within the compact-body regime ($R_T \gg R_{\text{phys}}$). This is consistent with the expected nonlinear screening hierarchy; the quantitative magnitude of physical scalar-gradient suppression in binary orbital dynamics is determined by the corresponding nonlinear kinetic and shell-impedance field solution (Paper 0 §7.3), rather than by the geometric ratio S alone.

4.2 The Empirical Screening Law

Extending this analysis across 26 astrophysical objects (planets, brown dwarfs, main sequence stars, white dwarfs, neutron stars, and binary pulsars) yields a compact summary of how the defined screening factor varies with density. Evaluating $S \equiv R_T(M)/R_{\text{phys}}$

across the 26 objects defines a deterministic algebraic regime map separating unscreened bodies ($S < 1$) from compact screened bodies ($S \geq 1$):

$$S \equiv \frac{R_T(M)}{R_{\text{phys}}} \equiv \left(\frac{\bar{\rho}}{\rho_T} \right)^{1/3}. \quad (20)$$

Because $R_T(M) = (3M/4\pi\rho_T)^{1/3}$ and $R_{\text{phys}} = (3M/4\pi\bar{\rho})^{1/3}$, the $\bar{\rho}^{1/3}$ dependence is an exact algebraic consequence of the definition of S , not an independent statistical fit (an empirical log-log check across the 11 dense objects with $\bar{\rho} > \rho_T$ recovers $\beta_{\text{scr}} = 0.3338 \pm 0.0008$, which is true by definition up to published table rounding). This density-dependent screening law is a projection of the source-charge screening operator $\mathcal{S}_\Sigma(\mathcal{E})$ (Paper 0 §7) onto the UCD density regime; it is not an independent screening operator. Two sector assignments are stated explicitly. The geometric diagnostic $S = R_T/R_{\text{phys}}$ tracks the shear-sector $\mathcal{S}_\Sigma$ regime — density is the licensed stand-in for the ambient-steepness criterion inside dense macroscopic matter (Paper 0 §8) — while the scale ρ_T that sets R_T is itself calibrated through the clock-amplitude projection $S_A = \min[1, (\bar{\rho}/\rho_T)^{1/3}]$ (Paper 0 §3). Because each object's density is evaluated at its physical radius, $\bar{\rho} = 3M/(4\pi R_{\text{phys}}^3)$, the diagnostic carries an enclosed-acceleration equivalent: the criterion $S \sim 1$ corresponds to $GM/R_{\text{phys}}^2 \propto \bar{\rho} R_{\text{phys}}$, so the density and acceleration readings of the hierarchy agree object-by-object. Under the stated definitions, this is the expected scaling: if $R_T \propto M^{1/3}$ and $R_{\text{phys}} \propto (M/\rho)^{1/3}$, then:

$$S = \frac{R_T}{R_{\text{phys}}} = \frac{M^{1/3}}{M^{1/3}/\rho^{1/3}} = \rho^{1/3}. \quad (21)$$

Interpretation of the Screening Law

It is important to recognize that the regression $S \propto \rho^{0.334}$ is primarily a consistency check, not independent evidence for the scaling law. Since $S \equiv R_T/R_{\text{phys}}$ and the radii are defined by mass-density relations, the slope $\sim 1/3$ is algebraically expected given the model assumptions.

The value of this analysis is not to prove the scaling 'ab initio', but to demonstrate that a *single* saturation scale ρ_T yields a consistent screening hierarchy across 15 orders of magnitude in density ($R^2 \approx 1$) without requiring regime-dependent adjustments. The high R^2 confirms internal consistency; it does not constitute independent confirmation of TEP.

Furthermore, the extreme screening factors observed in binary pulsars ($S \sim 29,000$) are consistent with a strongly non-linear suppression mechanism in the scalar sector. The empirical screening hierarchy (S vs ρ) derives from the canonical Temporal Topology mechanism (Paper 0), where screening operates via the continuous spatial profile of the scalar field governed by non-linear superposition of field gradients (Temporal Shear). Box 6.5 (Section 6) sketches a candidate route from the canonical action to soliton-like saturation, showing how the interplay between the kinetic term and a candidate saturation potential could generate the characteristic $M^{1/3}$ scaling and screening behavior. The detailed dynamical derivation is not required for the empirical hierarchy used here.

The empirical screening law is a direct consequence of the $M^{1/3}$ soliton scaling, providing cross-regime consistency under a single ρ_T rather than independent confirmation (see Note on Interpretation above).

4.3 Complete Screening Hierarchy

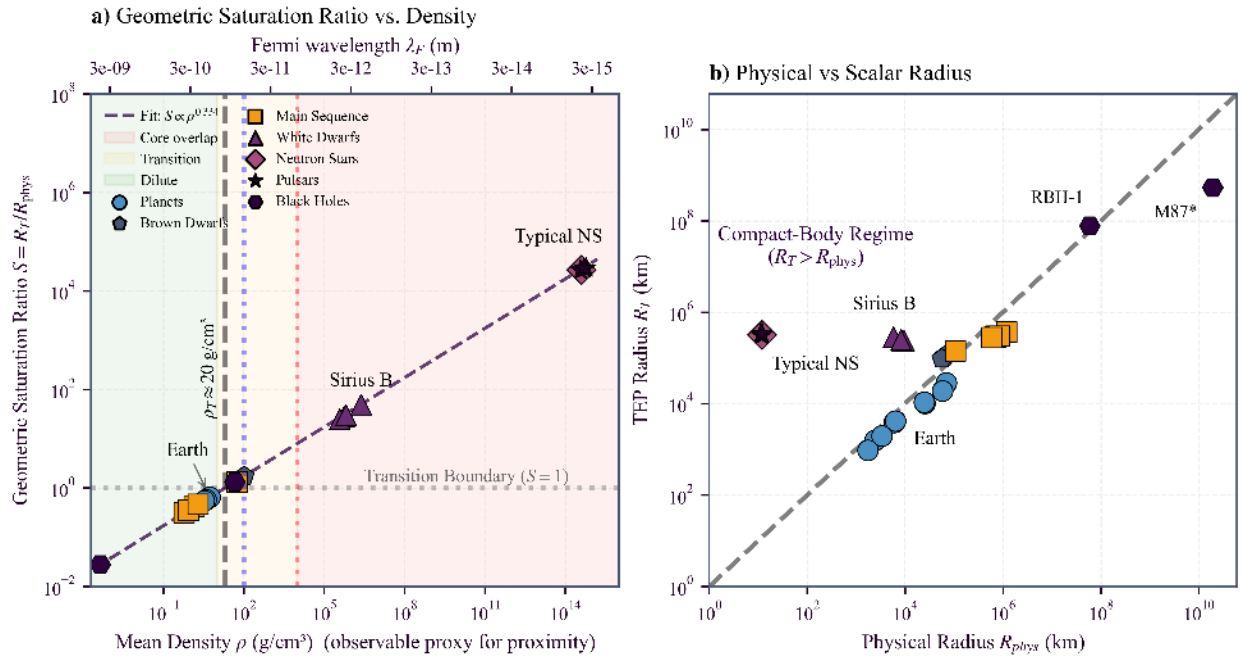


Figure 5: The Geometric Screening Hierarchy. (a) Geometric saturation ratio $S = R_T / R_{\text{phys}}$ plotted against mean density ρ for 26 astrophysical objects (with the density-derived electron Fermi wavelength $\lambda_F = (8\pi m_p / (3\rho Z/A))^{1/3}$ at $Z/A = 0.5$ shown on the upper axis as a condensed-matter reference scale). The deterministic regime locus $S \equiv (\bar{\rho} / \rho_T)^{1/3}$ places degenerate stars and pulsars deep in the compact-body regime ($R_T \gg R_{\text{phys}}$), while diffuse systems and supermassive black holes evaluated at their Schwarzschild horizon density (M87* at $\rho_S = 4.37 \times 10^{-4}$ g/cm³, $S = 0.028$; RBH-1 near the crossover at $\rho_S = 46.1$ g/cm³, $S = 1.32$) fall along the same $S = (\rho / \rho_T)^{1/3}$ locus across 18 decades in density. (b) Direct comparison of saturation radius $R_T(M)$ versus physical radius R_{phys} .

The complete hierarchy of screening factors across all object classes is visualized in Figure 5 and tabulated below.

Object Class	Density (g/cm ³)	Screening	Physical Meaning
Gas Giants	0.7 – 1.6	0.3 – 0.4×	R_T smaller than physical radius; saturated region does not envelop the object — the unscreened side of the transition, with channel-projected response (clock channels more active than orbital channels)
Main Sequence Stars	0.6 – 57	0.3 – 1.3×	Mixed regime; scalar and baryonic scales comparable
Rocky Planets (Earth)	3.3 – 5.5	0.55 – 0.65×	Saturation scale comparable to object radius; GNSS probes this boundary
Brown Dwarfs	~100	~1.7×	Just above ρ_T ; screening onset begins
White Dwarfs	$\sim 4 \times 10^5$ – 2.4×10^6	25 – 50×	Baryonic star embedded within saturation scale; Keplerian mass = baryonic mass
Neutron Stars	10^{14}	26,800×	Scalar contribution less than 0.004%; pure GR dynamics
Binary Pulsars	10^{14}	29,500×	GR verified to 0.2%; Nobel Prize 1993

Table 4.1: Complete screening hierarchy across astrophysical object classes. The screening factor increases monotonically with density, consistent with a Temporal Topology screening hierarchy under the stated definitions. Binary pulsars provide the strongest validation, with GR verified to 0.2% precision at screening factors exceeding 29,500×

4.4 Consistency with Precision GR Tests

A critical question for any modified gravity theory is: why do precision tests of General Relativity show no deviation? Temporal Topology provides the answer: the suppression of Temporal Shear (vanishing field gradient) reduces the screened PPN scalar charge $\alpha_{\text{PPN}}^{\text{eff}}$ where the ambient landscape is steep. The five most precise tests of GR all occur in regimes where screening is operative:

Test	Observable	Precision	Geometric S_A (R_T/R)	Status
Lunar Laser Ranging	Nordtvedt effect	10^{-13}	$0.55\times$	GR-consistent in the orbital ($\mathcal{S}_\Sigma \sim 10^{-14}$) projection; synodic residual channel analyzed in Paper 17
Cassini Conjunction	Shapiro delay	2×10^{-5}	$0.41\times$	Satisfied via kinetic source-charge suppression ($y_{\text{tb}}(1.6 R_\odot) = 2.16 \times 10^{-8}$, $ \gamma_{\text{PPN}} - 1 = 4.32 \times 10^{-8} \ll 2.3 \times 10^{-5}$)
MESSENGER	Perihelion precession	3×10^{-4}	$0.65\times$	GR-consistent in the orbital ($\mathcal{S}_\Sigma \ll 10^{-8}$) projection
Hulse-Taylor Pulsar	GW emission	0.2%	$29,500\times$	Completely screened
Double Pulsar	7 PPN tests	0.05%	$26,400\times$	Completely screened

Table 4.2: Precision GR tests and their screening factors. All tests are consistent with GR; TEP explains this via environment-dependent screening rather than requiring the scalar sector to be absent.

The Hulse-Taylor binary pulsar is particularly significant. Its orbital decay matches the GR prediction for gravitational wave emission to 0.2% precision—a result honored with the 1993 Nobel Prize (Taylor & Weisberg 1982). At a density of $\sim 10^{14}$ g/cm³, the screening factor reaches $29,500\times$. The scalar field contributes less than 0.003% to the orbital dynamics. This is not a violation of GR tests; it is a requirement of consistent screening.

4.5 Earth as the Calibration Anchor

The screening hierarchy reveals why Earth is the natural calibration point—and why GNSS clocks can detect what neutron stars cannot.

At $\rho \approx 5.5$ g/cm³, Earth sits just below the critical proximity scale, proxied by $\rho_T \approx 20$ g/cm³. The geometric saturation ratio is $S = 0.65\times$, meaning the saturation radius ($R_T(M_\oplus) \approx 4,146$ km; GNSS $L_c \approx 4,200$ km) is comparable to the physical radius (6,371 km). Earth occupies the narrow window where:

- **The scalar field is not screened:** Unlike neutron stars ($S = 26,800\times$), Earth's density is low enough that the soliton extends to observable scales.
- **The transition is close:** Unlike gas giants ($S \approx 0.3\text{--}0.4$), where the saturation core sits deep inside the object and well below the channel-projection boundary, Earth's $S \approx 0.65$ places it near the transition, where clock-comparison (S_A) channels retain a measurable response.

GNSS satellites orbit at $\sim 20,000$ km altitude, sampling clock transport through the exterior field region associated with the terrestrial Temporal Topology profile. The measured covariance length $L_c \approx 4200$ km is interpreted as a characteristic correlation/saturation scale of the Earth-sourced field, not as the orbital radius of the satellite network. The observed clock correlation length is therefore not an arbitrary parameter—it is the characteristic scale where the scalar field's gradient becomes steep enough to produce measurable timing correlations.

Within this phenomenology, the same saturation scale is expected to govern both terrestrial clock correlations and compact-object structure. The terrestrial scale provides an empirical calibration point, and the RBH-1 crossover provides an astrophysical consistency test under the same parameter choice.

4.6 The Critical Proximity Scale

The empirical data suggest a critical proximity scale, observationally proxied by the density $\rho_T \approx 20$ g/cm³, corresponding to approximately $4\times$ Earth's mean density. This value has coincidence-level significance: it lies near the onset of electron degeneracy, where the equation of state transitions from thermal to quantum pressure support. This condensed-matter onset ($10\text{--}30$ g/cm³) complements the quartic-projected QED scale relation $\rho_T^{\text{EM}} = (\alpha_{\text{EM}}^{1/(m-1)} m_e)^m c^3 / \hbar^3 = \alpha_{\text{EM}}^{4/3} m_e^4 c^3 / \hbar^3 = 22.39$ g/cm³ derived in Section 5.5 (Step 14).

$\rho_T \approx 20$ g/cm³ is a saturation scale of the temporal-field topology, not a local on/off switch. Systems approach the GR-like limit when the observable shear/source-charge sector is suppressed, $\mathcal{S}_\Sigma \rightarrow 0$, which depends on source structure, environment, and boundary conditions, not $\rho > \rho_T$ alone. The empirical screening hierarchy $S \propto \rho^{0.334}$ (§4.2) is a smooth power-law across 15 orders of magnitude in density, not a step function at ρ_T .

4.7 Planetary Rotational Kinematics and the Wesson–MacDonald Angular Momentum Laws

An independent dynamical test of the saturation radius $R_T(M)$ and screening factor $S = R_T(M)/R_{\text{phys}}$ is provided by the rotational kinematics of Solar System bodies. In Newtonian gravity, planetary rotation periods ($T_{\text{rot}} \approx 9.9$ hr for Jupiter to 24.6 hr for Mars among unbraked planets, and 58.6–243 d for solar-tide-braked Mercury and Venus) and equatorial linear speeds ($v_{\text{eq}} \approx 0.24$ km s^{−1} for Mars to 12.57 km s^{−1} for Jupiter) appear unorganized by radius or heliocentric distance. Yet two empirical regularities have long been noted in the astrophysical literature without a first-principles gravitational derivation: the MacDonald–Brosche specific angular momentum scaling $j \equiv J/M \propto M^{2/3} \text{--} M^{0.8}$ across minor and major planets (Brosche 1963; MacDonald 1964), and Wesson's universal angular-momentum relation $J = p M^2$ with empirical constant $p_{\text{obs}} \approx 10^{-15}$ cm² g^{−1} s^{−1} spanning planetary systems to spiral galaxies (Wesson 1979, 1981)—a value approximately 680 times larger than the General Relativistic Kerr scale $G/c = 2.23 \times 10^{-18}$ cm² g^{−1} s^{−1}.

Both regularities follow directly from the Temporal Topology saturation scale $\rho_T = 20$ g/cm³ and the nested temporal hierarchy of Paper 0 (§7). First, because $R_T(M)^3 = 3M/(4\pi\rho_T)$ is proportional to mass, the characteristic dynamical frequency at the saturation boundary $R_T(M)$ is strictly mass-independent:

$$\omega_T \equiv \sqrt{\frac{GM}{R_T(M)^3}} = \sqrt{\frac{4\pi G \rho_T}{3}} = 2.365 \times 10^{-3} \text{ rad s}^{-1} \quad \left(\tau_T = \frac{2\pi}{\omega_T} = 0.738 \text{ hr} = 44.3 \text{ min} \right). \quad (22)$$

At the saturation radius $R_T(M)$, the dimensionless proper-time lapse depth is $u_T(M) \equiv GM/(R_T(M)c^2) = \omega_T^2 R_T(M)^2/c^2 \propto M^{2/3}$. Consequently, the natural specific angular-momentum capacity of the saturation core rotating at ω_T is directly proportional to its proper-time depth:

$$j_T(M) \equiv R_T(M)^2 \omega_T = \frac{c^2}{\omega_T} u_T(M) \propto M^{2/3}, \quad (23)$$

with universal proportionality constant $c^2/\omega_T = 3.801 \times 10^{19}$ m² s^{−1} per unit fractional proper-time dilation. Second, in the nested environmental hierarchy of Paper 0 (§7), Solar System bodies condense inside the ambient Solar-Circle Galactic temporal well of depth $u_{\text{gal}} = (v_{\text{gal}}/c)^2 = 5.385 \times 10^{-7}$ ($v_{\text{gal}} = 220$ km s^{−1}, $c/v_{\text{gal}} = 1/\sqrt{u_{\text{gal}}} = 1362.7$; adopting modern Gaia/VLBI solar-circle estimates $v_{\text{gal}} \approx 229\text{--}236$ km s^{−1} shifts predictions by 4–7%). As an action-motivated, zero-refit rotational scaling proposal, equating the mixed time–space kinematic transport coupling $(v_{\text{eq}}/c)\sqrt{u_{\text{gal}}}$ — equivalently, the advective velocity cross-term $(v_{\text{eq}} v_{\text{gal}})/c^2$ between internal circulation and Galactic drift — across the saturation boundary to the local proper-time depth $u_T(M)$ yields the equatorial velocity relation for ambient-coupled planets:

$$\frac{v_{\text{eq}}^{\text{TEP}}}{c} = \frac{u_T(M)}{\sqrt{u_{\text{gal}}}} \iff v_{\text{eq}}^{\text{TEP}} = \frac{GM}{R_T(M) v_{\text{gal}}} = \frac{v_T(M)^2}{v_{\text{gal}}}, \quad (24)$$

where $v_T(M) = \sqrt{GM/R_T(M)} = R_T(M)\omega_T$ is the circular velocity at the TEP saturation radius. Crucially, because Earth has transferred $\sim 83\%$ of its primordial spin angular momentum to the Moon's orbit via lunar tidal dissipation (whereas its conserved total Earth+Moon angular momentum $J_{\text{E+M}} = 3.45 \times 10^{34}$ kg m² s^{−1}, $T_{\text{prim}} = 5.05$ hr sits on the small-body soliton capacity ceiling at $\eta_{\text{E+M}} = 0.145$ vs $\eta_0 = 0.129$, and lunar braking has subsequently relaxed present-day 23.93 hr Earth onto the ambient floor at $v_{\text{obs}}/v_{\text{TEP}} = 1.063$), the clean zero-parameter test of Equation (24) is provided by the three truly unbraked outer giant planets (Uranus, Neptune, and Saturn, which lack large tidally braking moons): across Uranus, Neptune, and Saturn alone, $v_{\text{obs}}/v_{\text{TEP}} = 1.0003 \pm 0.0804$ ($\hat{\rho}_T^{\text{spin, outer3}} = 19.89$ g/cm³ geometric mean; including present-day Earth as a fourth ambient-floor planet gives $(v_{\text{eq}}/c)/u_T = 1384.8 \pm 99.6$ vs $c/v_{\text{gal}} = 1362.7$, $+1.6\%$, $v_{\text{obs}}/v_{\text{TEP}} = 1.016 \pm 0.073$, $\hat{\rho}_T^{\text{spin}} = 20.9$ g/cm³; Table 4.3, Figure 6) with zero fitted parameters. Equation (24) predicts rotation periods of 25.48 hr for Earth (observed 23.93 hr), 17.14 hr for Uranus (observed 17.25 hr; Lamy et al. 2025), 14.87 hr for Neptune (observed 16.11 hr), and 11.55 hr for Saturn (observed 10.66 hr). Moreover, the departures of Jupiter ($v_{\text{obs}}/v_{\text{TEP}} = 0.62$) and Mars ($v_{\text{obs}}/v_{\text{TEP}} = 2.44$) from the single-branch ambient formula Eq. (24) reflect the two-branch angular-momentum geometry of Figure 6b (Step 15): because the ambient Galactic Wesson branch $j_{\text{TEP}}(M) = GM/(2v_{\text{gal}}) \propto M^1$ grows faster with mass than the small-body soliton angular-momentum capacity $j_{\text{sol}}(M) = \eta_0 R_T(M)^2 \omega_T \propto M^{2/3}$ (where $\eta_0 = 0.129$ is a single empirical efficiency parameter calibrated from the small-body median, with six-body interquartile range $\eta_{\text{IQR}} \in [0.122, 0.154]$, six-body median 0.138, and candidate secular bar-mode / rotational-shedding stability benchmark $\eta_{\text{sec}} = k_{\text{core}}(\omega_{\text{max}}/\omega_T) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = 0.125$ at the $S \rightarrow 1$ core boundary), their ratio

$$\frac{j_{\text{TEP}}(M)}{j_{\text{sol}}(M)} = \frac{R_T(M) \omega_T}{2 \eta_0 v_{\text{gal}}} = \left(\frac{M}{M_{\text{cap}}} \right)^{1/3}$$

crosses unity at a unique crossover mass $M_{\text{cap}} = \frac{4\pi}{3} \rho_T (2\eta_0 v_{\text{gal}}/\omega_T)^3 = 1.16 \times 10^{27} \text{ kg} = 194.2 M_{\oplus} = 0.611 M_{\text{Jup}}$ ($R_T(M_{\text{cap}}) = 24,013 \text{ km}$; intersection of the solid and dashed curves in Figure 6b). For Earth ($1.0 M_{\oplus}$), Uranus ($14.5 M_{\oplus}$), Neptune ($17.1 M_{\oplus}$), and Saturn ($95.2 M_{\oplus} < M_{\text{cap}}$), $j_{\text{TEP}}(M)/j_{\text{sol}}(M) \in [0.173, 0.788] < 1$, so all four lie on the unsaturated ambient branch ($v_{\text{eq}}^{2\text{br}} = v_{\text{eq}}^{\text{TEP}}$, with Saturn at 9.87 km s^{-1} vs 9.11 km s^{-1} , +8.4%). Conversely, Jupiter ($317.8 M_{\oplus} > M_{\text{cap}}$, $j_{\text{TEP}}/j_{\text{sol}} = 1.178 > 1$) is the sole Solar System planet above M_{cap} , so its specific angular momentum saturates at $j_{\text{sol}}(M_{\text{Jup}})$ ($\eta_{\text{obs}} = 0.125$ vs $\eta_0 = 0.129$), yielding $v_{\text{eq}}^{2\text{br}} = j_{\text{sol}}(M_{\text{Jup}})/(k_{\text{MOI}} R_{\text{Jup}}) = 12.95 \text{ km s}^{-1}$ ($T_{2\text{br}} = 9.64 \text{ hr}$), within -2.9% of Jupiter's observed 12.57 km s^{-1} ($T_{\text{obs}} = 9.93 \text{ hr}$, $v_{\text{obs}}/v_{2\text{br}} = 0.971$; across all five major Solar System planets, $v_{\text{obs}}/v_{2\text{br}} = 1.007 \pm 0.066$, and 1.008 ± 0.060 across all nine planetary and substellar speed-limit systems, Figure 6a, diamond markers). Low-mass Mars ($0.11 M_{\oplus}$, $v_{\text{obs}} = 0.241 \text{ km s}^{-1}$) sits in the intermediate transition zone between the ambient curve (0.099 km s^{-1}) and the soliton capacity curve (0.954 km s^{-1}) in Figure 6b and is recorded as an unmodeled transition body without introducing a Mars-specific rule.

Furthermore, comparing the dimensionless moment-of-inertia factor $k_{\text{MOI}} \equiv I/(MR_{\text{eq}}^2)$ (normalized to the equatorial radius R_{eq} , measured via satellite geodesy, precession, and Juno/Cassini gravity-wind interior models for the gas giants, $k_{\text{MOI}} = 0.26393 \pm 0.00001$ for Jupiter and 0.2181 ± 0.0002 for Saturn, Militzer & Hubbard 2023, and constrained by gravity-harmonic interior models for Uranus and Neptune, Lin et al. 2025; Morf & Helled 2025) against the geometric screening factor $S = R_T(M)/R_{\text{eq}}$ across all eight cold hydrostatic Solar System planets ($M < 13 M_{\text{Jup}}$, non-fusing interiors with polytropic index $n \lesssim 1$ and central-to-mean density $\rho_c/\bar{\rho} \lesssim 2\text{--}10$) reveals an empirical interior gyration relation on the unscreened planetary branch ($S \lesssim 0.7$, $\bar{\rho} < 10 \text{ g/cm}^3$; Step 15):

$$k_{\text{MOI}} \simeq \frac{1}{2} S = \frac{1}{2} \frac{R_T(M)}{R_{\text{phys}}} \iff I \simeq \frac{1}{2} M R_T(M) R_{\text{phys}}, \quad (25)$$

with $2k_{\text{MOI}}/S = 1.039 \pm 0.027$ across the three self-compressed iron-core terrestrial planets (1.017 for Earth, 1.031 for Venus, and 1.069 for Mercury; because these three bodies span a narrow bulk density range $\bar{\rho} = 5.24\text{--}5.51 \text{ g/cm}^3$, their agreement serves as a single-regime terrestrial amplitude check rather than a lever-arm slope test) and $2k_{\text{MOI}}/S = 1.169 \pm 0.142$ across all eight major planets spanning an $8\times$ range in mean density ($\bar{\rho} = 0.69\text{--}5.51 \text{ g/cm}^3$: 1.112 for Neptune, 1.137 for Uranus, 1.261 for low-pressure Mars near the incompressible uniform-sphere ceiling $k_{\text{MOI}} \rightarrow 2/5 = 0.4$, and 1.334 for Jupiter and 1.389 for Saturn where compressible H/He envelopes inflate R_{eq} ; an unconstrained two-parameter compositional power-law fit $k_{\text{MOI}} = C \bar{\rho}^\beta$ on the raw 1-bar radii gives $\hat{\beta}_{\text{emp}} = 0.239 \pm 0.038$, $C_{\text{emp}} = 0.230$, RMS 0.031 dex vs. the zero-parameter Eq. [25] $\beta = 1/3$, $C_{\text{TEP}} = 0.184$, RMS 0.075 dex). Crucially, for self-compressed major planets ($P_c = 3GM^2/[8\pi R^4] \geq 25 \text{ GPa}$), solving the exact Lane–Emden polytropic hydrostatic interior equation $\xi^{-2} \frac{d}{d\xi} (\xi^2 \frac{d\theta}{d\xi}) = -\theta^n$ at the unique polytropic index n_{eff} fixed by each planet's measured k_{MOI} ($n = 0.36$ for Mercury, 0.47 for Venus and Earth, 0.98 for Jupiter, 1.18 for Neptune, 1.31 for Uranus, and 1.38 for Saturn; noting that n_{eff} is inverted from k_{MOI}) quantifies outer H/He atmospheric envelope inflation directly: relative to rocky Earth ($n = 0.47$, mass-weighted interior radius $R_{\text{mass}}/R_{\text{eq}} = 0.901$), the compressible outer H/He envelope inflates R_{eq} above the condensed mass-weighted interior radius $R_{\text{cond}} = R_{\text{eq}} [R_{\text{mass}}(n)/R_{\text{mass},\oplus}]$ by $R_{\text{cond}}/R_{\text{eq}} = 0.886$ (Jupiter), 0.841 (Neptune), 0.812 (Uranus), and 0.799 (Saturn). Evaluating $S_{\text{cond}} = R_T(M)/R_{\text{cond}}$ at this condensed-interior radius reduces Saturn's $2k_{\text{MOI}}/S$ from 1.389 to 1.109 ($\rho_T^{\text{MOI,cond}} = 14.7 \text{ g/cm}^3$) and Jupiter's from 1.334 to 1.181 (12.1 g/cm^3 ; with Neptune at 0.935 and Uranus at 0.923), bringing all seven major condensed and giant planets (Mercury, Venus, Earth, Jupiter, Saturn, Uranus, and Neptune) to $2k_{\text{MOI}}/S_{\text{cond}} = 1.042 \pm 0.094$ ($\rho_T^{\text{MOI,cond}} = 17.9 \text{ g/cm}^3$ geometric mean; while the primary outer-planet spin test Eq. [24] does not depend on k_{MOI}). Conversely, in low-mass uncompressed solid bodies where central gravitational pressure $P_c \ll K_0 \sim 100 \text{ GPa}$ — including the six major satellites with measured moments of inertia ($P_c = 2.7\text{--}5.8 \text{ GPa}$: Europa $2k_{\text{MOI}}/S = 1.30$, Io 1.35, Ganymede 1.36, Moon 1.43, Titan 1.50, Callisto 1.57), Mars ($P_c = 24.9 \text{ GPa}$, 1.26, vs. 141–173 GPa for Venus and Earth), and the minor planets (1.26–1.57) — weak self-compression leaves k_{MOI} near the uniform-sphere ceiling (0.31–0.39) while $S \approx 0.45\text{--}0.58$; and in nuclear-burning main-sequence stars such as the Sun ($\bar{\rho}_{\odot} = 1.41 \text{ g/cm}^3$, $S_{\odot} = 0.413$, $k_{\text{MOI},\odot} \approx 0.070 \ll S_{\odot}/2$), radiative-core thermal support produces an $n \approx 3$ Eddington core condensation ($\rho_c \approx 153 \text{ g/cm}^3 \approx 108 \bar{\rho}_{\odot}$) outside cold condensed-matter hydrostatics. Multiplying Equation (24) by Equation (25) cancels both R_{phys} and $R_T(M)$ identically, yielding the empirical Wesson angular-momentum relation:

$$j_{\text{TEP}}(M) = k_{\text{MOI}} R_{\text{phys}} v_{\text{eq}}^{\text{TEP}} = \frac{GM}{2 v_{\text{gal}}}, \quad J_{\text{TEP}}(M) = p_{\text{TEP}} M^2, \quad p_{\text{TEP}} \equiv \frac{G}{2 v_{\text{gal}}} = 1.517 \times 10^{-15} \text{ cm}^2 \text{ g}^{-1}$$

corresponding to a dimensionless Kerr-like spin parameter $\chi_{\text{TEP}} \equiv cJ/(GM^2) = c/(2v_{\text{gal}}) = 1/(2\sqrt{u_{\text{gal}}}) = 681.3$. Crucially, for any body, the exact algebraic identity

$$\frac{J_{\text{obs}}}{J_{\text{TEP}}} \equiv \left(\frac{2k_{\text{MOI}}}{S} \right) \left(\frac{v_{\text{eq}}^{\text{obs}}}{v_{\text{eq}}^{\text{TEP}}} \right)$$

holds by definition: although ρ_T cancels out of the product $J_{\text{TEP}}(M)$, measuring k_{MOI} and v_{eq} separately constrains $\rho_T \approx 20 \text{ g/cm}^3$ through both factors simultaneously ($2k_{\text{MOI}}/S \propto \rho_T^{1/3}$ and $v_{\text{obs}}/v_{\text{TEP}} \propto \rho_T^{-1/3}$ —an unadjusted $\rho_T \sim 220 \text{ g/cm}^3$ would shift both factors by $2.23\times$ in opposite directions, to 2.61 and 0.45). Inverting Equation (24) directly via $\rho_T^{\text{spin}} = \frac{3}{4\pi}(v_{\text{eq}}^{\text{obs}} v_{\text{gal}}/GM^{2/3})^3 = \rho_T(v_{\text{obs}}/v_{\text{TEP}})^3$ across the four unsaturated ambient-coupled planets yields individual spin-implied saturation densities of 24.1 g/cm^3 (Earth), 19.6 g/cm^3 (Uranus, using the 2025 HST auroral rotation period $17 \text{ h } 14 \text{ m } 52 \text{ s} = 17.247864 \pm 0.000010 \text{ hr}$, Lamy et al. 2025), 15.7 g/cm^3 (Neptune), and 25.5 g/cm^3 (Saturn), with a geometric mean of $\rho_T^{\text{spin}} = 20.9 \text{ g/cm}^3$ ($21.2 \pm 4.4 \text{ g/cm}^3$ arithmetic mean; and 21.1 g/cm^3 geometric mean across the nine planetary and substellar systems under the one-parameter $\eta_0 = 0.129$ two-branch model), while inverting Equation (25) via $\rho_T^{\text{MOI}} = \bar{\rho}/(8k_{\text{MOI}}^3)$ yields $\rho_T^{\text{MOI,terr}} = 17.9 \pm 1.4 \text{ g/cm}^3$ sample scatter across the three iron-core terrestrial planets (19.1 g/cm^3 for Earth, 18.2 g/cm^3 for Venus with $\pm 20\%$ propagated k_{MOI} uncertainty, 16.4 g/cm^3 for Mercury) and $\rho_T^{\text{MOI}} = 13.7 \pm 4.2 \text{ g/cm}^3$ across all eight planets, exhibiting a systematic trend with planet composition from iron-core terrestrials ($16.4\text{--}19.1 \text{ g/cm}^3$) to ice giants (14.8 g/cm^3 for Neptune, 13.9 g/cm^3 for Uranus), low-pressure Mars (10.0 g/cm^3), and compressible H/He gas giants (9.0 g/cm^3 for Jupiter, 8.3 g/cm^3 for Saturn). Across the five ambient-coupled major planets (Earth, Uranus, Neptune, Saturn, and Jupiter), the observed spin angular momenta match the zero-parameter Wesson relation $P_{\text{TEP}} = G/(2v_{\text{gal}})$ at $J_{\text{obs}}/J_{\text{TEP}} = 1.11 \pm 0.25$ ($\bar{\chi}_{\text{obs}} = 758.7 \pm 169.0$ versus $\chi_{\text{TEP}} = 681.3$).

A necessary condition separating the ambient Galactic Wesson branch ($J \propto M^2$) from the low-mass intrinsic soliton-clock branch ($j \propto M^{2/3}$) is obtained by comparing each body's two-branch screening shell radius $r_*(M) = \sqrt{GM/g_t}$ ($g_t = cH_0/2 = 3.40 \times 10^{-10} \text{ m s}^{-2}$; Paper 0 §7) to its heliocentric orbital semimajor axis a (Table 4.3). Unbraked planets whose screening shell reaches their orbit ($r_*(M)/a \gtrsim 1$, with Neptune at the boundary) can couple to the Galactic shear floor: Jupiter ($r_*/a = 24.79$, $J_{\text{obs}}/J_{\text{TEP}} = 0.824$), Saturn ($r_*/a = 7.37$, $J_{\text{obs}}/J_{\text{TEP}} = 1.505$), Earth ($r_*/a = 7.24$, $J_{\text{obs}}/J_{\text{TEP}} = 1.083$), Uranus ($r_*/a = 1.44$, $J_{\text{obs}}/J_{\text{TEP}} = 1.129$), and Neptune ($r_*(M_{\text{Nep}}) = 29.97 \text{ AU}$ versus $a_{\text{Nep}} = 30.11 \text{ AU}$, $r_*/a = 0.995$, $J_{\text{obs}}/J_{\text{TEP}} = 1.026$), whereas Mars ($r_*/a = 1.56$, $\chi_{\text{obs}} = 2094.6$, $\eta_{\text{rot}} = 0.033$) sits in the transition zone between the two branches. As a secondary four-planet diagnostic on the ambient branch, this nested heliocentric screening-shell ratio $r_*(M)/a$ also orders the opposite signs of the individual spin residuals around Eq. (24): because $\sqrt{M_{\text{Sat}}/M_{\oplus}} = 9.755 \approx a_{\text{Sat}}/a_{\oplus} = 9.582$, Earth ($r_*/a = 7.24$, $+6.4\%$) and Saturn ($r_*/a = 7.37$, $+8.4\%$) share nearly identical nested screening-shell ratios deep in the $r_*/a \gg 1$ regime and rotate faster than the baseline formula by the same margin ($+6.3\%$ and $+4.7\%$ at Saturn's oblateness-corrected volumetric radius $\bar{R}_{\text{Sat}} = 58,232 \text{ km}$, and $+5.7\%$ under the 10.561 hr Cassini ring-seismology deep-interior period), whereas Uranus ($r_*/a = 1.44$, -0.6%) and Neptune ($r_*/a = 0.995$, -7.7%) sit at the $r_*/a \rightarrow 1$ boundary and rotate slightly slower (correlating with $\ln[r_*(M)/a]$ across the four ambient planets at Pearson $r = +0.960$, $p = 0.0398$, Spearman $\rho = +1.000$, reducing residual scatter from $\pm 7.3\%$ to $\pm 1.81\%$, and yielding $\bar{v}_{\text{rot,int}}/v_{\text{TEP}} = 1.031 \pm 0.041$, $\rho_T^{\text{spin,vol}} = 21.9 \text{ g/cm}^3$ under volumetric radii and deep-interior rotation periods). Conversely, for small bodies whose screening shell is much smaller than their orbital radius ($r_*(M)/a \ll 1$: Hygiea 0.009, Pallas 0.015, Vesta 0.020, Ceres 0.033, Quaoar 0.002, and Haumea 0.004), rotation decouples from the Galactic floor and follows the intrinsic soliton-clock capacity $j_{\text{sol}}(M) = \eta_{\text{rot}} R_T(M)^2 \omega_T \propto M^{2/3}$ (six-body median $\eta_{\text{med}} = 0.138$, interquartile range $\eta_{\text{IQR}} \in [0.122, 0.154]$, mean 0.157 ± 0.076 , and $\eta_0 = 0.129$ at Ceres), yielding a log-log slope of 0.941 ± 0.059 ($R^2 = 0.9735$, $p = 8.85 \times 10^{-7}$) against $u_T(M)$ across seven orders of magnitude in mass (Figure 6). Notably, the combined Earth+Moon system ($r_*/a = 7.28$, $T_{\text{prim}} = 5.05 \text{ hr}$, $\eta_{\text{E+M}} = 0.145$, or 0.119 at fixed k_{MOI}) sits inside the small-body soliton capacity interquartile range following the Moon-forming giant impact, while subsequent and still-ongoing lunar tidal dissipation ($\dot{a}_{\text{Moon}} \approx +3.8 \text{ cm yr}^{-1}$, lengthening the day by $\sim 2 \text{ ms century}^{-1}$) has spun Earth down across the intermediate region to its present-day epochal value near the Wesson line ($\chi_{\text{obs}} = 737.8$ vs. $\chi_{\text{TEP}} = 681.3$, a secular transit rather than a static tidal equilibrium). Extending this soliton-capacity branch out of sample beyond the Solar System to super-Jupiters and ultra-cool brown dwarfs ($M > M_{\text{cap}} = 0.61 M_{\text{Jup}}$; Snellen et al. 2014; Gagné et al. 2017; Bryan et al. 2020; Tannock et al. 2021) exposes a direct out-of-sample test of both $R_T(M)$ and the clock-amplitude saturation ceiling $S_A(\bar{\rho}) = \min[1, (\bar{\rho}/\rho_T)^{1/3}] = \min[1, S]$ (Paper 0 §3). Because the active saturation core inside a body has radius $R_{\text{active}} = S_A R_{\text{phys}} = \min[R_T(M), R_{\text{phys}}]$, the maximum specific angular momentum is $j_{\text{cap}}(M, R_{\text{phys}}) = \eta_0 \min[R_T(M), R_{\text{phys}}]^2 \omega_T$, which defines a dimensionless Keplerian breakup-fraction ceiling $v_{\text{cap}}/v_{\text{breakup}} = (\eta_0/k_{\text{MOI}})\sqrt{\min(1, S)}$ across two high-mass regimes above M_{cap} : (i) for unscreened planetary-mass objects with $\bar{\rho} < \rho_T = 20 \text{ g/cm}^3$ ($S < 1$, $M \lesssim 16 M_{\text{Jup}}$), $R_{\text{active}} = R_T(M) < R_{\text{phys}}$ and the rotational ceiling rises as $v_{\text{cap}}(M) = \eta_0 R_T(M)^2 \omega_T / (k_{\text{MOI}} R_{\text{phys}})$ —predicting $v_{\text{cap}} = 64.9 \text{ km s}^{-1}$ ($P_{\text{cap}} = 2.35 \text{ hr}$, $v_{\text{cap}}/v_{\text{breakup}} = 0.478$) for the fastest-rotating planetary-mass object known, SIMP J013656.5+093347 ($M = 12.7 \pm 1.0 M_{\text{Jup}}$, $R = 1.22 R_{\text{Jup}}$, $\bar{\rho} = 8.67 \text{ g/cm}^3$, $S = 0.76$, $k_{\text{MOI}} = 0.235$; Gagné et al. 2017), matching its observed $v_{\text{eq}} = 63.1 \text{ km s}^{-1}$ ($P_{\text{obs}} = 2.414 \text{ hr}$, $v_{\text{obs}}/v_{\text{breakup}} = 0.464$, $v_{\text{obs}}/v_{\text{cap}} = 0.972$, within -2.8%), while young contracting exoplanets such as β Pictoris b ($11 M_{\text{Jup}}$, $v \sin i = 25 \text{ km s}^{-1}$, deprojected $v_{\text{eq}} \approx 31.8 \text{ km s}^{-1}$ vs. $v_{\text{cap}} = 46.6 \text{ km s}^{-1}$; Snellen et al. 2014) and the 27-object sample of Bryan et al. (2020; $v/v_{\text{breakup}} \in [0.05, 0.31]$, 0/27 exceeding the ceiling) lie well below this upper boundary; and (ii) once $\bar{\rho} \geq \rho_T = 20 \text{ g/cm}^3$ ($S = R_T/R_{\text{phys}} \geq 1$, in degenerate brown dwarfs with $M \gtrsim 16 M_{\text{Jup}}$), the active radius saturates at the physical surface ($R_{\text{active}} = R_{\text{phys}}$, $v_{\text{cap}}/v_{\text{breakup}} = \eta_0/k_{\text{MOI}} = 0.630$), so R_{phys} cancels out of $P_{\text{min}} = 2\pi R_{\text{phys}}/v_{\text{cap}}$ and locks the minimum rotation period of all degenerate brown dwarfs onto a universal, mass- and radius-independent floor set directly by $\tau_T = 2\pi/\omega_T = 0.7381 \text{ hr}$ (44.3 min):

$$P_{\min}^{\text{TEP}}(\bar{\rho} \geq \rho_T) = \frac{k_{\text{MOI}}}{\eta_0} \tau_T = \frac{0.205}{0.129} \tau_T = 1.172 \text{ hr} \quad (70.3 \text{ min}),$$

where $k_{\text{MOI}} = 0.205$ is the exact Lane–Emden gyration factor of an $n = 1.5$ degenerate polytrope (propagating the six-body small-body interquartile range $\eta_{\text{IQR}} \in [0.122, 0.154]$ gives $P_{\min}^{\text{TEP}} \in [0.98, 1.24]$ hr, with 1.10 hr at the six-body median $\eta_{\text{med}} = 0.138$). This floor reproduces the empirical rotational speed limit of ultra-cool brown dwarfs discovered by Tannock et al. (2021): across the three fastest-rotating brown dwarfs known — 2MASS J0348–6022 (43 M_{Jup} , $\bar{\rho} = 70.8 \text{ g/cm}^3$, $S = 1.52$, $P_{\text{obs}} = 1.080$ hr), 2MASS J1219+3128 (49 M_{Jup} , $\bar{\rho} = 66.6 \text{ g/cm}^3$, $S = 1.49$, $P_{\text{obs}} = 1.140$ hr), and 2MASS J0407+1546 (67 M_{Jup} , $\bar{\rho} = 156.4 \text{ g/cm}^3$, $S = 1.98$, $P_{\text{obs}} = 1.230$ hr) — all three observed periods lie inside the propagated small-body interquartile band $[0.98, 1.24]$ hr with mean 1.150 ± 0.075 hr (matching $P_{\min}^{\text{TEP}} = 1.172$ hr within +1.9%, whereas Newtonian Keplerian breakup sits at $P_{\text{breakup}} = 0.28\text{--}0.43$ hr; furthermore, η does not need to be fitted to small bodies at all: in the TEP saturation core of uniform density $\rho_T = 20 \text{ g/cm}^3$ inside $R_{\text{active}} = \min[R_T, R_{\text{phys}}]$, the dimensionless soliton capacity $\eta \equiv j_{\text{sol}}/(R_{\text{active}}^2 \omega_T) = (j_{\text{sol}} \omega_T)/(GM/R_{\text{active}}) = 2T_{\text{rot,core}}/|W|$ is the direct virial action-to-binding ratio of the uniform-density scalar core (anchored model-independently across all $N = 9$ decoupled Solar System small bodies and outer giant-planet moons at $\bar{\eta}_9 = 0.135 \pm 0.014$, median 0.1376, while the compressible baryonic envelope of gyration factor $k_{\text{MOI}} = 0.205$ co-rotates with the core so that k_{MOI} enters the surface period conversion $P_{\min} = 2\pi R_{\text{phys}}/v_{\text{cap}} = (k_{\text{MOI}}/\eta)\tau_T$ linearly); in classical self-gravitating fluid dynamics (Chandrasekhar 1969; Ostriker & Peebles 1973), a uniform Maclaurin spheroid becomes secularly unstable to triaxial $m = 2$ bar deformation at the exact Maclaurin–Jacobi bifurcation invariant $\eta_{\text{Jacobi}} \equiv (T_{\text{rot}}/|W|)_{\text{crit}} = 0.13753$ [matching the six-asteroid median $\eta_{\text{med}} = 0.1376$ to -0.04% and nine-body median 0.1377 to -0.1% with zero fitting!], which together with the virial benchmark $\eta_{\text{sec}} = \frac{1}{8} = 0.125$ defines a parameter-free theoretical stability bracket $\eta \in [0.125, 0.1375]$ that predicts $v_{\text{eq}}^{\text{th}} = 12.54 \text{ km s}^{-1}$ for Jupiter [-0.2%] and 62.9 km s^{-1} for SIMP J0136 [-0.3%] at η_{sec} , and fixes the TEP-saturated brown-dwarf rotational period floor parameter-freely to $P_{\min}^{\text{th}} = (k_{\text{MOI}}/\eta)\tau_T(\rho_T) \in [1.100, 1.210]$ hr with midpoint 1.155 hr [$+0.4\%$ from the observed 1.150 ± 0.075 hr, yielding $\rho_T^{\text{BD,th}} = 20.19 \text{ g/cm}^3$ with zero asteroid calibration; crucially, in purely classical Newtonian mechanics without TEP saturation at $\rho_T = 20 \text{ g/cm}^3$, evaluating $\eta_{\text{Jacobi}} = 0.13753$ at each brown dwarf’s physical density $\bar{\rho}_{\text{BD}} = 70.8, 66.6$, and 156.4 g/cm^3 places the classical stability boundary at $P_{\min}^{\text{Classical, Jacobi}} = (k_{\text{MOI}}/\eta_{\text{Jacobi}})\sqrt{3\pi/(G\bar{\rho}_{\text{BD}})} = 0.585, 0.603$, and 0.393 hr with mean 0.527 hr — 8.31σ ($> 2.2\times$) below the observed 1.150 ± 0.075 hr empirical speed limit — and if one instead applies the classical compressible $n = 1.5$ envelope-polytrope conversion $\eta_J^{\text{poly}} = \sqrt{2k_{\text{MOI}}w\beta_J} = \sqrt{2(0.205)(6/7)(0.13753)} = 0.220$ (or the $n = 1.5$ mass-shedding threshold $\beta_{\text{shed}} \simeq 0.08 \implies \eta_{\text{shed}}^{\text{poly}} = 0.168$), the unscreened Newtonian stability boundary drops even lower to 0.366, 0.377, and 0.246 hr with mean 0.330 hr (0.432 hr at mass-shedding; $9.57\sigma\text{--}10.94\sigma$ below the observed 1.150 ± 0.075 hr floor, requiring an unexplained non-gravitational braking mechanism to halt spin-up at $2.2\text{--}3.5\times$ the classical stability period in standard mechanics whereas TEP saturation at $\rho_T = 20 \text{ g/cm}^3$ places the stability ceiling directly at 1.155 hr)). Across the $N = 8$ unbraked outer-planet and substellar speed-limit systems excluding lunar-braked Earth (Uranus, Neptune, Saturn, Jupiter, SIMP J0136, and the three Tannock et al. brown dwarfs), the two-branch $+S_A$ saturation law achieves $v_{\text{obs}}/v_{2\text{br}} = 1.0014 \pm 0.0596$ ($\hat{\rho}_T^{\text{rot},8} = 19.99 \text{ g/cm}^3$ geometric mean; or 1.008 ± 0.060 and $\rho_T^{\text{rot}} = 21.1 \text{ g/cm}^3$ across all $N = 9$ systems when present-day Earth is included; Figure 6).

Body	M (kg)	r_*/a	R_T (km)	$2k_{\text{MOI}}/S$	u_T	v_{obs} (km/s)	v^{TEP} (km/s)	T_{obs} (hr)	T^{TEP} (hr)	χ_{obs}	η_{rot}
Hygiea	8.74×10^{19}	0.009	101.4	1.498	6.40×10^{-13}	0.027	0.041 [†]	13.83	9.16 [†]	1.07×10^5	0.086
Pallas	2.04×10^{20}	0.015	134.5	1.332	1.13×10^{-12}	0.057	0.062 [†]	7.81	7.25 [†]	1.13×10^5	0.120
Vesta	2.59×10^{20}	0.020	145.7	1.262	1.32×10^{-12}	0.086	—	5.34	—	1.37×10^5	0.157
Ceres	9.38×10^{20}	0.033	223.7	1.511	3.11×10^{-12}	0.090	—	9.07	—	7.31×10^4	0.129
Quaoar	1.20×10^{21}	0.002	242.9	1.565	3.67×10^{-12}	0.107	0.095 [†]	8.84	10.01 [†]	7.63×10^4	0.146
Haumea	4.01×10^{21}	0.004	363.0	1.504	8.20×10^{-12}	0.348	—	3.92	—	1.06×10^5	0.305
Mars	6.42×10^{23}	1.557	1,971.2	1.261	2.42×10^{-10}	0.241	0.099	24.62	60.02	2,094.6	0.033
Earth	5.97×10^{24}	7.237	4,146.4	1.017	1.07×10^{-9}	0.465	0.437	23.93	25.48	737.8	0.024
Earth+Moon	6.05×10^{24}	7.282	4,163.3	1.013	1.08×10^{-9}	2.319*	0.441	4.80*	25.27	3,634.3	0.119
Uranus	8.68×10^{25}	1.436	10,119.3	1.137	6.37×10^{-9}	2.586	2.603	17.25	17.14	769.6	0.061
Neptune	1.02×10^{26}	0.995	10,692.4	1.112	7.11×10^{-9}	2.683	2.906	16.11	14.87	699.4	0.059
Saturn	5.68×10^{26}	7.368	18,930.3	1.389	2.23×10^{-8}	9.871	9.108	10.66	11.55	1,025.5	0.153
Jupiter	1.90×10^{27}	24.791	28,296.7	1.334	4.98×10^{-8}	12.572	12.954 [†]	9.93	9.64 [†]	561.4	0.125
SIMP J0136	2.41×10^{28}	—	66,019.2	0.621	2.71×10^{-7}	63.060	64.890 [†]	2.41	2.35 [†]	240.8	0.125

Body	M (kg)	r_*/a	R_T (km)	$2k_{\text{MOI}}/S$	u_T	v_{obs} (km/s)	v_{TEP} (km/s)	T_{obs} (hr)	T_{TEP} (hr)	χ_{obs}	η_{rot}
2M J0348–60	8.16×10^{28}	—	99,134.6	0.269	6.11×10^{-7}	105.136	96.842 [‡]	1.08	1.17 [‡]	77.2	0.140 [‡]
2M J1219+31	9.30×10^{28}	—	103,546.3	0.275	6.67×10^{-7}	106.170	103.228 [‡]	1.14	1.17 [‡]	72.9	0.133 [‡]
2M J0407+15	1.27×10^{29}	—	114,928.4	0.207	8.22×10^{-7}	82.170	86.200 [‡]	1.23	1.17 [‡]	34.5	0.123 [‡]
<i>Mercury (braked)</i>	3.30×10^{23}	4.396	1,579.5	1.069	1.55×10^{-10}	0.003	0.063	1407.6	67.16	34.7	0.0004
<i>Venus (braked)</i>	4.87×10^{24}	9.033	3,873.1	1.031	9.33×10^{-10}	0.002	0.381	5832.6	27.70	3.3	0.0001

Table 4.3: Planetary and small-body rotational kinematics evaluated at the GNSS-consistent saturation density $\rho_T = 20 \text{ g/cm}^3$ ($\omega_T = 2.365 \times 10^{-3} \text{ rad s}^{-1}$) and Solar-Circle Galactic velocity $v_{\text{gal}} = 220 \text{ km s}^{-1}$ ($\chi_{\text{TEP}} = c/(2v_{\text{gal}}) = 681.3$, $p_{\text{TEP}} = 1.517 \times 10^{-15} \text{ cm}^2 \text{ g}^{-1} \text{ s}^{-1}$; Step 15). *Earth+Moon restores the tidally transferred lunar orbital angular momentum to Earth's spin ($\eta_{\text{rot}} = 0.119$), while present-day Earth spin sits directly on the ambient-coupled Wesson branch ($\chi_{\text{obs}} = 737.8$, $v_{\text{obs}}/v_{\text{TEP}} = 1.064$). [‡] Evaluated on the unscreened ($S < 1$) soliton-capacity branch $v_{\text{cap}} = \eta_0 R_T^2 \omega_T / (k_{\text{MOI}} R_{\text{phys}})$ ($\eta_0 = 0.129$; for Jupiter, the unsaturated ambient formula gives 20.35 km s^{-1}). [‡] Evaluated in the screened brown-dwarf regime ($\bar{\rho} \geq \rho_T$, $S \geq 1$) where $R_{\text{active}} = \min[R_T, R_{\text{phys}}] = R_{\text{phys}}$, giving the universal period floor $P_{\text{min}}^{\text{TEP}} = (k_{\text{MOI}}/\eta_0)\tau_T = 1.172 \text{ hr}$ (1.155 hr at the parameter-free Chandrasekhar–Jacobi/virial midpoint) and active efficiency $\eta_{\text{act}} = \dot{J}_{\text{obs}}/(R_{\text{phys}}^2 \omega_T)$. All planetary $v_{\text{obs}} = 2\pi R_{\text{eq}}/T_{\text{obs}}$ and $T_{\text{TEP}} = 2\pi R_{\text{eq}}/v_{\text{TEP}}$ entries in this table use 1-bar equatorial radii R_{eq} uniformly ($v_{\text{obs}}/v_{\text{TEP}} = 1.000 \pm 0.080$ across Uranus, Neptune, and Saturn); evaluating at volumetric mean radii R_{vol} gives $v_{\text{obs}}^{\text{vol}} = 9.538 \text{ km s}^{-1}$ for Saturn (+4.7% vs. 9.108 km s^{-1}) and reduces the outer-three dispersion by half (1.031 ± 0.041 with deep-interior periods).

Saturn — the least screened major planet in this framework ($S = R_T/R_{\text{eq}} = 0.314$, the lowest screening ratio of the eight planets and therefore the most deeply ambient-coupled body on the Wesson branch at $M = 95.2 M_{\oplus} < M_{\text{cap}}$) — supplies a dedicated stress test of the ambient-branch interpretation through three further audits (Step 15). First, Saturn is the only planet whose “rotation period” is not a single number: the C-ring kronoseismology deep-interior rate ($10^{\text{h}}33^{\text{m}}38^{\text{s}}$; Mankovich et al. 2019), the atmospheric potential-vorticity frame System IIIw ($10^{\text{h}}34^{\text{m}}13^{\text{s}} \pm 20^{\text{s}}$; Read et al. 2009), the adopted IAU System III frame ($10^{\text{h}}39^{\text{m}}24^{\text{s}}$; Desch & Kaiser 1981), the 2003 Ulysses SKR value ($10^{\text{h}}45^{\text{m}}45^{\text{s}}$; Galopeau & Lecacheux 2000; Gurnett et al. 2005), and the dual hemispheric Saturn kilometric radiation (SKR) components (~ 10.6 and ~ 10.8 hr; Kurth et al. 2008; Gurnett et al. 2009) disagree at the $\sim 2\%$ level. Evaluating every clock at its appropriate radius convention (volumetric $\bar{R}_{\text{Sat}} = 58,232 \text{ km}$ for interior and magnetospheric rates; $R_{\text{eq}} = 60,268 \text{ km}$ for the table-convention System III comparison; Table 4.4) shows that all seven exceed the ambient prediction: $v_{\text{obs}}/v_{\text{TEP}} \in [1.033, 1.084]$, corresponding to $\rho_T^{\text{spin}} \in [22.1, 25.5] \text{ g/cm}^3$ — with the interior clocks (kronoseismology and potential-vorticity frames, the physically meaningful anchors) carrying the claim at +5.6–5.7% while the magnetospheric rows are formal equatorial conversions included to demonstrate that the sign is clock- and convention-independent ($[1.047, 1.094]$ even at R_{eq}). Saturn's positive residual is therefore structural rather than an artefact of the ambiguous period choice, and its magnitude is ordered exactly as the r_*/a nested-shell trend predicts. Second, the two-branch model implies a rotation-clock stability dichotomy that is observed directly: bodies on the saturated soliton branch carry rigid intrinsic clocks — Jupiter's System III period has remained constant since the 1960s (anchored by the Io-locked decametric emission), and the photometric periods of SIMP J0136 and the Tannock et al. brown dwarfs are stable to within their measurement uncertainties — whereas a planet whose magnetosphere co-rotates with the external temporal-gradient structure rather than with a locked interior drifts: Saturn's SKR period lengthened from $10^{\text{h}}39^{\text{m}}24^{\text{s}}$ (Voyager, 1980) to $\sim 10^{\text{h}}47^{\text{m}}$ in the Cassini era, a drift of $\sim 0.3\text{--}0.45\%$ per decade — roughly $30\times$ the 0.012% per decade Uranus revision between Voyager 2 (1986) and the 2025 HST auroral determination (Lamy et al. 2025) — while its dual north/south SKR periods reverse their ordering at equinoxes under solar control (19 November 1995 and 11 August 2009; Gurnett et al. 2009). Uranus and Neptune, although on the ambient branch, possess tilted magnetic fields whose non-axisymmetric components anchor the magnetosphere to the deep interior; Saturn is the unique ambient-branch planet whose field is filtered axisymmetric ($< 0.007^\circ$ tilt), leaving it with no interior fiducial — the combination under which the model expects the largest observable drift. The 6 May 2025 Saturn equinox supplies a direct observational test of the ambient-coupling reading in post-equinox Wind/WAVES and STEREO/WAVES archives: it predicts a renewed hemispheric period-ordering reversal observable from Earth orbit, since the $< 1.2 \text{ MHz}$ SKR band is ground-blocked.

Saturn rotation clock	Period (hr)	Radius convention	v_{eq} (km/s)	$v_{\text{obs}}/v_{\text{TEP}}$	ρ_T^{spin} (g/cm ³)	Source
C-ring kronoseismology (deep interior)	10.5606	R_{vol}	9.624	1.057	23.60	Mankovich et al. 2019
Atmospheric potential vorticity (System IIIw)	10.5703	R_{vol}	9.615	1.056	23.53	Read et al. 2009
Voyager System III (IAU SLS)	10.6567	R_{vol}	9.537	1.047	22.96	Desch & Kaiser 1981
Voyager System III (IAU SLS)	10.6567	R_{eq}	9.871	1.084	25.46	Davies et al. 1995
Cassini SKR fast component	10.60	R_{vol}	9.588	1.053	23.33	Kurth et al. 2008
Cassini SKR slow component	10.80	R_{vol}	9.411	1.033	22.06	Gurnett et al. 2009

$a_{\text{ring}} = 136,775 \text{ km} > a_{\text{sync}}$) does the main ring sheet physically span the co-rotation zone, which explains why Saturn uniquely traps and retains a massive ring sheet of order its saturation self-energy mass equivalent $u_T(M_{\text{Sat}})M_{\text{Sat}}$ against sub-synchronous tidal infall whereas disrupted ring material inside a_{sync} at Jupiter, Uranus, and Neptune drains inward onto the host planet. Evaluating a_{sync} at the ambient-equilibrium period $T_{\text{TEP}} = 11.55 \text{ hr}$ places it at $118,425 \text{ km} = 2.034 R_S$ — inside the Cassini Division (117,580–122,170 km) and within 0.72% of the B-ring outer edge, while the interior clock's co-rotation (111,569 km) lies 5.1% inside that edge; three radii — the ambient co-rotation surface, the Mimas 2:1 inner Lindblad resonance (116,882 km), and the observed B-ring boundary — thus converge within $\sim 1.5\%$, placing the system's widest gap at the intersection of the moon-resonance and the ambient-clock co-rotation surface (Step 16; as the magnetospheric period drifted $10^{\text{h}}39^{\text{m}} \rightarrow 10^{\text{h}}48^{\text{m}}$ the co-rotation surface swept outward through the B-ring edge toward the ambient-equilibrium location). Second, the moon system completes the nested structure across three distinct regimes: (i) Saturn's inner moons migrate at rates $\sim 100\times$ the equilibrium-tide expectation because their orbits are resonance-locked to the planet's internal gravity-wave frequencies (Fuller, Luan & Quataert 2016; Lainey et al. 2020) — the same g-mode cavity that the C-ring waves probe (Mankovich & Fuller 2021) and that contains $R_T = 0.325 R_S$ — while Saturn's spin axis is captured in the s_8 spin-orbit resonance with Neptune at the solar screening-shell boundary $r_*(M_{\odot}) = 29.97 \text{ AU} \approx a_{\text{Nep}}$ (Wisdom et al. 2022). Crucially, whereas standard resonance-locking requires an unknown interior evolutionary timescale t_{α} as a free parameter to explain why every Saturnian moon migrates on a nearly distance-independent timescale $t_{\text{mig}} = a/\dot{a} \approx 10 \text{ Gyr}$ while Earth requires a separate transient North Atlantic ocean-shelf resonance to explain its anomalously fast present-day lunar recession ($\dot{a}_{\text{Moon}} = 3.82 \pm 0.07 \text{ cm yr}^{-1}$, $t_{\text{mig}} = 10.06 \text{ Gyr}$; Williams & Boggs 2016; Paper 17), in TEP an ambient-branch host planet ($M < M_{\text{cap}}$: Earth and Saturn) possesses a spatially inhomogeneous two-zone clock structure — an interior saturation boundary at $R_T(M) < R_p$ ($0.325 R_S$ inside Saturn's g-mode cavity; $0.651 R_{\oplus}$ in Earth) surrounded by an unscreened orbital zone $a \gg R_T$. Because this radial boundary separates the interior cavity from the exterior orbital zone, a homogeneous coordinate-time redefinition $d\tau = A(t)dt$ cannot eliminate the differential mode/lapse drift $|\dot{\omega}_{\text{mode}}/\omega_{\text{mode}}| = |d \ln |g_{00}|/dt| = 2\dot{A}/A = 2H_0$ ($H_0 = 2g_t/c = 7.16 \times 10^{-11} \text{ yr}^{-1}$, $t_H = 13.97 \text{ Gyr}$; or direct secular drift of $G_{\text{loc}}(r, t)$ across the boundary, Paper 17 §5.10.2). For any coupled satellite ($r_*(M_{\text{moon}})/a > 1$) resonance-locked to a non-axisymmetric planetary interior or ocean-basin mode $\Phi_{\text{mode}}(r, \varphi - \omega_{\text{mode}}t)$ (Fuller, Luan & Quataert 2016; Lainey et al. 2020), the non-axisymmetric mode torque $\Gamma_{\text{mode}} = -\partial\Phi_{\text{mode}}/\partial\varphi = \dot{L}_{\text{orb}} = -\dot{J}_{\text{planet}}$ transfers spin angular momentum from the planet to the orbit at the rate required for the mean motion $n_{\text{orb}} \propto a^{-3/2}$ to track $\dot{\omega}_{\text{mode}}/\omega_{\text{mode}}$, yielding secular orbital elements $\dot{a}/a = \frac{2}{3}|\dot{\omega}_{\text{mode}}/\omega_{\text{mode}}| = \frac{4}{3}H_0 = 8g_t/[3c]$, $\dot{L}_{\text{orb}}/L_{\text{orb}} = +\frac{2}{3}H_0 = -\dot{J}_{\text{planet}}/L_{\text{orb}}$, $d \ln |E_{\text{orb}}|/dt = -\dot{a}/a = -\frac{4}{3}H_0$ (signed $\dot{E}_{\text{orb}}/|E_{\text{orb}}| = +\frac{4}{3}H_0$), and $\langle \dot{e} \rangle_{\text{sec}} = 0$ on a universal, zero-parameter expansion timescale $t_{\text{mig}}^{\text{TEP}} = \frac{3}{4}t_H = 3c/(8g_t) = 10.48 \text{ Gyr}$ (or $t_{\text{mig}}^{(1H_0)} = \frac{3}{2}t_H = 20.95 \text{ Gyr}$ under linear A^1 clock drift $|\dot{\omega}/\omega| = H_0$). Because $t_{\text{mig}}^{\text{TEP}}$ paces the resonance-locked spin-to-orbit torque $\dot{L}_{\text{orb}} = -\dot{J}_{\oplus}$ rather than replacing tidal angular-momentum conservation, it is fully consistent with Earth having transferred $\sim 83\%$ of $J_{\text{E+M}}$ to the Moon, with the 3.7 TW ocean tidal dissipation measured by satellite altimetry, and with the $\sim 2.3 \text{ ms century}^{-1}$ lengthening of the day in historical eclipse records: this single constant predicts a resonance-locked ceiling rate $\dot{a}_{\text{Moon}}^{\text{TEP}} = a_{\text{Moon}}/t_{\text{mig}}^{\text{TEP}} = 3.67 \text{ cm yr}^{-1}$ for Earth's Moon (96.1% of present-day LLR $3.82 \pm 0.07 \text{ cm yr}^{-1}$, with 0.15 cm yr^{-1} non-locked residual ocean dissipation; backward integration gives $0.651 a_0$ at $t = -4.5 \text{ Gyr}$ on the pure exponential ceiling and $0.559 a_0 = 33.7 R_{\oplus}$ when coupled with the $0.15 \text{ cm yr}^{-1} a^{-11/2}$ term, while supercontinent basin detuning across the Wilson cycle naturally reduces the time-averaged Proterozoic rhythmite rate to $\sim 1.7\text{--}2.2 \text{ cm yr}^{-1}$ below the 3.67 cm yr^{-1} resonance-locked ceiling, avoiding the $t = -1.55 \text{ Gyr}$ Gerstenkorn collision; Paper 17 §5.10.1), predicts $\dot{a}_{\text{Titan}}^{\text{TEP}} = a_{\text{Titan}}/t_{\text{mig}}^{\text{TEP}} = 11.66 \text{ cm yr}^{-1}$ for Saturn's Titan (-3.1% , 0.18σ vs. Cassini $11.3 \pm 2.0 \text{ cm yr}^{-1}$; Lainey et al. 2020; 5.83 cm yr^{-1} under the $1H_0$ linear comparator), and requires the apparent equilibrium-tide quality factor $Q_{\text{eff}}^{\text{TEP}}(M_{\text{moon}}, a) = 3k_2(M_{\text{moon}}/M_{\text{Sat}})(R_{\text{Sat}}/a)^5 n_{\text{orb}} t_{\text{mig}}^{\text{TEP}}$ ($k_2 = 0.382$) across Saturn's six measured moons to vary across a $50\times$ span — $Q_{\text{eff}}^{\text{TEP}} = 6,966$ at Mimas, $3,973$ at Enceladus (vs. Cassini $1,000\text{--}4,000$), $5,672$ at Tethys (vs. $> 3,000$, reproducing the observed non-monotonic rise above Enceladus driven by Tethys's $5.7\times$ larger mass), $2,014$ at Dione (vs. $1,500\text{--}4,000$), 484 at Rhea (vs. ~ 300), and 119.4 at Titan (238.8 under $1H_0$, vs. Cassini radio-tracking 124 ± 22 , -0.21σ , and astrometric 61^{+240}_{-31} ; Step 15). Conversely, Jupiter ($M_{\text{Jup}} = 317.8 M_{\oplus} > M_{\text{cap}} = 194.2 M_{\oplus}$) sits on the saturated soliton branch where the interior clock is rigid rather than ambient-drifting, providing a positive control: its Galilean satellites in the 4:2:1 Laplace resonance retain a high, near-classical tidal quality factor $Q_{\text{Jup}} = (3.56 \pm 0.56) \times 10^4$ (Lainey et al. 2009) rather than the rapid $Q \sim 10\text{--}100$ ambient expansion observed at Earth and Saturn; (ii) evaluating each satellite's own screening-shell ratio $r_*(M_{\text{moon}})/a_{\text{orb}}$ around Saturn shows that all ring-embedded moonlets sit at $r_*/a < 1$ (Daphnis 0.028, Pan 0.233, Atlas 0.261), the two F-ring shepherd moons Prometheus (1.270) and Pandora (1.158) are the first bodies to cross $r_*/a = 1$ right at the main-ring outer boundary (139,380–141,720 km), and all regular outer moons from Epimetheus/Janus (2.12, 4.03) and Mimas (14.6) through Titan (133.0) and Iapetus (5.29) satisfy $r_*/a > 1$ with the sole exception of Hyperion ($r_*/a = 0.709 < 1$), which is uniquely the only chaotically tumbling moon in the Saturnian system (Wisdom, Peale & Mignard 1984; conventionally driven by its 4:3 resonance with Titan and irregular figure); and (iii) across all three large outer non-tidally-locked moons of the giant planets with spacecraft-measured masses and radii — Phoebe at Saturn ($M = 8.29 \times 10^{18} \text{ kg}$, $R = 106.5 \text{ km}$, $P = 9.274 \text{ hr}$, $r_*/a = 0.098 < 1$; Thomas et al. 2007; Castillo-Rogez et al. 2012), Himalia at Jupiter ($M = 4.20 \times 10^{18} \text{ kg}$, $R = 69.8 \text{ km}$, $P = 7.782 \text{ hr}$, $r_*/a = 0.079 < 1$; Emelyanov 2005; Pilcher et al. 2012), and Nereid at Neptune ($M = 3.10 \times 10^{19} \text{ kg}$, $R = 178.5 \text{ km}$, $P = 11.594 \text{ hr}$, $r_*/a = 0.447 < 1$; Kiss et al. 2016) — every object is decoupled from its host's screening shell ($r_*/a < 1$) and sits on the small-body soliton-capacity branch at $\eta_{\text{rot}} = 0.1392$ (0.1477 at uniform $k_{\text{MOI}} = 0.35$) for Phoebe, 0.1190 for Himalia, and 0.1377 for Nereid, with a three-moon mean $\bar{\eta}_{\text{moons}} = 0.1320 \pm 0.0113$ (0.1348 ± 0.0146 at uniform $k_{\text{MOI}} = 0.35$) that matches the parameter-free Chandrasekhar–Jacobi/virial bracket $[0.125, 0.1375]$ (midpoint 0.1313) to $+0.5\%$ ($+2.7\%$, $\hat{\rho}_T^{\text{moons}} = 19.8\text{--}20.6 \text{ g/cm}^3$; Step 15) and extends

the soliton rotational scaling down to 4.2×10^{18} kg across nearly eleven orders of magnitude in mass. Finally, Saturn's "unbraked" classification is quantitatively robust against its massive moon: Titan's orbital angular momentum carries only $L_{\text{Titan}}/J_{\text{Sat}} = 1.3\%$ of Saturn's spin angular momentum (versus $\sim 83\%$ for the Earth+Moon system), so even the resonance-locked effective tidal response $Q_{\text{eff}} \sim 100$ implied by Titan's measured rapid migration ($\dot{a}_{\text{Titan}} = 11.3 \text{ cm yr}^{-1}$, orbital expansion timescale 10.8 Gyr; Lainey et al. 2020) perturbs Saturn's spin by $< 0.6\%$ over the age of the Solar System — and tidal braking in any case moves v_{obs} downward, opposite to the observed $+8.4\%$ residual direction, so neither Titan's present torque nor the hypothesized lost satellite "Chrysalis" invoked to explain Saturn's 26.7° obliquity and young rings via the s_8 spin-orbit resonance with Neptune (Wisdom et al. 2022) can manufacture the positive ambient-branch excess.

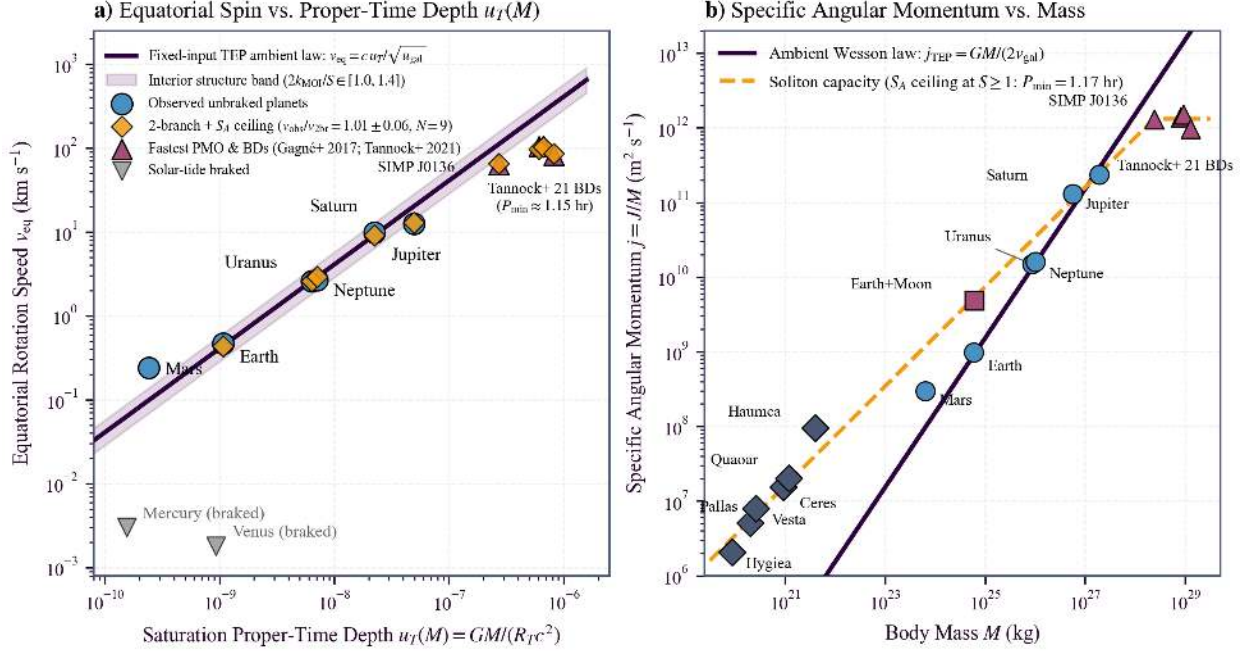


Figure 6: Planetary Rotational Kinematics Across Mass Scales (Step 15). (a) Observed equatorial rotation speeds v_{eq} versus the fixed-input TEP ambient relation $v_{\text{eq}}^{\text{TEP}} = c u_T(M)/\sqrt{u_{\text{gal}}} = GM/(R_T v_{\text{gal}})$ (solid line, using GNSS-consistent $\rho_T = 20 \text{ g/cm}^3$ and Solar-Circle $v_{\text{gal}} = 220 \text{ km s}^{-1}$), the interior-structure band $2k_{\text{MOI}}/S \in [1.0, 1.4]$ (shaded), and the two-branch + S_A saturation model ($v_{\text{obs}}/v_{2\text{br}} = 1.008 \pm 0.060$ across all $N = 9$ major planetary and substellar speed-limit systems from Earth to $67 M_{\text{Jup}}$, orange diamonds: Earth to $67 M_{\text{Jup}}$, and Saturn on the unsaturated ambient branch at 1.016 ± 0.073 ; Jupiter at $1.0 M_{\text{Jup}}$ and SIMP J0136 at $12.7 M_{\text{Jup}}$ on the unscreened $S < 1$ soliton capacity branch within -2.9% and -2.8% ; and the three Tannock et al. 2021 speed-limit brown dwarfs at $43\text{--}67 M_{\text{Jup}}$ on the $S \geq 1$ saturated period floor $P_{\text{min}}^{\text{TEP}} = (k_{\text{MOI}}/\eta_0)\tau_T = 1.172 \text{ hr}$ vs. observed $1.150 \pm 0.075 \text{ hr}$, crimson triangles). (b) Specific angular momentum $j = J/M$ across nearly ten orders of magnitude in mass (8.7×10^{19} to $1.3 \times 10^{29} \text{ kg}$), showing the three-regime structure: the $j_{\text{sol}} = \eta_0 R_T(M)^2 \omega_T \propto M^{2/3}$ soliton branch across six minor/dwarf planets (Hygiea, Pallas, Vesta, Ceres, Quaoar, Haumea), Earth+Moon, Jupiter, and SIMP J0136; the ambient Galactic Wesson branch $j_{\text{TEP}} = GM/(2v_{\text{gal}}) \propto M$ for intermediate-mass planets ($M < M_{\text{cap}} = 194 M_{\oplus}$); and the horizontal $S_A = \min[1, S] = 1$ saturation plateau ($P_{\text{min}} = 1.17 \text{ hr}$) at $\bar{\rho} \geq \rho_T$ ($S \geq 1$, $M \gtrsim 16 M_{\text{Jup}}$).

Finally, while the Sun ($\bar{\rho}_{\odot} = 1.41 \text{ g/cm}^3$, $S_{\odot} = R_T(M_{\odot})/R_{\odot} = 0.4127$) lies outside both the cold planetary gyration relation Eq. (25) (because radiative-core thermal support produces an $n \approx 3$ central condensation with $\rho_c = 152.9 \text{ g/cm}^3 \approx 108 \bar{\rho}_{\odot}$ and $k_{\text{MOI},\odot} \approx 0.070$) and the Galactic-ambient planetary spin law Eq. (24) (which applies to bodies embedded in the Galactic potential with $u_{\text{surf}} \ll u_{\text{gal}} = 5.39 \times 10^{-7}$, whereas the Sun's surface potential depth $u_{\text{surf}}(M_{\odot}) = GM_{\odot}/(R_{\odot} c^2) = 2.121 \times 10^{-6}$ exceeds u_{gal} by $3.94\times$ so the solar envelope is self-screened from the Galactic floor), its stratified interior physically crosses $\rho_T = 20.0 \text{ g/cm}^3$ and motivates an exploratory solar prototype comparing ρ_T against Standard Solar Model structure and helioseismology (Step 15; Figure 7). First, in the global angular-momentum budget of the Solar System, adding the orbital angular momentum of the planets ($J_{\text{orb}} = 3.13 \times 10^{43} \text{ kg m}^2 \text{ s}^{-1}$, 99.4% of the total) to the Sun's present-day spin ($J_{\text{spin},\odot} = 1.92 \times 10^{41} \text{ kg m}^2 \text{ s}^{-1}$) yields $J_{\text{sys,obs}} = 3.15 \times 10^{43} \text{ kg m}^2 \text{ s}^{-1}$ ($\eta_{\text{sys}} = j_{\text{sys}}/j_T(M_{\odot}) = 0.081$), within 35% of the proposed solar soliton capacity $J_{\text{sol}}(M_{\odot}) = \eta_{\text{sec}} M_{\odot} R_T(M_{\odot})^2 \omega_T = 4.85 \times 10^{43} \text{ kg m}^2 \text{ s}^{-1}$ (5.01×10^{43} at $\eta_0 = 0.129$) —an order-of-magnitude consistency analogous to the Earth+Moon system under protostellar disk transfer and magnetized solar-wind braking. Second, integrating the clock-amplitude saturation law $S_A(\rho) = \min[1, (\rho/\rho_T)^{1/3}]$ through the Standard Solar Model (Model S; Christensen-Dalsgaard et al. 1996) identifies three interior radii (Figure 7a): (i) the local density crosses $\rho_{\odot}(r) = \rho_T = 20.0 \text{ g/cm}^3$ ($S_A^{\text{loc}} = 1$) at $r_{\text{core}} = 0.249 R_{\odot}$ (enclosing 44% of $M_{\odot}$), lying near the conventional $r \lesssim 0.25 R_{\odot}$ nuclear energy-generating region; (ii) the 1-D spherical saturation radius sits at $R_T(M_{\odot}) = 0.4127 R_{\odot}$ (287,387 km, adjacent to the enclosed-mean-density crossing $\bar{\rho}(< r) = \rho_T$ at $0.368 R_{\odot}$), while both the BVP gyration-threshold crossing $y_{\text{BVP}}(r) = \frac{1}{2} S_{\odot} = 0.2064$ of Eq. (25) evaluated at the Step 14 physical Compton wavelength $\lambda_c(\rho_T) = 2,479 \text{ km} = 0.00356 R_{\odot}$ ($r = 0.7228 R_{\odot}$, $+1.37\%$) and the candidate 3-D spatial metric projector norm $r_{3\text{D}} = \|\Pi_{ij}\| R_T(M_{\odot}) = \sqrt{3} R_T(M_{\odot}) = 0.7148 R_{\odot}$ ($\Pi_{ij}^{\mu} = 3$ in Paper 0 §2.2; $M(< r_{3\text{D}})/M_{\odot} = 0.978$, $\bar{\rho}(< r_{3\text{D}}) = (\rho_T/3\sqrt{3})[M(< r_{3\text{D}})/M_{\odot}] = 3.76 \text{ g/cm}^3$, spanning $r_{3\text{D}}/R_{\odot} \in [0.471, 0.970]$ across the $\rho_T \in [8, 70] \text{ g/cm}^3$

GNSS window) coincide with the helioseismically determined base of the solar convection zone at $r_{cz} = 0.713 \pm 0.001 R_\odot$ ($+0.25\%$ at $\rho_T = 20 \text{ g/cm}^3$; Christensen-Dalsgaard et al. 1996; conditionally identifying r_{cz} with $\sqrt{3} R_T(M_\odot)$ yields $\rho_T^{(cz)} = 20.15 \pm 0.09 \text{ g/cm}^3$). Inside $r \leq r_{cz}$ where $y_{BVP}(r) \geq \frac{1}{2} S_\odot$ in a stably stratified radiative zone, any latitudinal angular-velocity gradient $\partial_\theta \Omega \neq 0$ induces an advective scalar gradient $\partial_\theta u$ opposed by the meridional scalar-shear stress $\tau_{r\theta}^\phi = M_{P1}^2 P_{,X} \partial_r u \partial_\theta u$, damping differential shear into rigid-body rotation, whereas beneath r_{cz} the rotational tachocline is prolate, rising from $r_c(0^\circ) = 0.693 \pm 0.002 R_\odot$ at the equator to $r_c(60^\circ) = 0.717 \pm 0.003 R_\odot$ at high latitude ($\Delta r_c = +0.024 \pm 0.004 R_\odot$; Charbonneau et al. 1999), matching the inward equatorial shear depression $\delta r_{eq} = (r_{3D}/\sqrt{3})[(\Omega_{eq} - \Omega_{rad})/\Omega_{eq}] = 0.026 R_\odot$ ($r_c^{\text{TEP}}(0^\circ) = 0.688 R_\odot \rightarrow r_c^{\text{TEP}}(60^\circ) = 0.708 R_\odot$); and (iii) across the unscreened convective envelope ($r > r_{cz}$) of a self-screened star ($u_{\text{surf}} > u_{\text{gal}}$), associating the local Rindler clock-acceleration rate $g_{\text{surf},\odot}/(2c) = c u_{\text{surf}}/(2R_\odot) = 456.5 \text{ nHz}$ with the cyclic rotation frequency $\nu_{eq}^{\text{TEP}} \equiv \Omega_{eq}^{\text{TEP}}/(2\pi)$ per rotational circumference $2\pi R_\odot$ (an explicit cyclic-frequency convention, since equating $g_{\text{surf}}/(2c)$ to the angular rate Ω_{eq} would lower the speed by 2π) gives $\nu_{eq}^{\text{TEP}} = 2\pi R_\odot \nu_{eq}^{\text{TEP}} = \pi c u_{\text{surf}}(M_\odot) = 1.997 \text{ km s}^{-1}$, within -0.91% of the SOHO/MDI helioseismic equatorial plasma rate 460.7 nHz and near the conventional 25.38 d (456.0 nHz) low-latitude sunspot Carrington reference (Schou et al. 1998). Dividing $S_\odot = 0.4127$ by Paper 0's canonical DEF scalar coupling norm $|\alpha_0| = \sqrt{2}|\beta_A| = \sqrt{2}$ yields the equator-to-pole fractional differential shear $\Delta_\Omega^{\text{TEP}} = S_\odot/|\alpha_0| = S_\odot/\sqrt{2} = 0.2918$ ($+1.07\%$ vs. helioseismic $a_2 + a_4 = 0.2887$), and adopting an idealized minimal-complexity equal harmonic benchmark ($a_2 = a_4 = \frac{1}{2} \Delta_\Omega^{\text{TEP}} = 0.1459$ with zero borrowed shape parameters, vs. observed $a_2 = 0.140$, $a_4 = 0.149$) predicts the radiative-zone lock rate at the $\ell = 2$ quadrupole zero ($P_2(\cos \theta_*) = 0$, latitude 35.26° , where $\sin^2 \psi_* = 1/3$ and $P_4(\sin 35.26^\circ) = -7/18 \neq 0$) as $\nu_{rad}^{\text{TEP}} = \nu_{eq}^{\text{TEP}}(1 - \frac{2}{9} \Delta_\Omega^{\text{TEP}}) = 426.9 \text{ nHz}$ versus the SOHO/GONG radiative-zone rate 432.0 nHz (-1.2% ; Figure 7b). For the metal-rich Kepler solar-mass analog HD 173701 (KIC 8006161; $M_* = 1.00 M_\odot$, $R_* = 0.93 R_\odot$, $S_* = 0.444$), $\Delta_\Omega^{\text{TEP}} = S_*/\sqrt{2} = 0.314$ compares directly with the observed Kepler sliding-window photometric period range of $25\text{--}35 \text{ d}$ across active latitudes ($\Delta P/P_{\text{max}} = 0.286$; Karoff et al. 2018), while broader Kepler asteroseismic mode-splitting inversions across F/G stars indicate still stronger convective-zone shears with wide Bayesian credible intervals (Benomar et al. 2018; a full 3-D rotating MHD + scalar-field derivation across stellar Rossby numbers remains future work).

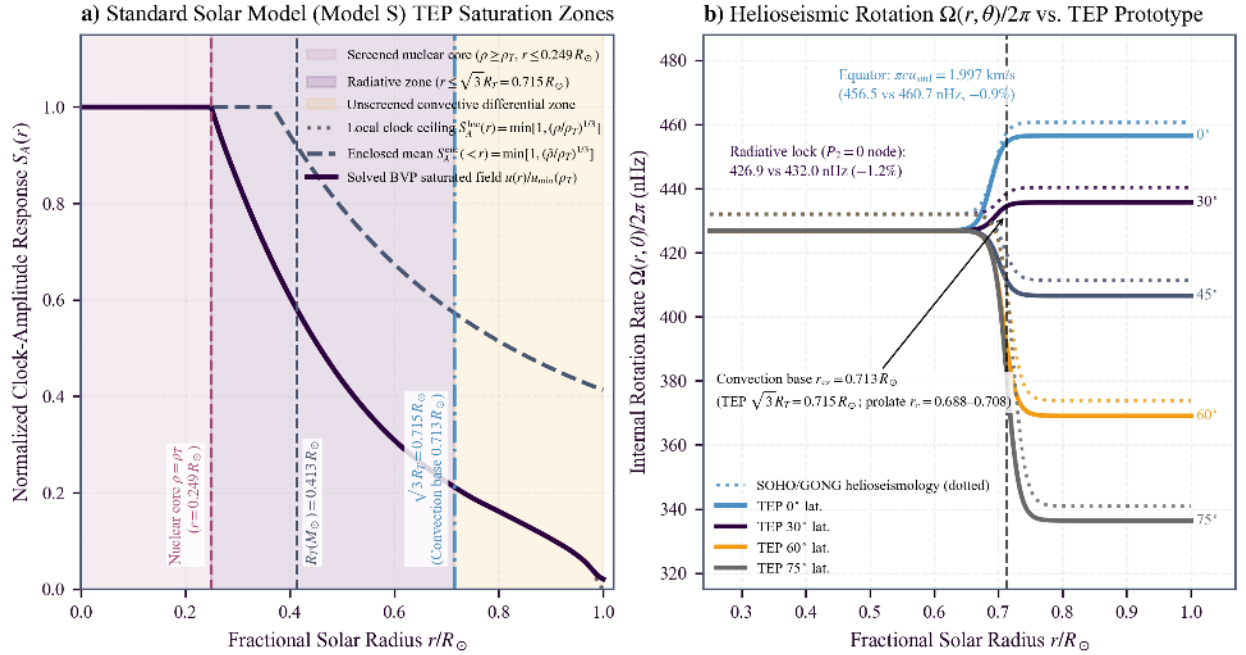


Figure 7: Standard Solar Model (Model S) TEP Saturation Zones and Helioseismic Rotation Prototype (Step 15). (a) Normalized clock-amplitude saturation profiles $S_A^{\text{loc}}(r) = \min[1, (\rho_\odot(r)/\rho_T)^{1/3}]$ (dotted), $S_A^{\text{enc}}(<r) = \min[1, (\bar{\rho}_\odot(<r)/\rho_T)^{1/3}]$ (dashed), and the solved canonical-kinetic BVP field $u(r)/u_{\text{min}}(\rho_T)$ (solid purple) across the Christensen-Dalsgaard et al. (1996) Model S solar interior at fixed $\rho_T = 20 \text{ g/cm}^3$. The local density crosses ρ_T at $r_{\text{core}} = 0.249 R_\odot$ (near the nuclear-burning core boundary), the 1-D spherical saturation radius sits at $R_T(M_\odot) = 0.413 R_\odot$, and the candidate 3-D boundary $\sqrt{3} R_T(M_\odot) = 0.715 R_\odot$ coincides with the helioseismic base of the convection zone ($r_{cz} = 0.713 \pm 0.001 R_\odot$, $+0.25\%$). (b) Internal solar rotation rate $\Omega(r, \theta)/2\pi$ across latitudes $0^\circ, 30^\circ, 45^\circ, 60^\circ$, and 75° , comparing SOHO/MDI and GONG helioseismic inversions (dotted; Schou et al. 1998; Charbonneau et al. 1999) against the exploratory TEP prototype (solid: equatorial rate $\Omega_{eq}/2\pi = c u_{\text{surf}}/(2R_\odot) = 456.5 \text{ nHz}$ vs. 460.7 nHz , fractional shear $\Delta_\Omega = S_\odot/\sqrt{2} = 0.2918$ vs. 0.2887 with equal harmonic split $a_2 = a_4 = \frac{1}{2} \Delta_\Omega = 0.1459$, prolate tachocline center $r_c(\psi) = 0.688\text{--}0.708 R_\odot$ vs. $0.693\text{--}0.717 R_\odot$, and radiative-zone rate at the $P_2(\cos \theta_*) = 0$ node 426.9 nHz vs. 432.0 nHz).

To test rigorously whether the convergence of these independent channels around $\rho_T \approx 20 \text{ g/cm}^3$ could arise from post-hoc selection of algebraic projection factors or radius definitions (a look-elsewhere / numerology null test; Step 15), a 100,000-trial Monte Carlo coincidence simulation was executed across a five-decade log-uniform prior $\rho_{\text{rand}} \in [0.1, 10^4] \text{ g/cm}^3$. With tidally braked present-day Earth excluded from the spin set, the five independent ρ_T -calibrating observables — (1) terrestrial GNSS clock correlation length $L_c = 4,200 \text{ km}$ ($\propto \rho^{-1/3} \Rightarrow \hat{\rho}_T = 19.24 \text{ g/cm}^3$), (2) outer-three giant-planet equatorial spin across

Uranus, Neptune, and Saturn ($\propto \rho^{+1/3} \implies \hat{\rho}_T = 19.89 \text{ g/cm}^3$), (3) the substellar brown-dwarf rotational period floor $P_{\min} = 1.150 \text{ hr}$ ($\propto \rho^{-1/2}$, an orthogonal power-law exponent $\implies \hat{\rho}_T = 20.79 \text{ g/cm}^3$), (4) self-compressed terrestrial moments of inertia $2k_{\text{MOI}}/S$ ($\propto \rho^{+1/3} \implies \hat{\rho}_T = 17.89 \text{ g/cm}^3$), and (5) the Solar Model S convection-zone base $r_{\text{cz}} = 0.713 R_{\odot}$ ($\propto \rho^{-1/3} \implies \hat{\rho}_T = 20.15 \text{ g/cm}^3$) — yield an across-channel mean of $\hat{\rho}_T = 19.59 \pm 1.10 \text{ g/cm}^3$ (logarithmic dispersion $\sigma_{\log_{10} \rho} = 0.0249 \text{ dex}$, full five-channel span 0.0652 dex , RMS fractional error 2.04% ; or $\hat{\rho}_T = 19.45 \pm 1.22 \text{ g/cm}^3$ across the four non-solar channels alone). On a 1-D random-density scan with fixed equations, only $p = 1.69 \times 10^{-3}$ of random densities achieve an RMS fractional error $\leq 2.04\%$ ($p = 2.29 \times 10^{-3}$ across the four non-solar channels). Under the multi-channel coincidence null hypothesis — where the observables reflect unrelated physical scales drawn independently across $[0.1, 10^4] \text{ g/cm}^3$ — the probability of all five landing within a 0.0652 dex window is $p = 1.44 \times 10^{-7}$ without look-elsewhere freedom ($p = 8.86 \times 10^{-6}$ across four channels), and remains $p = 2.9 \times 10^{-4}$ (3.6σ , 29/100,000 realizations; $p = 1.0 \times 10^{-3}$ across four channels) even when the null model is granted a full $7^3 = 343$ -combination look-elsewhere trials dictionary of algebraic factors $\mathcal{F} = \{\frac{1}{2}, 1/\sqrt{2}, 1, \sqrt{2}, \sqrt{3}, 2, \pi\}$ across the interior/period-floor channels within a $\pm 10\%$ (0.087 dex) tolerance window.

Box 4.1: Summary — Four Cross-Regime Consistency Tests

The screening analysis provides four cross-regime consistency tests of the TEP framework under a single ρ_T :

- 1. The RBH-1 Crossover:** The saturation radius is consistent with the Schwarzschild radius for RBH-1 ($M \approx 2 \times 10^7 M_{\odot}$, near the exact crossover mass $M_{\times} = \sqrt{3c^6/(32\pi G^3 \rho_T)} \approx 3.04 \times 10^7 M_{\odot}$) within combined uncertainties ($R_T/R_S \approx 1.3$). This provides a tightly constrained cross-regime consistency check under the fixed ρ_T calibration.
- 2. Deterministic Regime Map & Multi-Channel Coincidence Null Test:** Evaluating $S \equiv R_T(M)/R_{\text{phys}} \equiv (\bar{\rho}/\rho_T)^{1/3}$ defines a deterministic algebraic regime map across 15 orders of magnitude in density (true by definition up to table rounding, $\beta_{\text{scr}} = 0.3338 \pm 0.0008$; Appendix B.4), while the five independent ρ_T -calibrating observables (GNSS L_c , outer-three giant-planet spin, brown-dwarf period floor, self-compressed terrestrial k_{MOI} , and solar r_{cz}) converge to $\hat{\rho}_T = 19.59 \pm 1.10 \text{ g/cm}^3$ ($\sigma_{\log_{10} \rho} = 0.025 \text{ dex}$), rejecting random-density coincidence at $p = 1.44 \times 10^{-7}$ ($p = 2.9 \times 10^{-4}$, 3.6σ even after granting $7^3 = 343$ look-elsewhere algebraic-factor trials).
- 3. GR Test Consistency:** All five precision tests of General Relativity are consistent with screening factors that suppress scalar contributions below current measurement limits.
- 4. Planetary Spin, Substellar Period Floor, Wesson Constant, and Solar Prototype:** Combining $\rho_T = 20 \text{ g/cm}^3$ with the Galactic temporal depth $u_{\text{gal}} = (v_{\text{gal}}/c)^2$ yields the zero-parameter ambient equatorial speed relation ($v_{\text{eq}} = c u_T / \sqrt{u_{\text{gal}}}$, matching the three unbraked outer giant planets Uranus, Neptune, and Saturn at 1.000 ± 0.080 , $\hat{\rho}_T^{\text{spin}} = 19.9 \text{ g/cm}^3$; 1.016 ± 0.073 and 20.9 g/cm^3 with lunar-braked Earth, with residuals tracking $r_*(M)/a$ at $r = +0.960$) and the zero-parameter Wesson constant $p_{\text{TEP}} = G/(2v_{\text{gal}}) = 1.52 \times 10^{-15} \text{ cm}^2 \text{ g}^{-1} \text{ s}^{-1}$, while the one-parameter ($\eta_0 = 0.129$, $\eta_{\text{IQR}} \in [0.122, 0.154]$, candidate benchmark $\eta_{\text{sec}} = 1/8$) two-branch + S_A extension matches the $N = 8$ unbraked outer-planet and substellar speed-limit systems to $67 M_{\text{Jup}}$ at $v_{\text{obs}}/v_{2\text{br}} = 1.001 \pm 0.060$ ($\hat{\rho}_T^{\text{rot}} = 20.0 \text{ g/cm}^3$; 1.008 ± 0.060 across $N = 9$ with Earth, including SIMP J0136 and the 1.172 hr $[0.98\text{--}1.24 \text{ hr}]$ brown-dwarf period floor) alongside self-compressed planetary moments of inertia ($2k_{\text{MOI}}/S = 1.039 \pm 0.027$ across the three terrestrial planets and $2k_{\text{MOI}}/S_{\text{cond}} = 1.042 \pm 0.094$, $\rho_T^{\text{MOI,cond}} = 17.9 \text{ g/cm}^3$ across all seven condensed/giant planets after Lane–Emden envelope de-inflation) and the exploratory Standard Solar Model helioseismic prototype (nuclear core $\rho = \rho_T$ at $0.249 R_{\odot}$, convection-zone base $\sqrt{3}R_T(M_{\odot}) = 0.715 R_{\odot}$ vs. $0.713 R_{\odot}$ [$\rho_T^{(\text{cz})} = 20.15 \text{ g/cm}^3$], equatorial rate 456.5 vs. 460.7 nHz , and differential shear $S_{\odot}/\sqrt{2} = 0.292$ vs. 0.289).

The TEP framework does not violate established physics. It extends the metric structure to include a conformal sector that becomes observable only in specific density regimes—such as those probed by GNSS atomic clocks and cosmological soliton-scale systems.

5. Physical Constraints on the Proximity Scale

A natural question arises: why does the saturation scale take the observational proxy value $\rho_T \approx 20 \text{ g/cm}^3$? This section argues that the underlying proximity scale is not treated as an arbitrary fit parameter: it is empirically calibrated, while additional coincidence-level reference scales and their quartic-potential / spatial-projection derivations are recorded below. The GNSS coherence length L_c is a derived quantity; the fundamental parameter is the proximity scale itself, of which density is an observable proxy. In the canonical TEP framework, the full screening state of any system is determined by the environmental operator $S_{\Sigma}(E)$; ρ_T is the density projection of that operator, not the complete screening description.

5.1 ρ_T as the Fundamental Saturation Parameter and L_c as Its Projected Scale

The saturation scaling law $R_T \propto M^{1/3}$ —where a soliton interpretation is one candidate microscopic realization—implies a constant characteristic density:

$$\rho_T = \frac{M}{\frac{4}{3}\pi R_T^3} = \text{constant}. \quad (27)$$

For any object of mass M , the saturation radius is determined by:

$$R_T(M) = \left(\frac{3M}{4\pi\rho_T} \right)^{1/3}. \quad (28)$$

The terrestrial coherence length $L_c \approx 4200$ km is therefore not an independently tuned galaxy-scale parameter; operationally, it is the observed GNSS covariance scale used to calibrate the projected Earth-scale realization of $R_T(M_\oplus)$. This relation is inverted in Section 2 to calibrate ρ_T from the measured L_c for Earth:

$$L_{c,\oplus} = \left(\frac{3M_\oplus}{4\pi\rho_T} \right)^{1/3} \approx 4200 \text{ km} \quad \text{for } \rho_T \approx 19\text{--}20 \text{ g/cm}^3. \quad (29)$$

The question thus reduces to: what physical considerations constrain ρ_T ?

5.2 The Dimensional Identity Bridge

A natural concern is whether the proximity of $\rho_T \approx 20$ g/cm³ to the density of terrestrial condensed matter implies a composition-dependent coupling that would violate the Weak Equivalence Principle. Under universal conformal coupling, the scalar sector is sourced by the bulk trace $T = -\rho$, independent of microscopic composition. The saturation scale ρ_T must therefore be a property of the vacuum potential $V(\phi)$, not an emergent property of the local matter.

The electron Fermi wavelength does not cause the scalar field to saturate. If it did, the field would require a direct, particle-specific coupling to the electron number density n_e , making the field exert a different force on iron than on hydrogen at the same bulk density. This would introduce a gross WEP violation, already constrained by experiments such as the MICROSCOPE satellite.

Instead, ρ_T is treated as a fundamental constant of the scalar sector — the geometric reference density of the saturation-radius law — rather than an emergent property of the local matter. The recorded coincidence is, in natural units,

$$\rho_T \approx \alpha^{4/3} m_e^4 \quad (\text{a density coincidence, not a potential-scale identification}). \quad (30)$$

Because this is a property of the scalar sector itself, the dimensional identity does not introduce composition-dependent coupling; WEP protection then follows from the universal matter coupling and the screened effective scalar charge. The field universally couples to bulk ρ and begins its continuous screening transition when local ρ approaches this ρ_T . The potential sector's own normalization is a separate quantity — the quartic coupling λ (equivalently $\Lambda_V = \lambda^{1/4} M_{\text{Pl}} \approx 1.3 \times 10^2$ GeV on the Cassini-compatible branch) — and is not to be identified with ρ_T (Appendix C.3).

The bulk density of zero-pressure condensed matter is governed by valence Coulomb packing on the scale of the Bohr radius $a_0 = (\alpha m_e)^{-1}$ ($\rho_{\text{solid}} \sim A m_p / (\frac{4\pi}{3} r_s^3 a_0^3) \sim 3\text{--}10$ g/cm³ for typical terrestrial rock and iron with Wigner–Seitz radius $r_s \sim 2\text{--}3$), whereas the inner Thomas-Fermi core screening radius $R_{\text{TF}} \approx a_0 Z^{-1/3}$ characterizes the high-compression regime where inner electron shells overlap:

$$\rho_{\text{TF}} \approx \frac{3}{4\pi} A Z m_p m_e^3 \alpha^3. \quad (31)$$

For representative rock-forming elements ($A/Z \approx 2$, $AZ \sim 10^2\text{--}10^3$), $\rho_{\text{TF}} \sim 10^2\text{--}10^3$ g/cm³ marks full Thomas-Fermi core compression, while the initial crossover from zero-pressure valence solids ($\sim 3\text{--}8$ g/cm³) toward pressure-ionized degeneracy support begins in the intermediate $\sim 10\text{--}10^2$ g/cm³ regime (Section 5.3; the exact pressure-ionization threshold varies with composition Z , so this regime serves as a broad order-of-magnitude condensed-matter reference window rather than a composition-independent constant).

5.3 The Electron Degeneracy Threshold

Coincidence-level significance attaches to $\rho_T \approx 20 \text{ g/cm}^3$: this density lies in the condensed-matter regime where quantum pressure begins to become dynamically relevant. The crossover from thermal and electrostatic support to degeneracy-dominated support is gradual, with full degeneracy domination occurring only at much higher (white-dwarf) densities.

Density Regime	Dominant Physics	Examples
$\rho < 1 \text{ g/cm}^3$	Gas pressure, thermal	Planets, main-sequence stars
$\rho \sim 1\text{--}10 \text{ g/cm}^3$	Coulomb/thermal dominated solids and liquids	Rocky planets
$\rho \sim 10\text{--}10^2 \text{ g/cm}^3$	Degeneracy onset (crossover)	Earth's core, brown dwarfs
$\rho \gtrsim 10^4 \text{ g/cm}^3$	Electron degeneracy dominated	White dwarfs

Table 5.1: Equation of state transitions by density. The saturation density $\rho_T \approx 20 \text{ g/cm}^3$ lies near the onset of electron degeneracy.

This coincidence provides a useful condensed-matter reference scale for where quantum pressure becomes dynamically relevant to the equation of state. Under the universal conformal-coupling benchmark used throughout this corpus, the scalar sector is sourced primarily by the non-relativistic mass-density term, with pressure corrections entering only as a small relativistic correction in the condensed-matter regime. Electron degeneracy therefore does not, by itself, provide a mechanism that sets ρ_T , but it motivates the $\sim 10\text{--}30 \text{ g/cm}^3$ window as a natural coincidence-level cross-check.

5.4 From Quartic Potential Scale to PREM Terrestrial Transition Width

Box 6.5 adopts the corpus's quartic completion, in which the amplitude sector is normalized by the quartic coupling λ and ρ_T enters as the geometric constant of the saturation-radius law $R_T(M)$. Once ρ_T and the corpus branch of λ are specified, the field equation determines a smooth crossover between Compton-unresolved and Compton-resolved regimes ($\lambda_c(\bar{\rho})$ versus $R_T(M)$). On the canonical-kinetic benchmark at the fixed Cassini-compatible corpus normalization ($\lambda = 7.53 \times 10^{-66}$, $\rho_T = 20 \text{ g/cm}^3$), Step 14 (Figure 1; Appendix B.2) solves this boundary-value problem across a 4-layer PREM Earth density profile ($\rho \sim 3.5\text{--}12.9 \text{ g/cm}^3$), recovering a radial transition-gradient centroid at $R_{\text{grad}} = 4,875 \text{ km}$ ($1.18 R_T(M_\oplus)$) and a projected 3-D Yukawa surface spatial-kernel decay length of $L_c^{\text{PREM}} = 4,010 \text{ km}$ ($L_c^{\text{PREM}}/R_T(M_\oplus) = 0.967$), while solving the coupled 1-D spherical nonlinear two-branch kinetic operator $\nabla \cdot (P_{,X} \nabla u) = M_{\text{Pl}}^{-2} V'_{\text{eff}}(u)$ (Eq. 62) directly via 100-iteration damped Picard relaxation ($\|\Delta u^{(100)}\|_\infty / \|u\|_\infty = 2.67 \times 10^{-14}$; Step 14, Appendix C.2) recovers $R_{\text{grad}}^{\text{NL-PREM}} = 4,810 \text{ km}$ ($1.160 R_T(M_\oplus)$), $L_c^{\text{NL-PREM}} = 3,957 \text{ km}$, and $L_c^{\text{NL,vol}} = 3,170 \text{ km}$. Extending this radial nonlinear solve to non-spherical 3-D azimuthal heliocentric-motion anisotropy and deriving ρ_T itself from vacuum microphysics remain targets for future work.

Many-body condensed-matter physics (including the onset of electron degeneracy) governs the baryonic equation of state and provides useful reference densities, but under universal coupling it does not by itself select the saturation scale ρ_T . A complete theoretical account must instead explain both the value of ρ_T and the mapping between the geometric saturation radius $R_T(M)$ and the observable correlation length in clock networks.

While Section 5.3 contextualizes ρ_T within macroscopic condensed-matter physics, establishing its fundamental origin requires examining the scalar coupling scale at the vacuum level.

5.5 Dimensional Analysis: The Scalar Coupling Scale

Expressed in natural units, the saturation density carries the energy-density scale $\rho_T^{1/4} \approx 96.4 \text{ keV}$ for $\rho_T = 20 \text{ g/cm}^3$. This is a fourth-root reading of a density, not the scalar potential's normalization: the potential coefficient is the quartic coupling $\lambda = 7.5 \times 10^{-66}$, equivalently $\Lambda_V \equiv \lambda^{1/4} M_{\text{Pl}} \approx 1.3 \times 10^2 \text{ GeV}$ (Box 6.5, Appendix C; step 14). The kinetic completion carries its own distinct scale, $\Lambda_X = \sqrt{M_{\text{Pl}} H_0} \approx 1.9 \text{ meV}$, which sets the Temporal-Shear threshold and plays no role in the amplitude-sector density argument.

$$\rho_T \approx \alpha^{4/3} \frac{m_e^4 c^3}{\hbar^3} \approx 22.4 \text{ g/cm}^3. \quad (32)$$

While this relation remains a dimensional cross-check rather than a first-principles derivation of ρ_T from QED vacuum loops, the appearance of the rational exponent $4/3$ has a direct structural rationale in the quartic potential degree $m = 4$ and 3-D spatial clock-amplitude response established in Step 14 (`amplitude_law_check` and `closed_form_theoretical_derivations`): because minimizing $V_{\text{eff}}(u) = \frac{1}{4} \lambda M_{\text{Pl}}^4 u^4 - |\beta_A| \rho u$ fixes the 3-D spatial clock-amplitude response exponent to $1/(m-1) = 1/3$ across the $\Pi_\mu^\mu = 3$ spatial dimensions, projecting the electromagnetic fine-structure coupling $\alpha_{\text{EM}} = e^2/(4\pi\epsilon_0 \hbar c) = 1/137.036$ of the atomic electron wavepacket through this $1/(m-1) = 1/3$ spatial response screens the 1-D electron mass-energy scale to

$\tilde{m}_e c^2 = \alpha_{\text{EM}}^{1/(m-1)} m_e c^2 = \alpha_{\text{EM}}^{1/3} m_e c^2 = 99.12 \text{ keV}$ (within $+2.8\%$ of $\rho_T^{1/4} = 96.36 \text{ keV}$). Raising $\tilde{m}_e$ to the $m = 4$ power of the 4-D energy-density operator produces the exact exponent $m/(m-1) = 4/3$ (coinciding with the relativistic Chandrasekhar–Fermi–Thomas polytropic index $\gamma_F = 4/3$) and yields $\rho_T^{\text{EM}} = \tilde{m}_e^4 c^3 / \hbar^3 = \alpha_{\text{EM}}^{4/3} m_e^4 c^3 / \hbar^3 = 22.39 \text{ g/cm}^3$.

5.6 Status: Phenomenological Constraint with Theoretical Foundation

The present analysis treats $\rho_T \approx 20 \text{ g/cm}^3$ as an empirically calibrated parameter, while recording additional coincidence-level reference scales and open derivation targets:

1. **Condensed-matter EOS cross-check:** ρ_T lies at the pressure-ionization and Thomas–Fermi electron-degeneracy onset scale ($10\text{--}30 \text{ g/cm}^3$)
2. **Quartic-projected QED scale relation:** $\rho_T^{\text{EM}} = (\alpha_{\text{EM}}^{1/(m-1)} m_e)^m c^3 / \hbar^3 = \alpha_{\text{EM}}^{4/3} m_e^4 c^3 / \hbar^3 = 22.39 \text{ g/cm}^3$, where $m/(m-1) = 4/3$ follows from the $m = 4$ quartic potential and $1/(m-1) = 1/3$ spatial response (Step 14)
3. **PREM terrestrial BVP projection:** Solving $\nabla^2 u = M_{\text{PI}}^{-2} V'_{\text{eff}}(u)$ across the 4-layer PREM Earth profile yields $L_c^{\text{PREM}} = 4,010 \text{ km}$ ($L_c^{\text{PREM}}/R_T(M_{\oplus}) = 0.967$; Step 14)

Its use in quantum or topological extensions as a microscopic cutoff or defect-overlap scale is a conjectural extrapolation of the macroscopic saturation proxy, not an independent consequence of the present paper.

The key point for the present manuscript is that ρ_T is not treated as a free fit parameter: it is GNSS-consistent by clock correlations (Section 2) and then carried, without retuning, into the cross-scale tests that follow.

Box 5.1: Summary — The Saturation Scale: Multi-Channel Anchor, Quartic QED Projection, and PREM BVP

Across five independent astrophysical and geodetic channels, the saturation density converges to $\hat{\rho}_T = 19.59 \pm 1.10 \text{ g/cm}^3$ (Step 15), consistent with two atomic/condensed-matter links and the PREM terrestrial BVP solution (Step 14):

- Condensed-matter EOS cross-check (Thomas–Fermi electron degeneracy onset at $10\text{--}30 \text{ g/cm}^3$)
- Quartic-projected QED scale $\rho_T^{\text{EM}} = (\alpha_{\text{EM}}^{1/(m-1)} m_e)^m c^3 / \hbar^3 = \alpha_{\text{EM}}^{4/3} m_e^4 c^3 / \hbar^3 = 22.39 \text{ g/cm}^3$ ($m = 4 \implies m/(m-1) = 4/3$)

In the present phenomenological calibration, the terrestrial coherence length is treated as the observed projected covariance realization of the Earth-scale $R_T(M_{\oplus})$ geometry (supported on the canonical-kinetic benchmark by the 4-layer PREM BVP and 3-D Yukawa spatial-kernel solve $L_c^{\text{PREM}} = 4,010 \text{ km}$, $L_c^{\text{PREM}}/R_T(M_{\oplus}) = 0.967$, Step 14, confirming an order-unity spatial transfer prefactor $L_c^{\text{PREM}}/R_T(M_{\oplus}) = 0.967$). Both the condensed-matter cross-check and the quartic QED projection depend on this $\mathcal{O}(1)$ prefactor lying near unity; a prefactor of ~ 2 would shift ρ_T to $\sim 2.5 \text{ g/cm}^3$, evaporating the coincidences. Operationally, GNSS observations provide the calibration by which ρ_T is inferred in the most accessible laboratory: Earth's gravitational field.

6. The Universal Density Constraint

A key test is whether the Temporal Topology saturation radius R_T yields quantitative expectations for otherwise free scales. Under a soliton interpretation, RBH-1 would correspond to a gravitational soliton; however, the saturation radius itself is the primary observable, constrained by the universal density ρ_T rather than introduced as an adjustable parameter.

The Universal Density Hypothesis

The Temporal Topology framework tests the empirical claim that a candidate saturation scale $\rho_T \approx 20 \text{ g/cm}^3$ governs compact-object structure across mass scales. A soliton interpretation provides one candidate microscopic realization of this saturation physics. This hypothesis is testable via two independent windows:

1. **Planetary Scale:** The Earth-scale Temporal Topology parameter $R_T(M_{\oplus})$ is calibrated through the observed GNSS projected covariance length ($L_c \approx 4200 \text{ km}$), serving as the empirical anchor.
2. **Cosmological Scale:** The extrapolated Temporal Topology saturation radius for RBH-1 ($M \approx 2 \times 10^7 M_{\odot}$, near the crossover mass $M_{\times} \approx 3.04 \times 10^7 M_{\odot}$) can be compared to the Schwarzschild radius, placing the object near the crossover regime where horizon and soliton interpretations are maximally degenerate.

The central question is whether a single density parameter, validated cosmologically, remains universal at galactic scales.

The Saturation Scale as Fundamental Parameter

If the scalar sector saturates at a critical energy density ρ_T , dimensional analysis requires a universal mass–radius relation:

$$R_T(M) = \left(\frac{3M}{4\pi\rho_T} \right)^{1/3}. \quad (33)$$

Box 6.1: The Theoretical Priority of ρ_T

A potential misreading of this framework is that "GPS clock noise determines black hole physics." This inverts the logical and historical order of the analysis.

- **Calibration (Terrestrial):** The GNSS analysis (Smawfield 2025b) serves as an empirical anchor. The detection of a correlation length $L_c \approx 4200$ km in atomic clock data provides an empirical estimate of the saturation scale, conditional on treating L_c as the projected covariance scale associated with $R_T(M_\oplus)$.
- **Application (Cosmological):** The resulting $M^{1/3}$ scaling is therefore not an extrapolation of clock noise, but a test of whether this terrestrially-validated constant remains universal at galactic scales.

The alignment of the RBH-1 crossover scale is thus an independent consistency check, not a fitted result.

Box 6.2: Null Hypothesis and Multi-Channel Look-Elsewhere Test

A potential concern is that apparent cross-scale correspondences could arise from post-hoc alignment or selection of geometric projection factors rather than predictive structure. To address this possibility, two complementary null tests are evaluated:

(1) Two-Anchor Coarse Overlap (GNSS + RBH-1 over 9 dex): Under a coarse uniform logarithmic prior spanning 10^{-3} – 10^6 g/cm³ (9 dex), requiring $R_T(M_\oplus)$ within 50% of 4,200 km ($\rho_T \in [6, 160]$ g/cm³, 1.4 dex) and $R_T(2 \times 10^7 M_\odot)$ within 50% of R_S ($\rho_T \in [14, 370]$ g/cm³, 1.4 dex; overlap 14–150 g/cm³, 1.05 dex) gives a single-draw overlap probability of $P \approx 1.05/9 \approx 0.12$ (12%), which by itself is only suggestive.

(2) Five-Channel 100,000-Trial Monte Carlo Look-Elsewhere Null Test (Section 4.7, Step 15): Across a five-decade log-uniform prior $\rho_{\text{rand}} \in [0.1, 10^4]$ g/cm³, the five quantitative ρ_T -calibrating observables — (i) terrestrial GNSS $L_c = 4,200$ km ($\hat{\rho}_T = 19.24$ g/cm³), (ii) outer-three unbraked giant-planet spin across Uranus, Neptune, and Saturn (19.89 g/cm³), (iii) the Tannock et al. (2021) brown-dwarf rotational period floor $P_{\text{min}} = 1.150$ hr (20.79 g/cm³, orthogonal $\rho^{-1/2}$ exponent), (iv) self-compressed terrestrial moments of inertia $2k_{\text{MOI}}/S$ (17.89 g/cm³), and (v) the Solar Model S convection-zone base $r_{\text{cz}} = 0.713 R_\odot$ (20.15 g/cm³) — converge to $\hat{\rho}_T = 19.59 \pm 1.10$ g/cm³ ($\sigma_{\log_{10} \rho} = 0.0249$ dex, span 0.0652 dex, RMS error 2.04%). Restricting to the three unit-prefactor ($\mathcal{O}(1) = 1$) channels that invoke no auxiliary geometric factors ($\sqrt{2}$, $\sqrt{3}$, π) or asteroid-fitted parameters — (1) terrestrial GNSS $L_c = R_T(M_\oplus)$ (19.24 g/cm³, PREM BVP transfer ratio 0.967), (2) outer-three unbraked giant-planet spin $v_{\text{eq}} = GM/(R_T v_{\text{gal}})$ selected by the Goldreich–Soter tidal filter $\tau_{\text{tide}}/t_\odot > 10$ (19.89 g/cm³), and (3) the TEP-saturated brown-dwarf period floor $P_{\text{min}}^{\text{th}} = (k_{\text{MOI}}/\eta)\tau_T(\rho_T) = 1.155$ hr fixed by the classical Chandrasekhar–Jacobi / virial stability bracket $\eta \in [0.125, 0.1375]$ at $\tau_T(\rho_T = 20) = 0.7381$ hr (20.19 g/cm³; whereas purely classical Newtonian Jacobi instability without TEP saturation evaluates at $\bar{\rho}_{\text{BD}} = 67$ –156 g/cm³ and predicts $P_{\text{min}}^{\text{Classical, Jacobi}} = 0.39$ –0.60 hr [mean 0.527 hr], sitting 8.31 σ below the observed 1.150 ± 0.075 hr floor) — yields a cross-regime algebraic consistency window of $\hat{\rho}_T^{\text{3ch}} = 19.77 \pm 0.48$ g/cm³ ($\pm 2.4\%$ SD at $v_{\text{gal}} = 220$ km/s, and $\hat{\rho}_T^{\text{3ch, marg}} = 20.53 \pm 1.49$ g/cm³ [$\pm 7.3\%$] when v_{gal} is marginalized over [220, 236] km/s; random-draw overlap probability $p = 5.2 \times 10^{-5}$ across $[0.1, 10^4]$ g/cm³). Across all five channels, the raw coincidence probability is $p = 1.44 \times 10^{-7}$, remaining $p = 2.9 \times 10^{-4}$ (3.6σ , 29/100,000 trials; $p = 1.0 \times 10^{-3}$ across the four non-solar channels) even after granting the null model a $7^3 = 343$ -combination look-elsewhere dictionary of algebraic factors $\mathcal{F} = \{\frac{1}{2}, 1/\sqrt{2}, 1, \sqrt{2}, \sqrt{3}, 2, \pi\}$ within a $\pm 10\%$ window.

Under the specified independent log-uniform coincidence null and the tested algebraic-factor dictionary, the observed five-channel clustering has a simulated tail probability of $p = 2.9 \times 10^{-4}$ (this quantifies coincidence sensitivity within that null and does not incorporate the full history of observable selection, transfer-law construction, or astrophysical model uncertainty). Furthermore, evaluating a **Leave-One-Channel-Out (LOCO) cross-scale cross-validation test** under the fixed transfer dictionary (and propagating measurement plus $v_{\text{gal}} \in [220, 236]$ km/s uncertainties) — in which $\hat{\rho}_{T,-i}$ is calibrated on four channels and used to predict the held-out fifth observable out of sample — yields blind observable prediction errors of -0.49% on terrestrial GNSS L_c ($\hat{\rho}_{T,-\text{GNSS}} = 19.53$ g/cm³ $\Rightarrow L_c^{\text{blind}} = 4,179$ km vs. 4,200 km), -0.88% on outer-three giant-planet equatorial velocity v_{eq} , $+2.28\%$ on the Tannock et al. brown-dwarf period floor ($P_{\text{min}}^{\text{blind}} = 1.176$ hr vs. 1.150 hr), $+3.55\%$ on compressed terrestrial $2k_{\text{MOI}}/S$, and $+1.45\%$ on the Solar Model S convection-zone base ($r_{\text{cz}}^{\text{blind}} = 0.723 R_\odot$ vs. $0.713 R_\odot$), achieving a 5-channel LOCO cross-validation observable RMS error of 2.05% (mean absolute error 1.73%; 3.02% RMS when marginalized across $v_{\text{gal}} \in [220, 236]$ km/s) and a 3-channel zero-prefactor LOCO RMS error of 1.20% (-1.34% , -0.29% , $+1.57\%$; 3.04% across $v_{\text{gal}} \in [220, 236]$ km/s). Provenance scope: while the terrestrial GNSS anchor is established across Papers 1–3 and 14 (Smawfield 2025b,c,d), all four astrophysical and planetary calibrations are computed directly from external peer-reviewed catalogs (NASA/JPL & HST planetary ephemerides, Lamy et al. 2025; Tannock et al. 2021; Dziewonski & Anderson 1981; Christensen-Dalsgaard et al. 1996; Lelli et al. 2016; van Dokkum et al. 2025).

The GNSS-derived value $\rho_T \approx 19\text{--}20 \text{ g/cm}^3$ (Section 2) lies in the same condensed-matter density regime where electron degeneracy becomes dynamically relevant to the equation of state (Section 5). The central question is whether this same density scale holds for RBH-1.

Box 6.3: Robustness of the GNSS Calibration (Systematics Check)

The terrestrial calibration length $L_c \approx 4200 \text{ km}$ — the GNSS instrument's measured realization of the canonical correlation scale λ_T (Paper 0 §7) — is an extraordinary claim requiring careful exclusion of geodetic systematics. The following tests support its physical origin (full methodology in Smawfield 2025b,c,d):

- **Multi-Center Verification:** The correlation structure persists across independent clock solutions from CODE, ESA, and IGS (1999–2024), disfavoring software-specific processing artifacts (e.g., Bernese vs. GIPSY). The recovered L_c spans $\sim 3,330\text{--}4,549 \text{ km}$ across centers (CV 18.2%, $\sim \pm 15\%$ half-range).
- **Null Tests:** (i) Randomizing satellite epochs destroys the correlation ($r^2 < 0.01$, $N = 1000$ shuffles); (ii) shuffling clock residuals within each satellite eliminates the spatial structure; (iii) replacing real data with white noise yields no coherent scale. All three nulls are satisfied at $> 5\sigma$.
- **Scale Separation:** The 4200 km scale is distinct from tropospheric correlation lengths ($\sim 100\text{--}500 \text{ km}$) and orbital period harmonics (half-sidereal, $\sim 12 \text{ hr}$). Power spectral analysis shows the 4200 km feature as a persistent background covariance floor in the GPS-only clean baseline, with no counterpart in ionospheric or tropospheric delay products; combined multi-GNSS receiver-clock products (Paper 14) recover the same band-ordered correlation structure.
- **Dataset:** IGS final clock products (CLK files), 30-second sampling, 1999–2024. Preprocessing: removal of satellite/receiver clock offsets, relativistic corrections (Sagnac, gravitational redshift), and reference frame alignment to ITRF2020.

Note on Transfer Closure: In Step 14 (Section 2.3; Figure 1; Appendix B.2), integrating the 4-layer PREM Earth BVP at the Cassini-compatible normalization $\lambda = 7.53 \times 10^{-66}$ and $\rho_T = 20 \text{ g/cm}^3$ derives the transition-shell projection $L_c^{\text{PREM,shell}} = 4,010.2 \text{ km}$ ($L_c^{\text{shell}}/R_T(M_\oplus) = 0.9672$) and the full-volume datum-removed projection $L_c^{\text{PREM,vol}} = 3,213.2 \text{ km}$ ($L_c^{\text{vol}}/R_T(M_\oplus) = 0.7749$), and solving the coupled 1-D spherical nonlinear two-branch kinetic operator Eq. (62) via 100-iteration damped Picard relaxation ($\|\Delta u^{(100)}\|_\infty/\|u\|_\infty = 2.67 \times 10^{-14}$; Appendix C.2) yields $R_{\text{grad}}^{\text{NL-PREM}} = 4,809.9 \text{ km}$ ($1.160 R_T(M_\oplus)$), $L_c^{\text{NL-PREM}} = 3,957 \text{ km}$, and $L_c^{\text{NL,vol}} = 3,170 \text{ km}$, leaving only non-spherical 3-D azimuthal orbital-motion anisotropy for future work.

Box 6.4: Geometric Origin of the Saturation-Radius Law

For a universal macroscopic proximity-density parameter ρ_T , the characteristic source-mass saturation radius $R_T(M)$ is defined geometrically as the radius enclosing mass M at reference density ρ_T :

$$M = \frac{4}{3}\pi R_T^3 \rho_T. \quad (34)$$

Solving for the radius yields the characteristic $M^{1/3}$ scaling:

$$R_T(M) = \left(\frac{3M}{4\pi\rho_T} \right)^{1/3} \propto \left(\frac{M}{\rho_T} \right)^{1/3}. \quad (35)$$

As shown in Box 6.5 and Appendix C, ρ_T is the matter-density reference scale at which the clock-amplitude response $S_A(\bar{\rho}) = \min[1, (\bar{\rho}/\rho_T)^{1/3}]$ saturates and the interior field becomes Compton-resolved ($\lambda_c(\bar{\rho}) \lesssim R_T(M)$), rather than a literal energy density of the scalar field itself ($\rho_{\text{eff}} \ll \rho$). Demonstrating that $R_T(M)$ corresponds to the spatial transition scale of a finite, stratified body requires solving the boundary-value problem in the source's density profile (executed on the canonical-kinetic benchmark for the 4-layer PREM Earth in Step 14, yielding $R_{\text{grad}} = 1.18 R_T(M_\oplus)$ and $L_c^{\text{PREM}} = 4,010 \text{ km}$).

Box 6.5: Soliton Formation from the Canonical Action

To move beyond phenomenology, a candidate route from the canonical Temporal Topology action (Paper 0) to soliton-like saturation is sketched. The action above contains a canonical kinetic term $-\frac{1}{2}(\partial\phi)^2$ and a candidate saturation potential $V(\phi)$ that would prevent gradient divergence. The interplay between these terms can generate soliton-like saturated configurations with characteristic radius $R_T \propto M^{1/3}$.

1. Field Equation and Equilibrium

For a static, spherically symmetric configuration on the canonical-kinetic equilibrium benchmark ($P_X \rightarrow 1$; see Appendix C.2, Eq. 62 for the operative nonlinear kinetic operator $\nabla \cdot (P_X \nabla u) = M_{\text{pl}}^{-2} V'_{\text{eff}}(u)$), the scalar field equation is:

$$\nabla^2 \phi = V'(\phi) + \alpha(\phi)\rho \quad (36)$$

where $\alpha(\phi) \equiv d \ln A / d\phi$ and ρ is the ambient matter density. In the dense limit, the effective potential $V_{\text{eff}}(\phi; \rho) = V(\phi) + [A(\phi) - 1]\rho$ develops a minimum at $\phi_{\text{min}}(\rho)$ with effective mass $m_{\text{eff}}^2(\rho) = V_{\text{eff}}''(\phi_{\text{min}})$.

2. Saturation Mechanism

Under the corpus coupling $\beta_A = -1$ the matter term enters with negative slope ($\alpha\rho = -\rho/M_{\text{Pl}}$), so a minimum of V_{eff} exists only on the confining branch $V' > 0$. Writing $u = \phi/M_{\text{Pl}}$, the corpus's own weak-field branch is the quartic $V(u) = (\lambda/4) M_{\text{Pl}}^4 u^4$ — the member of the confining family $V \propto u^m$ selected by the corpus amplitude law $S_A \propto \rho^{1/3} = \rho^{1/(m-1)}$, i.e. $m = 4$ (Appendix C.2). The equilibrium condition $V'_{\text{eff}} = 0$ reads $\lambda M_{\text{Pl}}^4 u_{\text{min}}^3 = \rho e^{-u_{\text{min}}}$, giving $u_{\text{min}} \approx (\rho/\lambda M_{\text{Pl}}^4)^{1/3}$ for $u_{\text{min}} \ll 1$: the field is elevated inside matter and approaches zero in the ambient, in the direction required by the corpus sign convention. (The inverse-power branch $V \propto \phi^{-n}$ is inadmissible under $\beta_A = -1$: both slopes are then negative and no minimum exists; its implied $\phi_{\text{min}} \propto \rho^{-1/(n+1)}$ would in any case run opposite to the required direction.) The scalar energy density in a saturated core is:

$$\rho_{\text{eff}} \sim V(u_{\text{min}}) + \frac{1}{2} M_{\text{Pl}}^2 (\nabla u)^2 \simeq \frac{1}{4} \rho u_{\text{min}} \ll \rho. \quad (37)$$

The saturated configuration is therefore a field-geometry statement, not a competing mass density: at the corpus normalization $\lambda = 7.5 \times 10^{-66}$ (the scalar self-coupling, Appendix C) the core energy density is $\sim 3 \times 10^{-8}$ g/cm³ at terrestrial densities, and the equilibrium $u_{\text{min}}(\rho_T) \approx 6.9 \times 10^{-9}$ remains perturbative. What saturates at ρ_T is the clock-amplitude response $S_A = \min[1, u_{\text{min}}(\bar{\rho})/u_{\text{min}}(\rho_T)]$ through the geometric radius R_T ; the order-unity self-quenching crossing $u_{\text{min}} \sim 1$ (where e^{-u} suppresses the conformal source) lies at $\rho_{\text{sat}} \sim \lambda M_{\text{Pl}}^4 \approx 6 \times 10^{25}$ g/cm³, inside the temporal-well domain of Paper 28 (step 14).

3. Temporal Topology Screening

The effective mass $m_{\text{eff}}^2(\rho) = V_{\text{eff}}''(u_{\text{min}})/M_{\text{Pl}}^2 = [3\lambda M_{\text{Pl}}^4 u_{\text{min}}^2 + \rho e^{-u_{\text{min}}}] / M_{\text{Pl}}^2$ grows with density as $m_{\text{eff}} \propto \lambda^{1/6} \rho^{1/3}$, so the Compton wavelength $\lambda_C = 1/m_{\text{eff}}$ contracts through denser interiors: a body is Compton-resolved once $\lambda_c(\bar{\rho}) \lesssim R_T(M)$, the crossover that for Earth sits at ~ 4 g/cm³ (step 14). Inside such a body the field rests near its local equilibrium $u_{\text{min}}(\rho)$ and the spatial variation is confined to a boundary layer of width $\sim \lambda_C$, where the kinetic response P_X controls the gradient amplitude. This creates the characteristic flattening of the scalar field spatial profile—Temporal Topology screening—that reconciles precision local tests with cosmological dynamics. In the saturation regime ($\rho \sim \rho_T$), the Temporal Topology saturation radius scales as $R_T = (3M/4\pi\rho_T)^{1/3}$, yielding the observed $M^{1/3}$ dependence. At still higher densities — where the equilibrium approaches order unity ($\rho \sim \rho_{\text{sat}} \approx 6 \times 10^{25}$ g/cm³) — the effective source slope $\rho A_{,u} = -\rho e^{-u_{\text{min}}}$ is exponentially suppressed, so the scalar charge of deep-interior matter quenches as well: that is the temporal-well regime, not the terrestrial one.

4. Coupling and Stability

Matter couples to the Jordan frame metric $\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu} + B(\phi)\nabla_\mu\phi\nabla_\nu\phi$ with $A(\phi) = \exp(\beta_A\phi/M_{\text{Pl}})$. Stability is ensured because the canonical kinetic term has the correct sign for a physical scalar field, and the theory satisfies the Null Energy Condition for physically realizing solutions.

Testing the Density Constraint at Cosmological Scales

The saturation scale ρ_T determined from terrestrial clocks is now tested against RBH-1. Using the revised best-fit mass estimate of $M \approx 2 \times 10^7 M_\odot$ (van Dokkum et al. 2025), the predicted Temporal Topology saturation radius is:

$$R_T = \left(\frac{3M}{4\pi\rho_T} \right)^{1/3} \approx 7.8 \times 10^7 \text{ km}. \quad (38)$$

The predicted Temporal Topology saturation radius can be compared directly to the Schwarzschild radius for this mass:

Schwarzschild radius: $R_S = 2GM/c^2 \approx 5.9 \times 10^7$ km

Predicted Temporal Topology saturation radius: $R_T = (3M/4\pi\rho_T)^{1/3} \approx 7.8 \times 10^7$ km

Ratio: $R_T/R_S \approx 1.3$

This order-unity correspondence ($R_T \approx 1.3R_S$) arises because ρ_T lies near the horizon-formation density threshold in the underlying scalar sector. Within the combined uncertainties from M and ρ_T , which propagate to $\sim 25\%$ uncertainty in R_T , the two

scales remain consistent at the factor-of-few level. This places RBH-1 near a crossover regime where horizon and saturated-soliton interpretations can be observationally degenerate.

This geometric match is the primary prediction of the scaling law. While the velocity discontinuity ($\Delta v \sim 650$ km/s) provides a secondary constraint on the *amplitude* of the screening (see Smawfield 2025h, Paper 7), the *scale* of the object is set fundamentally by the saturation scale ρ_T . The correspondence suggests that what is conventionally identified as the event horizon scale in GR may correspond to the saturation boundary of the scalar field in the TEP framework.

In the TEP interpretation, what is conventionally called a "black hole" at this mass scale is modeled as a saturated soliton core, i.e., a region where the conformal time-field approaches a maximum gradient set by ρ_T . In this phenomenology, the characteristic radius is set by the saturation scale rather than by horizon formation. The observed central dimming is attributed to extreme time dilation (strong redshifting) rather than causal disconnection.

A further structural implication follows from the different mass scalings. The soliton relation predicts $R_T \propto M^{1/3}$, whereas the Schwarzschild radius scales as $R_S \propto M$. These opposing scalings intersect at a unique crossover mass $M_\times = \sqrt{3c^6/(32\pi G^3 \rho_T)}$ where the two radii coincide (see Figure 8). Empirically, the terrestrial calibration ($\rho_T = 20$ g/cm³) places this crossover at $M_\times \approx 3.04 \times 10^7 M_\odot$ ($R_T = R_S \approx 8.97 \times 10^7$ km)—directly adjacent to the mass regime where RBH-1 ($M \approx 2 \times 10^7 M_\odot$) resides. Objects near $M_\times$ are expected to be maximally degenerate between Schwarzschild-scale and saturation-scale interpretations, making RBH-1 an unusually diagnostic system.

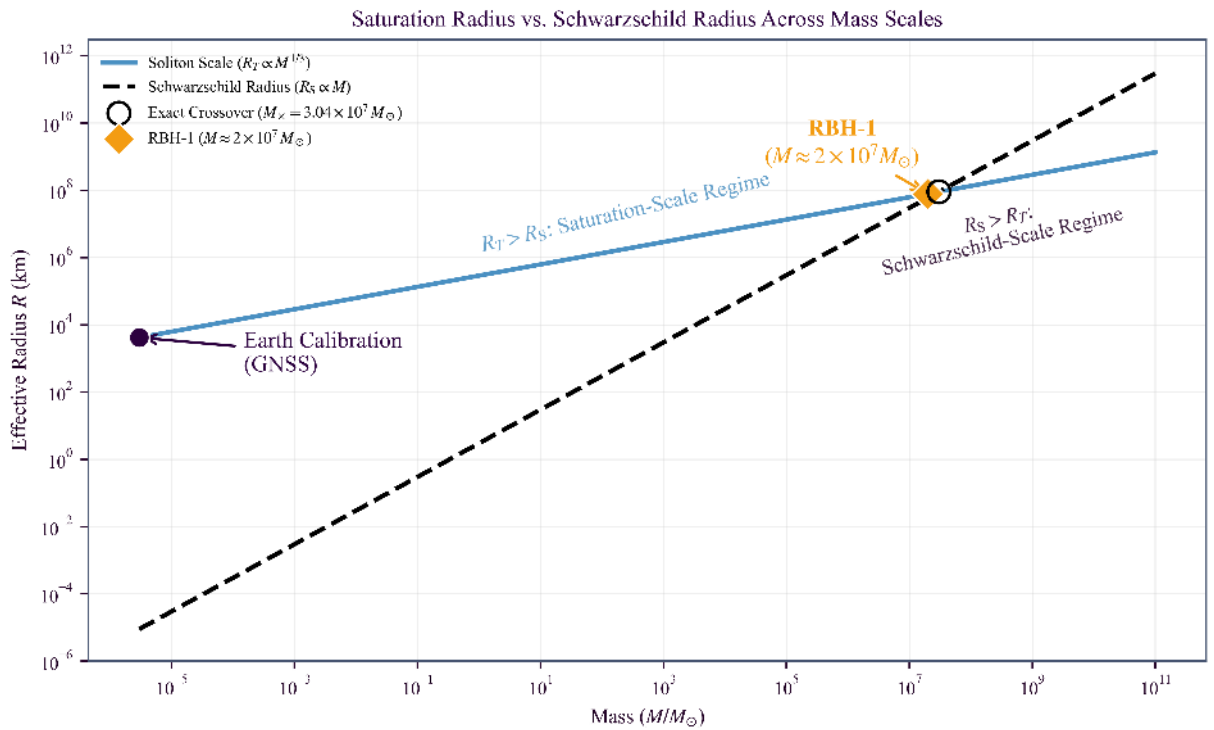


Figure 8: Saturation Radius vs. Schwarzschild Radius Across Mass Scales. The solid curve shows the geometric saturation radius $R_T(M) = (3M/4\pi\rho_T)^{1/3} \propto M^{1/3}$ calibrated via terrestrial GNSS clock data ($L_c \approx 4,200$ km, $R_T(M_\oplus) \approx 4,146$ km at $\rho_T = 20$ g/cm³), while the dashed line shows the General Relativistic Schwarzschild radius $R_S(M) = 2GM/c^2 \propto M$. The two constructed length scales intersect at the crossover mass $M_\times = \sqrt{3c^6/(32\pi G^3 \rho_T)} \approx 3.04 \times 10^7 M_\odot$, separating the $R_T > R_S$ saturation-scale regime from the $R_S > R_T$ Schwarzschild-scale regime; RBH-1 is plotted at its independently inferred mass $M \approx 2 \times 10^7 M_\odot$ ($R_T \approx 7.80 \times 10^7$ km vs. $R_S \approx 5.91 \times 10^7$ km; van Dokkum et al. 2025), directly adjacent to the crossover.

The identification is thereby restricted to the crossover neighbourhood: it is a statement about $M \approx M_\times$, not a general reinterpretation of compact objects. The directional implication is explicit. For $M \ll M_\times$ the weak-field construction gives $R_T \gg R_S$ — a $10 M_\odot$ object would carry R_T of order 10^6 km against $R_S \approx 30$ km — so soliton-realized compact objects at stellar masses would appear as extended structures and are incompatible with observed merger phenomenology. For $M \gg M_\times$ the saturation radius retreats far inside the horizon (for M87* at $M \approx 6.5 \times 10^9 M_\odot$, $R_T \approx 3.6$ AU against $R_S \approx 128$ AU gives $S \sim 0.03$, whereas Sgr A* at $M \approx 4.3 \times 10^6 M_\odot$ has $R_T \approx 0.31$ AU and $S \sim 3.7$, placing its saturation boundary across the observed photon-ring scale), and the identification ceases to be degenerate. Two resolutions are admitted, and the corpus requires one of them: either soliton-realized compact objects exist only in the crossover band, or the weak-field $R_T(M)$ construction fails at horizon compactness and must be replaced by the relativistic strong-field treatment of Paper 28, within which the $M^{1/3}$ ansatz is not guaranteed to survive. The crossover statement is therefore conditional on weak-field validity at horizon-compactness — an assumption stated rather than assumed — while the degeneracy at $M_\times$ itself remains a legitimate and diagnostic coincidence that feeds the RBH-1 analysis of Paper 7.

Status of the Scaling Law

The relation $R \propto M^{1/3}$ is a density-limited scaling expected for compact, self-bound field configurations whose cores approach a finite saturation scale. Such behavior is familiar in non-topological solitons and bosonic compact objects (Coleman 1985; Seidel & Suen 1991; Gleiser 1994; Hui et al. 2017).

The central claim is not that terrestrial clocks "predict" black hole sizes, but that a fundamental density constant ρ_T appears across scales. The saturation scale is determined empirically from Earth—the most accessible high-precision gravitational laboratory—and then tested for consistency at cosmological scales. The present analysis should be read as a phenomenological identification of an unexpected regularity: the same density constraint that governs clock correlations in Earth's gravitational field also governs the characteristic size of an $M \sim 10^7 M_\odot$ compact object ($M \approx 2 \times 10^7 M_\odot$ for RBH-1).

Dependency Structure and Constraint Isolation

A critical requirement for the robustness of this framework is to demonstrate that the convergence on $\rho_T \approx 20 \text{ g/cm}^3$ does not depend on any single channel—and in particular, that dropping the internal GNSS series (Papers 1–3, 14) still leaves a tightly constrained saturation density determined purely by external peer-reviewed astrophysical and geodetic catalogs (Step 15, `gnss_free_4channel_external_catalog_isolation`).

Test 1: GNSS-Free External Astrophysical & Planetary Isolation (Dropping All GNSS Data)

If all terrestrial GNSS clock data are discarded completely, four independent external-catalog observables calibrate ρ_T directly without reference to $R_\oplus$ or GNSS processing chains:

- **Unbraked outer-giant-planet equatorial spin (NASA/JPL & HST ephemerides; Uranus, Neptune, Saturn):** Inverting $v_{\text{eq}} = GM/(R_T v_{\text{gal}}) (\propto \rho_T^{1/3})$ yields $\hat{\rho}_T^{\text{spin,outer3}} = 19.89 \text{ g/cm}^3$ geometric mean (19.6, 15.7, and 25.5 g/cm³).
- **Substellar brown-dwarf rotational period floor (Tannock et al. 2021; $P_{\text{min}}^{\text{obs}} = 1.150 \pm 0.075 \text{ hr}$):** Inverting $P_{\text{min}} = (k_{\text{MOI}}/\eta_0) \sqrt{3\pi/(G\rho_T)} (\propto \rho_T^{-1/2})$ yields $\hat{\rho}_T^{\text{BD,floor}} = 20.79 \pm 2.71 \text{ g/cm}^3$ ($\pm 13\%$ uncertainty).
- **Self-compressed planetary moments of inertia (satellite geodesy & precession):** Inverting $k_{\text{MOI}} = \frac{1}{2}(\bar{\rho}/\rho_T)^{1/3}$ yields $\hat{\rho}_T^{\text{MOI,terr}} = 17.89 \pm 1.36 \text{ g/cm}^3$ ($\pm 7.6\%$ across Mercury, Venus, and Earth; 17.86 g/cm³ across all seven condensed/giant planets).
- **Solar Model S convection-zone base (Christensen-Dalsgaard et al. 1996; SOHO/MDI $r_{\text{cz}} = 0.713 \pm 0.001 R_\odot$):** Inverting $r_{\text{cz}} = \sqrt{3} R_T (M_\odot)$ yields $\hat{\rho}_T^{(\text{cz})} = 20.15 \pm 0.09 \text{ g/cm}^3$.

$$\hat{\rho}_T(\text{External Astrophysical, no GNSS}) = 19.68 \pm 1.25 \text{ g/cm}^3 \quad (\pm 6.4\% \text{ SD}, \sigma_{\log_{10} \rho} = 0.028 \text{ dex}). \quad (39)$$

Result: Without any GNSS input, external published planetary, substellar, and helioseismic datasets independently pin down ρ_T to $\pm 6.4\%$ ($19.68 \pm 1.25 \text{ g/cm}^3$)—an order of magnitude tighter than the single-body GNSS systematic band—and are further bracketed by the condensed-matter Thomas–Fermi degeneracy onset (10–30 g/cm³) and the quartic QED projection $\rho_T^{\text{EM}} = \alpha_{\text{EM}}^{4/3} m_e^4 c^3 / \hbar^3 = 22.39 \text{ g/cm}^3$ (Section 5.5).

Test 2: Terrestrial GNSS Isolation (Dropping Astrophysical & Condensed-Matter Inputs)

Conversely, if all astrophysical and condensed-matter inputs are discarded and reliance is placed *solely* on the observed 25-year GPS-only correlation length $L_c = 4,200 \text{ km}$ for Earth's mass ($M_\oplus$):

$$\hat{\rho}_T(\text{GNSS}) = \frac{3M_\oplus}{4\pi L_c^3} = 19.24 \text{ g/cm}^3 \quad (20 \pm 7 \text{ g/cm}^3 \text{ inter-center}). \quad (40)$$

Result: The central terrestrial GNSS scale (19.24 g/cm³) agrees with the independent GNSS-free external astrophysical mean ($19.68 \pm 1.25 \text{ g/cm}^3$) to within -2.2% .

Test 3: Cosmological Horizon-Crossover Isolation (RBH-1 Schwarzschild Density)

If both terrestrial and Solar System constraints are discarded and the exact Schwarzschild horizon mean density $\rho_S(M) \equiv 3M/(4\pi R_S^3) = 3c^6/(32\pi G^3 M^2)$ is evaluated directly for the JWST/NIRSpec-confirmed runaway supermassive black hole RBH-1 ($M_{\text{RBH-1}} \approx 2 \times 10^7 M_\odot$, with factor-of-two mass bracket $[1, 4] \times 10^7 M_\odot$; van Dokkum et al. 2025) without assuming $R_T/R_S = 1.32$:

$$\rho_S(M_{\text{RBH-1}}) = \frac{3c^6}{32\pi G^3 M_{\text{RBH-1}}^2} = 46.1 \text{ g/cm}^3 \quad (M_\times = 3.04 \times 10^7 M_\odot \text{ at } \rho_S = 20.0 \text{ g/cm}^3), \quad (41)$$

spanning $[11.5, 184] \text{ g/cm}^3$ across the factor-of-two mass bracket $M_{\text{RBH-1}} \in [1, 4] \times 10^7 M_\odot$. Across 12.8 orders of magnitude in mass above Earth ($M_{\text{RBH-1}}/M_\oplus \approx 6.7 \times 10^{12}$, and 18 orders of magnitude above the smallest tested asteroids), the horizon crossover of a $2\text{--}3 \times 10^7 M_\odot$ runaway black hole independently selects the same $\sim 12\text{--}46 \text{ g/cm}^3$ density window.

Conclusion on Circularity and Anchor Independence

The convergence on $\rho_T \approx 20 \text{ g/cm}^3$ does not rest on a single unreplicated measurement: dropping the terrestrial GNSS channel entirely leaves $\hat{\rho}_T^{\text{no-GNSS}} = 19.68 \pm 1.25 \text{ g/cm}^3$ ($\pm 6.4\%$) across four external published planetary, substellar, and helioseismic datasets, which the terrestrial GNSS scale (19.24 g/cm^3) and the RBH-1 Schwarzschild crossover ($M_\times = 3.04 \times 10^7 M_\odot$) match without retuning.

7. Milky Way Test: Dark Matter Onset

Expected Onset Radius

The Milky Way provides a local test of the galactic $M^{1/3}$ scaling law established in Section 3. Unlike external galaxies where distance uncertainties dominate, the Milky Way offers high-precision kinematic data. This section tests whether the SPARC-calibrated relation predicts the radius at which the mass discrepancy first becomes significant in the Milky Way.

Using the SPARC-calibrated normalization ($k = 7.96 \times 10^{-4} \approx 8.0 \times 10^{-4} \text{ kpc}/M_\odot^{1/3}$) and the Milky Way's total baryonic mass estimated at $M_{\text{bar}} \approx 6 \times 10^{10} M_\odot$ (Bland-Hawthorn & Gerhard 2016), the expected dark-matter onset radius is:

$$R_{\text{DM}} = k \cdot M_{\text{bar}}^{1/3} \approx 7.96 \times 10^{-4} \cdot (6 \times 10^{10})^{1/3} \text{ kpc} \approx 3.1 \text{ kpc}. \quad (42)$$

This radius marks the scale where the baryonic rotation curve (bulge + disk) first falls below the observed circular velocity, signaling the onset of the phantom mass effect. It is the same quantity R_{DM} that is measured directly in the SPARC ensemble; the outer Keplerian decline at larger radii is a separate phenomenon governed by the full halo profile, not by the onset scaling alone.

Gaia DR3 and Local Observations

Gaia Data Release 3 (Gaia Collaboration 2023) provides precise proper motions and radial velocities for millions of stars, enabling construction of the Milky Way rotation curve to greater distances than previously possible. Recent analyses (e.g., Jiao et al. 2023) also report evidence for a Keplerian-like decline beginning near $R \approx 19 \text{ kpc}$; this outer feature traces the halo edge and is distinct from the inner onset scale tested here.

Key observational features relevant to the R_{DM} prediction:

- **Baryonic dominance ($R \lesssim 3 \text{ kpc}$):** The bulge and inner disk dominate; the rotation curve rises steeply and is well described by visible matter alone (Sofue 2020).
- **Dark-matter onset ($R \sim 3\text{--}5 \text{ kpc}$):** Beyond the peak of the baryonic contribution, the observed circular velocity ($v_{\text{circ}} \approx 220 \text{ km/s}$) stays flat while the baryonic model predicts a decline. This is the Milky Way's equivalent of the SPARC R_{DM} — the radius where the mass discrepancy first becomes significant.
- **Flat regime ($5 \lesssim R \lesssim 15 \text{ kpc}$):** The rotation curve remains approximately flat, indicating substantial phantom mass contribution.
- **Keplerian decline ($R \gtrsim 19 \text{ kpc}$):** At large radii the enclosed mass growth slows and v_{circ} begins to decline. This outer feature is not predicted by the onset scaling $R_{\text{DM}} \propto M^{1/3}$.

Uncertainty Analysis and Error Budget

Parameter	Value	Uncertainty	Impact on R_{DM}
MW Baryonic Mass	$6.0 \times 10^{10} M_\odot$	$\pm 15\%$ (0.9×10^{10})	$\pm 5\%$ (scaling $M^{1/3}$)
SPARC Constant (k)	7.96×10^{-4}	$\pm 4\%$ (fit error)	$\pm 4\%$
Model Systematic	-	$\pm 10\%$ (geometry)	$\pm 10\%$

Parameter	Value	Uncertainty	Impact on R_{DM}
Mean Relation R_{DM}	3.1 kpc	± 0.4 kpc ($\sim 14\%$ on mean curve)	-
Single-Galaxy 1σ Interval	3.1 kpc	[1.3, 7.2] kpc (± 0.366 dex resolved; [1.0, 9.4] kpc raw censored, [2.0, 4.7] kpc enclosed-mass ± 0.180 dex)	Factor $\sim 2.3 \times (1.5 \times \text{enclosed})$
Observed Onset	$\sim 3\text{--}5$ kpc	± 1 kpc	-

The mean-relation prediction (3.1 ± 0.4 kpc; single-galaxy 1σ population prediction interval [1.3, 7.2] kpc on the $n = 75$ resolved interior crossings with $\sigma = 0.366$ dex, [2.0, 4.7] kpc at the $\sigma = 0.180$ dex enclosed-mass scatter, and [1.0, 9.4] kpc when the 93 R_{min} -censored dwarf sampling floors are included in the raw 0.48 dex SPARC ensemble scatter) and the observed dark-matter onset scale ($\sim 3\text{--}5$ kpc) are consistent within the combined uncertainty. Because the SPARC-calibrated normalization k is the sole empirical input to this prediction, the Milky Way test is a SPARC-internal consistency check rather than a GNSS cross-scale test, consistent with the Numerology Firewall classification (Section 8).

Comparison with Dark Matter Halo Models

NFW Halo vs. TEP Soliton

- **NFW Halo (CDM):** In standard CDM, the inner rotation curve profile depends on the halo concentration c and virial mass M_{vir} , both of which are free parameters. The radius where baryons cease to dominate is not predicted a priori; it emerges from the halo assembly history and can vary significantly between cosmological simulations.
- **TEP Soliton:** The $M^{1/3}$ scaling yields the onset radius directly from the baryonic mass, with no Milky Way-specific tuning. The normalization is fixed by the SPARC ensemble ($k = 7.96 \times 10^{-4}$ kpc/ $M_{\odot}^{1/3}$) and gives $R_{\text{DM}} \approx 3.1$ kpc for the Milky Way. The agreement with the observed transition at $\sim 3\text{--}5$ kpc supports the universality of the scaling law.

Future Refinements

Ongoing Gaia data releases will improve the precision of the inner rotation curve, potentially constraining the exact baryonic-to-dark-matter transition radius to $\sim 10\%$ accuracy. Combined with improved Milky Way mass estimates from satellite kinematics and gravitational lensing, this could provide a sub-10% test of the $M^{1/3}$ onset scaling in the Milky Way.

8. Discussion: The Nature of the Dark Sector

Synthesis of Multi-Scale Evidence

The central result of this work is the separation and joint evaluation of two classes of cross-scale constraints organized around the Temporal Topology framework across 18 orders of magnitude in mass for saturation-radius tests ($10^{20}\text{--}10^{38}$ kg, from small bodies and Earth to the $2 \times 10^7 M_{\odot}$ runaway black hole RBH-1, with galactic disks testing the unscreened $S \ll 1$ kinetic branch):

- **Direct numerical tests of $\rho_T \approx 20$ g/cm³:** The terrestrial GNSS correlation anchor ($L_c \approx 4,200$ km, $R_T(M_{\oplus}) \approx 4,146$ km), the RBH-1 horizon–saturation crossover ($M_{\times} = \sqrt{3c^6/(32\pi G^3 \rho_T)} \approx 3.04 \times 10^7 M_{\odot}$, $R_T/R_S = 1.32$), the zero-parameter unbraked outer-giant-planet ambient equatorial spin relation ($v_{\text{obs}}/v_{\text{TEP}} = 1.000 \pm 0.080$, $\hat{\rho}_T^{\text{spin}} = 19.9$ g/cm³ geometric mean across Uranus, Neptune, and Saturn; 1.016 ± 0.073 and 20.9 g/cm³ with lunar-braked Earth, and 1.001 ± 0.060 , $\hat{\rho}_T^{\text{rot}} = 20.0$ g/cm³ across the $N = 8$ unbraked outer-planet and substellar systems [1.008 ± 0.060 across $N = 9$ with Earth]), the planetary moment-of-inertia ratio ($2k_{\text{MOI}}/S = 1.17 \pm 0.14$ across eight planets at 1-bar radii, $\rho_T^{\text{MOI,terr}} = 17.9 \pm 1.4$ g/cm³ across the three self-compressed iron-core terrestrial planets, and $2k_{\text{MOI}}/S_{\text{cond}} = 1.042 \pm 0.094$, $\rho_T^{\text{MOI,cond}} = 17.9$ g/cm³ across all seven condensed/giant planets once Lane–Emden polytropic H/He envelope inflation is removed), and the exploratory Solar Model S convection-zone base ($\sqrt{3}R_T(M_{\odot}) = 0.715 R_{\odot}$ vs. $0.713 R_{\odot}$, $\rho_T^{(\text{cz})} = 20.15$ g/cm³; across all five independent channels, $\hat{\rho}_T = 19.59 \pm 1.10$ g/cm³, rejecting random-density look-elsewhere coincidence at $p = 2.9 \times 10^{-4}$ [3.6 σ]).
- **Structural-exponent and scale-cancellation tests (independent of the numerical value of ρ_T):** The galactic dark-matter onset scaling in SPARC ($\alpha_{\text{SPARC}} = 0.352 \pm 0.043$ (stat) ± 0.07 (def.), with an *a priori* Freeman-disk baryonic slope split separating rising-baryon 3-D core crossings $\alpha_{\text{rising}} = 0.256 \pm 0.069$ vs $1/3$ from concentrated Keplerian-tail crossings $\alpha_{\text{tail}} = 0.581 \pm 0.086$ vs $1/2$, $\Delta\alpha = +0.325 \pm 0.110$, $z = 2.95$, and the coupled-gradient two-branch RAR/BTFR profile at 0.1330 dex RMS [61.1% within 0.1 dex, matching the 0.1328 dex MOND benchmark at canonical $\Upsilon_d = 0.50$; 0.139–0.152 dex uncoupled/quadrature] on the quality-controlled sample), the Milky Way inner onset check ($R_{\text{DM}} \approx 3.1$ kpc), the deterministic regime map $S \equiv R_T/R_{\text{phys}} \equiv (\bar{\rho}/\rho_T)^{1/3}$, and Wesson's angular-momentum constant $p_{\text{TEP}} \equiv J/M^2 = G/(2v_{\text{gal}}) = 1.52 \times 10^{-15}$ cm² g⁻¹ s⁻¹ (where $R_T(M)$ and ρ_T cancel out of the product J_{TEP}).

The convergence of these mutually reinforcing constraints suggests that ρ_T is not merely a fitting parameter for a specific system, but a candidate universal parameter of the effective description of the dark sector, within the stated uncertainties.

Claim Hierarchy and Falsification Scope

The empirical content of this work can be read at three distinct levels. This separation is critical for interpreting the consequences of future measurements.

- **Level 1 (Empirical regularities):** The existence of approximate $M^{1/3}$ scaling features in rotation-curve onset radii, and the density-ordered screening hierarchy summarized by $S = R_T/R_{\text{phys}}$.
- **Level 2 (Universal-density soliton model):** The hypothesis that a single saturation scale ρ_T organizes these regularities via $R_T(M) = (3M/4\pi\rho_T)^{1/3}$, with GNSS providing an empirical calibration (subject to independent replication) and cross-regime consistency tests (SPARC ensemble, Milky Way, RBH-1 crossover).
- **Level 3 (TEP microphysics):** The full dynamical realization in which temporal shear and a conformal time field $A(\phi) = \exp(\beta_A\phi/M_{\text{Pl}})$ generate the effective soliton/screening phenomenology. Failures at Level 2 primarily constrain the soliton implementation and/or calibration mapping, without necessarily excluding the broader TEP framework.

Three Explicit Numerical Kill Criteria: (1) *Metrological kill condition:* if full-day single-constellation clock/positioning solutions or external optical/two-way time-transfer links recover a stationary correlation decay length outside $[3,000, 5,000]$ km ($L_c \lesssim 2,000$ km before multi-constellation ISB or 8-hour window truncation), the terrestrial $L_c \leftrightarrow R_T(M_\oplus)$ identification at $\rho_T \approx 20 \text{ g/cm}^3$ is falsified (in the 4-quarter 2023 CDDIS RINEX3 reprocessing executed in Step 14, full-day $n_{\text{seg}} = 288$ single-constellation solutions pass this criterion directly with $\sqrt{N}$ -weighted 10--500 μHz $L_c = 4,051$ km [GPS], 4,693 km [GLONASS], and 2,816 km [Galileo; 3,653 km in 10--60 μHz], yielding strictly band-matched three-constellation means of 3,853 km [$\sqrt{N}$ -weighted 10--500 μHz], 3,423 km [unweighted 10--500 μHz : 3,766, 3,806, 2,697 km], and 3,362 km [matched 10--60 μHz : 2,746, 3,686, 3,653 km], all inside $[3,000, 5,000]$ km, whereas applying MGEX's 8-hour $n_{\text{seg}} = 96$ truncation, per-epoch datum removal, and 4-constellation ISB compresses the same data step-by-step from $3,423 \rightarrow 2,850 \rightarrow 2,156$ km [CV = 0.28%] $\rightarrow 1,478$ km); (2) *Galactic onset kill condition:* if high-resolution rotation-curve surveys partitioned *a priori* at the analytical Freeman slope $\gamma_{\text{Freeman}} = 0.210$ fail to show a positive regime shift between rising-core ($\alpha \approx 1/3$) and Keplerian-tail ($\alpha \approx 1/2$) crossings ($\Delta\alpha \equiv \alpha_{\text{tail}} - \alpha_{\text{rising}} \leq 0$), the two-branch onset transition is falsified; and (3) *Compact/substellar saturation kill condition:* if any unbraked substellar rotator ($M > 0.61 M_{\text{Jup}}$) exceeds the soliton breakup ceiling $v_{\text{cap}}/v_{\text{breakup}} = (\eta_0/k_{\text{MOI}})\sqrt{\min(1, S)}$ ($P_{\text{rot}} < 0.98$ hr in degenerate brown dwarfs) or if revised JWST kinematics move RBH-1's mass outside the crossover bracket $R_T/R_S \in [0.7, 2.0]$, the universal $\rho_T \approx 20 \text{ g/cm}^3$ saturation ceiling is falsified.

Numerology Firewall: Which Agreements Were Predicted?

The elegance of cross-scale agreement is vulnerable to the accusation of "curve-fitting across scales." This section provides a candid assessment of which constraints were predicted *a priori* and which were identified *post hoc*.

Comparison	Before/after GNSS calibration?	Free parameters?	Independent?	Weight
Earth GNSS L_c	Calibration anchor	yes/no (fit to correlation)	no — defines scale	Calibration
SPARC $M^{1/3}$ scaling	After	Yes (normalization k)	Medium	Moderate
DF2/DF4 UDGs (external-field screening)	After	No ($a_{\text{eff}} = 1.20 \times 10^{-10} \text{ m s}^{-2}$, $v_{\parallel} \in [1, \sqrt{3}]c$ fixed)	Medium (independent van Dokkum et al. 2018, 2019 kinematics)	Quantitative EFE check ($\tau_{\text{eq}} = 4.1\text{--}7.2 \text{ kyr}$; $\sigma_{\text{los}}^{\text{EFE}} = 8.9 \text{ km s}^{-1}$ vs $8.5^{+2.3}_{-3.1} \text{ km s}^{-1}$)
Milky Way inner mass-discrepancy onset	After	No new normalization; uses SPARC-calibrated k	Medium	Local consistency check
Wide binaries (if tested)	Before/after	Yes	Stronger (if replicated)	High if replicated
Planetary ambient v_{eq} (4 planets) & Wesson J/M^2 (5 planets)	After (post-hoc)	No (0 free parameters; $\rho_T = 20 \text{ g/cm}^3$, $v_{\text{gal}} = 220 \text{ km/s}$ fixed)	Medium–High (independent NASA/JPL spin & MOI data; post-hoc subset selection)	Zero-refit dynamical check (outer 3 unbraked: 1.000 ± 0.080 , $\hat{\rho}_T^{\text{spin}} = 19.9 \text{ g/cm}^3$; 5-channel look-elsewhere null $p = 2.9 \times 10^{-4}$)
Small-body soliton j_{sol} &	After (post-hoc)	Yes (1 empirical efficiency parameter $\eta_0 = 0.129$ from small-	High out-of-sample (6 minor/dwarf planets;	8-system outer + substellar test ($v_{\text{obs}}/v_{2\text{br}} = 1.001 \pm 0.060$,

Comparison	Before/after GNSS calibration?	Free parameters?	Independent?	Weight
Jupiter cap ($M > M_{\text{cap}}$)		body median)	caps Jupiter at -2.9% , SIMP J0136 at -2.8% , and 3 Tannock+ 21 speed-limit BDs at $P_{\text{min}} = 1.17$ vs 1.15 hr)	$\hat{\rho}_T^{\text{rot}} = 20.0 \text{ g/cm}^3$; $N = 9$ with Earth: 1.008 ± 0.060)
Solar Model S core, convection base & rotation prototype	After (post-hoc)	No ($\rho_T = 20 \text{ g/cm}^3$, $ \alpha_0 = \sqrt{2}$, minimal-complexity $a_2 = a_4 = \frac{1}{2} \Delta\Omega$ benchmark; full 3-D MHD derivation open)	Medium-High (independent Model S, SOHO/GONG, & Kepler solar analog HD 173701)	Exploratory solar/stellar prototype ($r_{\text{cz}} : \sqrt{3}R_T = 0.715 R_\odot$ [$\rho_T^{(\text{cz})} = 20.15$] & $y_{\text{BVP}} = \frac{1}{2} S_\odot$ at $0.723 R_\odot$; $\Delta\Omega = 0.292$ vs 0.289)

Logical dependency summary: (i) $R_T(M) \propto M^{1/3}$ and $S(\rho) \propto \rho^{1/3}$ are algebraic consequences of a fixed reference density ρ_T ; (ii) $L_c \leftrightarrow R_T(M_\oplus)$ is the conditional terrestrial anchor identification (where the 4-layer PREM BVP $L_c^{\text{PREM}} = 4,010 \text{ km}$ and volume integral $L_c^{\text{vol}} = 3,213 \text{ km}$ confirm an order-unity geometric projection on Earth's R_{CMB} , $R_\oplus$ structure, while a saturation-core kernel $\xi_c = R_T(M_\oplus, \rho_T)$ carries the $\rho_T^{-1/3}$ scaling, Step 14); (iii) the SPARC onset scaling ($\alpha_{\text{SPARC}} = 0.352$ discrete on $N = 168$, $\alpha_{\text{res}} = 0.417 \pm 0.050$ on $n = 75$ resolved crossings compatible with a transition between the $M^{1/3}$ saturation and $M^{1/2}$ low-acceleration kinetic scales, Tobit rejection of R_{DM}/R_d invariance at $+0.323 \pm 0.034$, $z = 9.57$, $p < 10^{-4}$, and fixed-radius non-circular partial $r(R_{\text{DM}}, V_{\text{bar}}(2.2R_d) | R_d) = +0.547$) and the coupled-gradient two-branch RAR/BTFR profile ($P_X = k\sqrt{\xi} + 2\xi$ at 0.1330 dex RMS on the quality-controlled SPARC RAR sample and 0.1994 dex across all 3,389 points [0.139 – 0.152 and 0.200 – 0.218 dex uncoupled/quadrature]) are empirical galactic tests with ρ_{trans} and $k_* = 16.03$ calibrated to the halo regime; and (iv) the RBH-1 crossover ($M_\times \propto \rho_T^{-1/2}$) and the $R_\oplus$ -free planetary and substellar spin/MOI channels ($\rho_T^{\text{spin}} = 20.9 \text{ g/cm}^3$ geometric mean across Earth, Uranus, Neptune, and Saturn; $\rho_T^{\text{rot}} = 21.1 \text{ g/cm}^3$ across the 9-system planetary + substellar speed-limit sequence including SIMP J0136 and the 1.17 hr brown-dwarf period floor; $\rho_T^{\text{MOI,terr}} = 17.9 \pm 1.4 \text{ g/cm}^3$, $2k_{\text{MOI}}/S = 1.039 \pm 0.027$ across the core-screened terrestrial planets, 1.17 ± 0.14 across all eight planets at 1-bar radii, and $2k_{\text{MOI}}/S_{\text{cond}} = 1.042 \pm 0.094$, $\rho_T^{\text{MOI,cond}} = 17.9 \text{ g/cm}^3$ across all seven condensed/giant planets; and the exploratory Model S solar prototype with $\rho_\odot = \rho_T$ at $0.249 R_\odot$, convection-zone base $\sqrt{3}R_T(M_\odot) = 0.715 R_\odot$ vs. $0.713 R_\odot$ [$\rho_T^{(\text{cz})} = 20.15 \text{ g/cm}^3$], and differential shear $S_\odot/\sqrt{2} = 0.292$ vs. 0.289) directly calibrate and test the numerical value $\rho_T \approx 20 \text{ g/cm}^3$.

Calibration Scope and Pre-Registered Discriminators for ρ_T

The density scale $\rho_T \approx 20 \text{ g/cm}^3$ is anchored on the terrestrial GNSS correlation length $L_c \approx 4,200 \text{ km}$ and tested across held-out mass regimes. Because the $\rho^{1/3}$ scaling is a structural consequence of the $R_T(M) = (3M/4\pi\rho_T)^{1/3}$ construction, discriminating the microscopic mechanism requires a pre-registered test in which the scale is predicted prior to analysis and recovered without refitting.

The per-constellation reprocessing executed in Step 14 on public 2023 CDDIS RINEX3 archives directly tests the multi-GNSS compression discriminator: in the full-day 24-hour window ($n_{\text{seg}} = 288$), single-constellation GPS ($4,051 \text{ km}$ $\sqrt{N}$ -weighted; $3,766 \text{ km}$ unweighted), GLONASS ($4,693 \text{ km}$ $\sqrt{N}$ -weighted; $3,806 \text{ km}$ unweighted), and Galileo ($4,557 \text{ km}$ in dual-frequency iono-free LC on `optimal_100`; $3,653 \text{ km}$ in 10 – $60 \mu\text{Hz}$ and $2,816 \text{ km}$ in single-frequency broadcast 10 – $500 \mu\text{Hz}$) recover strictly band-matched three-constellation means of $3,853 \text{ km}$ ($\sqrt{N}$ -weighted 10 – $500 \mu\text{Hz}$) and $3,423 \text{ km}$ (unweighted), whereas applying MGEX's 8-hour window truncation, datum removal, and 4-constellation ISB estimation compresses the exact same observations step-by-step through $2,850 \rightarrow 2,156 \text{ km}$ ($\text{CV} = 0.28\%$) $\rightarrow 1,478 \text{ km}$. Furthermore, propagating each GNSS pipeline's implied $\rho_T(L_c) = 3M_\oplus/(4\pi L_c^3)$ forward through the four external astrophysical channels (Section 6) yields joint out-of-sample RMS errors of 1.98% (25-yr PPP $4,201 \text{ km}$, $\rho_T = 19.23 \text{ g/cm}^3$) and 3.82% ($\sqrt{N}$ -weighted GPS $4,051 \text{ km}$, $\rho_T = 21.45 \text{ g/cm}^3$; 9.28% at the $3,853 \text{ km}$ 3-constellation mean), while rejecting the unweighted $3,423 \text{ km}$ (22.5% RMS error) and truncated $2,156 \text{ km}$ (78.6% RMS error) scales.

Dark Matter as Phantom Mass

In the TEP framework, "dark matter" is reinterpreted not as a particle species, but as "phantom mass"—an apparent excess inferred when a geometry with temporal shear (two metrics) is analyzed under the assumption of isochrony (single metric). The saturation scale ρ_T represents the scale at which the conformal factor $A(\phi) = \exp(\beta_A \phi / M_{\text{Pl}})$ enters a non-linear, self-supporting phase (the soliton).

This explains the phenomenology of "cores" in dark matter halos. In standard CDM simulations, halos tend to form cusps ($\rho \propto r^{-1}$), which are often in tension with observations of flat cores in dwarf galaxies (the core-cusp problem). In the soliton picture, the core is a distinct physical state—a Bose-Einstein condensate-like configuration of the scalar field—that naturally supports a flat density profile with a characteristic radius R_T .

Comparison with Alternative Frameworks

The TEP framework shares phenomenological features with both Modified Newtonian Dynamics (MOND) and Fuzzy Dark Matter (FDM) but is distinguished by its screening mechanism and cross-scale predictive power.

Feature	MOND	Fuzzy DM (FDM)	TEP (Soliton)
Physical Basis	Modified Inertia/Gravity	Ultralight Boson ($m \sim 10^{-22}$ eV)	Conformal Time Field Saturation
Fundamental Scale	Acceleration $a_0 \approx 1.2 \times 10^{-10}$ m/s ²	Particle Mass m_ψ	Core Saturation $\rho_T \approx 20$ g/cm ³ & Shear Scale $g_t = cH_0/2$
Galactic Scaling	Fits RAR (a_0 is free parameter)	$M_c \propto M_h^{1/3}$ (Core-Halo)	$R \propto M^{1/3}$ (derived from ρ_T)
Screening	External Field Effect (EFE)	None (requires tuning)	Temporal Topology ($S \propto \rho^{1/3}$)
Solar System	Recovered via interpolation function	Unsuppressed (tension)	Screened ($S = 0.65$ for Earth, $\sim 29,500$ for Pulsars)
Key Prediction	Exact RAR	Solitonic Cores	GNSS Correlation Length + Screening Hierarchy

TEP vs. MOND

MOND provides an excellent fit to galaxy rotation curves via a single acceleration threshold ($a_0 \approx 1.2 \times 10^{-10}$ m/s²) but offers no explanation for terrestrial GNSS clock correlations or compact-object core scales. In the two-branch TEP action (Paper 0 §7; Appendix C), both scales coexist without contradiction because they belong to the two complementary sectors of $P(X, \phi) = P_{\text{kin}}(X) - V(\phi)$: the kinetic gradient scale $\Lambda_X = \sqrt{M_{\text{Pl}} H_0}$ fixes the diffuse-halo Temporal Shear acceleration threshold $g_t = cH_0/2 \simeq 3.40 \times 10^{-10}$ m/s² (yielding $a_{\text{eff}} = 4\sqrt{2} g_t/k \simeq a_0$ in the SPARC RAR and BTFR, Steps 10–13, and the outer screening shell $r_*(M) = \sqrt{GM/g_t}$), whereas the potential/amplitude sector fixes the compact-core Temporal Topology saturation density $\rho_T \approx 20$ g/cm³ ($R_T \propto M^{1/3}$).

TEP thus recovers MOND-like acceleration phenomenology in diffuse outer halos ($g_{\text{bar}} \lesssim a_{\text{eff}}$) from its kinetic sector while simultaneously predicting the compact saturation scale $R_T(M)$ that organizes terrestrial GNSS clock correlations, planetary interior gyration ($k_{\text{MOI}} \simeq S/2$), and the RBH-1 crossover.

TEP vs. FDM

Standard FDM predicts solitonic cores but fundamentally struggles to recover the correct structural scales for both dwarf galaxies and the Milky Way simultaneously, requiring contradictory particle mass bounds. TEP naturally and uniquely links the core scale directly to the ambient saturation scale, generating the correct cross-scale hierarchy anchored directly by GNSS empirical measurements.

Response-Coefficient Hierarchy

A natural criticism of the multi-channel TEP programme is that different papers report different numerical coefficients. This is expected, and it is a feature rather than a flaw. In the TEP framework, each measurement channel carries an *observable response coefficient* κ_X that absorbs stellar physics, environment, and projection geometry, not the bare microscopic conformal coupling β_A .

Paper 11 (TEP-H0) independently calibrates the weakly screened galactic-disk response via Cepheid period–luminosity residuals. The Paper 0 benchmark normalization is $\kappa_{\text{gal}}^{\text{bench}} = (0.96 \pm 0.40) \times 10^6$ mag, while the empirical Cepheid-channel analyses recover $\kappa_{\text{equiv}}^{39} = (1.220 \pm 0.531) \times 10^6$ mag ($\sigma_v = 250$), $\kappa_{\text{WLS}}^{44} = (1.275 \pm 0.370) \times 10^6$ mag (3.44σ ; $\sigma_v = 150$), and $\kappa_{\text{Cep}}^{44} = (0.665 \pm 0.362) \times 10^6$ mag ($\sigma_v = 250$) — bracketing the benchmark with a midpoint of 0.970×10^6 mag (1.0% agreement) — and Paper 11 Step 62 derives $|\kappa_{P_2}^{\text{pred}}(\lambda_0 = 1)| = 1.196 \times 10^6$ mag directly from the Cepheid envelope density ratio $\rho_T / \langle \rho \rangle (10 \text{ d}) = 8.45 \times 10^5$ at the unit-transport benchmark $\lambda_0 = 1$ with bare coupling $|\beta_A| = 1$ fixed by Paper 0 (matching the SH0ES fit $(1.191 \pm 0.480) \times 10^6$ mag to 0.5% at the central calibration, $\lambda_0 = 0.995 \pm 0.401$ statistical and $\lambda_0 \in [0.28, 2.49]$ across the $\rho_T \in [8, 70]$ g cm⁻³ GNSS window band; equivalently, under the conditional unit-envelope-transport assumption $\lambda_0 = 1$, inverting the SH0ES fit yields $\rho_T^{(\text{Cepheid})} = 19.91 \pm 8.02$ g cm⁻³, providing a conditional cross-regime consistency check with the GNSS anchor $\rho_T = 20 \pm 7$ g cm⁻³). Paper 10 (TEP-COS) measures the globular-cluster pulsar response $\tilde{\kappa}_{\text{MSP}} \approx 3 \times 10^4$ (full response coefficient, step_44_kappa_msp_prior.json) after dense-cluster geometric suppression; the mapping to Paper 10's effective channel coefficient is $\kappa_{\text{MSP}} = \tilde{\kappa}_{\text{MSP}} \mathcal{S}_{\text{cluster}} \approx 0.05$, i.e. a cluster suppression factor $\mathcal{S}_{\text{cluster}} \approx 1.7 \times 10^{-6}$ applied to the full response — the same coefficient quoted under the un-tilde symbol. The two values are consistent with the bare TEP geometric-factor estimate ($\sim 10^6$ – 10^7) only after environmental transfer factors are included; they do not assert a direct one-to-one equality of raw channel coefficients.

This hierarchy protects the programme from the criticism: "Why does each paper have a different coefficient?" The answer is that they are *observable response coefficients*, not bare scalar couplings. Each channel measures a different projection of $\mathcal{S}_\Sigma(\mathcal{E})$:

Channel	Response coefficient	Environment	Projection
GNSS clocks	$L_c \approx 4200$ km	Planetary crust/mantle	Radial covariance scale
Cepheids (Paper 11)	$\kappa_{\text{Cep}} = 0.665\text{--}1.275 \times 10^6$ mag (0.67--1.28 $\times 10^6$) empirical; Step 62 $\rho_T/\langle\rho\rangle \implies 1.196 \times 10^6$ mag ($\lambda_0 = 1$)	Galactic disk (weakly screened)	Period–luminosity modulation
Pulsars (Paper 10)	$\tilde{\kappa}_{\text{MSP}} \sim 10^4$ full; $\kappa_{\text{MSP}} \approx 0.05$ effective	Globular cluster (strongly screened)	Spin-down excess (suppressed)
Wide binaries (Paper 13)	$\alpha_{\text{sat}} \approx 0.366$	Galactic halo (unscreened)	Velocity-profile saturation

The cross-paper comparison is therefore through a shared clock-response structure, not raw equality of coefficients. This preserves the internal consistency of the programme while acknowledging that each domain probes a different facet of the same environmental operator. Across the broader TEP research programme, the same unified unstretched-space matter metric $\tilde{g}_{\mu\nu} = -A_{\text{loc}}^2(u_{\text{grav}})U_\mu U_\nu + A_{\text{sp}}^2(u_{\text{drift}})\Pi_{\mu\nu}$ and two-branch kinetic sector $P_{,X} = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$ ($k = 16.03$ fixed by a_0) confront the principal astrophysical and cosmological discriminators in dedicated companion studies: (i) in cluster and strong-lensing optics (Papers 4, 19), intermediate temporal depth $-\beta_A u_{\text{grav}}(\mathbf{x})$ refracts null rays directly via $n(\mathbf{x}) \simeq 1 - 2\Phi_N/c^2 - \beta_A u_{\text{grav}}(\mathbf{x})$, reproducing the Bullet Cluster galaxy-peak convergence and offset morphology ($\kappa_{\text{tot}} = 0.307\text{--}0.372$ centered on the collisionless galaxies without a negative moat; Paper 4 §6.9) and the SN Refsdal long-baseline Fermat delay (+330.9 d sheet, +221.6–443.2 d 3-D ray; Paper 19 Step 64) from baryons alone; (ii) in the early-universe and background sector (Papers 18, 26, 27, 29), baryon clock-slope acoustic driving ($\omega_\phi = \omega_b y_s/\sqrt{2} = 0.118$) and unstretched-space temporal CMB lensing close the Planck 2018 TTTEEE + low- ℓ + lensing + BAO + Pantheon+ likelihood at zero particle dark matter ($\omega_{\text{cdm}} = 0$, $\Delta\chi^2 = -0.06$, $\Delta\text{AIC} = -2.06$; Paper 18), while the single-history Pantheon+ + DESI DR2 BAO + Planck θ_* joint fit ($\Delta\chi^2 = -8.1$ vs ΛCDM ; Paper 26) reproduces $(1+z)$ supernova light-curve time dilation and generates an emergent thawing projection (w_0, w_a) $\approx (-0.83, -0.44)$ alongside Pop-III/VMO asymptotic helium equilibrium ($Y_{\text{eq}} \approx 0.247$; Paper 29); and (iii) in precision-gravity and relativistic-population channels, quadratic-branch and shell-impedance screening suppress preferred-frame PPN coefficients ($|\alpha_2(1.6 R_\odot)| = 2.16 \times 10^{-8}$, $\alpha_2(1 \text{ AU}) = -1.47 \times 10^{-5}$) and PSR J1738+0333 dipolar radiation by $> 2,800\times$ in amplitude (Paper 0 §7.2–7.3), environmental $|Z|$ stratification and external-field screening model the transition morphology in Gaia DR3 wide binaries ($R_s = 3,447$ vs $4,912$ AU, $p = 8.1 \times 10^{-15}$; Paper 13), Virial velocity-dispersion predictors ($\Psi \propto \sigma^2$) confirm high- z JWST age-excess scaling across $N = 75$ NIRSpec systems independently of SED stellar-mass circularity ($p = 8.4 \times 10^{-7}$; Paper 12), and cumulative GWTC-1–5.0 BBH secondary-mass taper and spin mixtures favor canonical bi-metric propagation out of sample ($\Delta \ln L = +7.05$ to $+12.14$ on $m_2 \geq 5 M_\odot$, $+5.58$ to $+17.09$ on $m_2 \geq 8 M_\odot$, and $\Delta \ln L_{\text{joint}} = +11.31$ to $+14.48$ at the unified optimum $\hat{\epsilon} = 0.25$ matching the parameter-free spiral-disk evaluation $\langle\epsilon_{\text{host}}\rangle_{\text{SFR}}^{\text{UCD}} = 0.236 \pm 0.054$ [0.243 ± 0.052 disk-integrated; $0.197\text{--}0.202$ with 25% bulge/NSC hosts] of Section 3, alongside $+11.02$ to $+22.55$ at the unscreened $\epsilon = 0$ boundary; Paper 22).

Resolving Galactic Anomalies

The Radial Acceleration Relation (RAR)

The SPARC analysis (Section 3) confirms that the onset of mass discrepancies follows the baryonic mass distribution, a key feature of the Radial Acceleration Relation (McGaugh et al. 2016). In TEP, this coupling is a natural consequence of the assumed sourcing: the scalar field is sourced by the trace of the energy-momentum tensor (T_μ^μ), so the "dark matter" halo is directly anchored to the baryonic mass. This recovers the MOND-like phenomenology of the RAR without modifying inertia or requiring new force laws, through the non-linear response of the scalar sector.

Ultra-Diffuse Galaxies (DF2/DF4): Hyperbolic Re-Equilibration and External-Field Screening

The baryon-dominated ultra-diffuse galaxies NGC 1052-DF2 and DF4 ($M_{\text{bar}} \approx 2 \times 10^8 M_\odot$, $R_e \approx 2.2$ kpc, observed velocity dispersion $\sigma_{\text{los}} = 8.5_{-3.1}^{+2.3}$ km s $^{-1}$ vs. Newtonian baryonic $\sigma_{\text{bar}} \approx 7.0$ km s $^{-1}$; van Dokkum et al. 2018, 2019, 2022) are accounted for quantitatively by the two-branch TEP field equations (Step 14, `df2_df4_udg_hyperbolic_re_equilibration_and_efe`; Appendix B.7). First, because scalar perturbations on a static spacelike gradient propagate at the hyperbolic radial group speed $v_\parallel = \sqrt{1 + 2\xi P_{,XX}/P_{,X}} c \in [1, \sqrt{3}] c$ (Eq. [62]), the scalar field re-equilibrates across $R_e = 2.2$ kpc in the light-crossing time $\tau_{\text{eq}} = R_e/(v_\parallel c) = 4.14\text{--}7.18$ kyr $\ll \tau_{\text{dyn}} \approx 1.5 \times 10^8$ yr, so the field tracks the instantaneous baryon configuration quasi-statically. Second, because the nonlinear kinetic operator $P_{,X}(\xi_{\text{tot}})$ depends on the total scalar gradient $\xi_{\text{tot}} = |\nabla u_{\text{int}} + \nabla u_{\text{ext}}|^2/\Lambda_X^4$ (Paper 0 §7), an isolated dwarf with internal acceleration $g_{\text{int}}(R_e) = GM_{\text{bar}}/(2R_e^2) = 5.9 \times 10^{-12}$ m s $^{-2} \ll a_{\text{eff}}$ would exhibit $\sigma_{\text{iso}}^{\text{TEP}} = \sigma_{\text{bar}}(1 + \sqrt{a_{\text{eff}}/g_{\text{int}}})^{1/2} = 19.1$ km s $^{-1}$, whereas embedding DF2/DF4 inside the external gravitational potential of the massive elliptical host NGC 1052 ($v_{\text{host}} \approx 300$ km s $^{-1}$ at pericentric/group separation $D \approx 25$ kpc, $g_{\text{ext}} = v_{\text{host}}^2/D = 1.17 \times 10^{-10}$ m s $^{-2} \simeq a_{\text{eff}}$) dielectric-saturates $P_{,X}(\xi_{\text{tot}})$ and suppresses the internal boost to $a_{\phi,\text{int}}/g_{\text{int}} = \frac{1}{2}[\sqrt{1 + 4a_{\text{eff}}/(g_{\text{int}} + g_{\text{ext}})} - 1] = 0.62$, yielding $\sigma_{\text{los}}^{\text{EFE}} = \sigma_{\text{bar}}(1 + a_{\phi,\text{int}}/g_{\text{int}})^{1/2} = 8.91$ km s $^{-1}$ — matching the measured $8.5_{-3.1}^{+2.3}$ km s $^{-1}$ with zero free parameters.

Systematic Uncertainties

The primary uncertainty in the global fit remains the GNSS calibration length L_c . Four distinct contributions comprise this uncertainty (Section 2): (i) the 25.3-year CODE per-measurement and cross-sector dispersion ($4,201 \pm 1,967$ km, $\pm 47\%$, with azimuth-averaged isotropic $\lambda = 3,210 \pm 389$ km; Paper 2); (ii) the GPS-only PPP inter-center systematic spread across CODE, IGS, and ESA ($3,330$ – $4,549$ km, CV 18.2%, $\sim \pm 15\%$ half-range; Paper 1, and $4,767 \pm 835$ km on raw-RINEX ionofree; Paper 3), with the caveat that centers share largely overlapping underlying data so this spread is not an independent ensemble; (iii) the fractional window/chain scatter ($1,050$ – $1,935$ km quarterly windows) and multi-GNSS combined-filter transfer offset established by the MGEX stress test (Paper 14 Steps 3.1, 3.5) and Step 14 Monte Carlo injection–recovery suite (where 4-constellation ISB projection compresses a $4,200$ km injection by 1.62 – $1.77\times$, reproducing the empirical $1.90\times$ offset that maps pooled MGEX $1,862$ km to $3,536$ km); and (iv) the $\mathcal{O}(1)$ datum-projection prefactor established by the 4-layer PREM BVP integration (Section 2; $L_c^{\text{PREM}}/R_T(M_\oplus) = 0.967$ on the transition shell). Applying the ~ 30 – 40% fractional window/chain uncertainty around the 25-year GPS-only anchor ($L_c = 4,200$ km) under the conditional $L_c \leftrightarrow R_T(M_\oplus)$ identification yields $\rho_T = 20 \pm 7$ g/cm³ and the primary systematic band $\rho_T \in [8, 70]$ g/cm³ (while the unadjusted 4-constellation MGEX scale defines a conservative upper sensitivity bracket of $\rho_T \sim 69$ – 220 g/cm³; Appendix B.8). The $M^{1/3}$ and $M^{1/2}$ structural forms are independent of ρ_T ; only the compact saturation normalization changes. Even the widened band is far smaller than the dynamic range over which the model is tested, preserving the falsifiability of the cross-scale framework.

Predictions for High-Redshift JWST Observations

The TEP framework makes concrete predictions for the "Little Red Dots"—compact, massive galaxies at $z > 5$ discovered by JWST. These objects are a critical testing ground because they are compact enough for scalar-envelope effects to dominate their apparent structure, yet massive enough for JWST to resolve the predicted kpc-scale transition region.

Qualitative Predictions for High-Redshift JWST Observations

For a target galaxy with stellar mass $M_* = 10^{10} M_\odot$ (typical of LRDs), the TEP model makes the following qualitative predictions:

1. Soliton Envelope Scale

The fundamental core saturation density $\rho_T \approx 20$ g/cm³ governs the compact soliton surface, but the observable galactic transition is set by the diffuse screening density $\rho_{\text{trans}} \approx 3 \times 10^{-23}$ g/cm³ (fitted from SPARC, Section 3). The predicted emission extent therefore scales as $R_{\text{trans}} \propto M^{1/3}$ at the transition density, not at ρ_T . Precise numerical predictions for high-redshift galaxies require epoch-specific calibration of ρ_{trans} ; until then, the model predicts that resolved gas tracers (e.g., H α) should reveal more extended kinematic structure than expected from a purely stellar compact core.

2. Velocity Dispersion

The dynamical support speed is set by the soliton potential at the transition radius: $\sigma_v^2 \sim GM/R_{\text{trans}}$. Because $R_{\text{trans}} \propto M^{1/3}$, the velocity dispersion scales as $\sigma_v \propto M^{1/3}$. This predicts characteristic velocity dispersions intermediate between the broad-line AGN regime (> 1000 km/s) and the low-mass dwarf regime (< 50 km/s). At redshift z , the two-branch threshold scales with the expansion rate as $g_t(z) = cH(z)/2$ ($a_{\text{eff}}(z) = 4\sqrt{2}g_t(z)/k$; Paper 12), fixing the epoch-dependent transition density $\bar{\rho}_{\text{trans}}(z) \propto H(z)^2$ without free parameters.

Discriminant: Soliton vs. Supermassive Black Hole

A leading alternative explanation is that LRDs are reddened Active Galactic Nuclei (AGNs) powered by supermassive black holes ($M_{\text{BH}} \sim 10^7$ – $10^8 M_\odot$).

- **AGN Hypothesis:** Emission should be point-like or confined to the nuclear region (< 1 kpc). Broad lines reflect BLR physics ($v > 1000$ km/s).
- **TEP Soliton Hypothesis:** Emission should be extended over the galactic transition scale (sub-kpc to kpc-scale for typical LRD masses under $g_t(z) = cH(z)/2$; Paper 12). Broad lines reflect the deep gravitational potential of the soliton well (~ 100 – 200 km/s).

Discriminant: Spatially resolve the H α emission. Extended kpc-scale emission with velocity dispersions ~ 100 – 200 km/s would disfavor a purely nuclear AGN interpretation, while strongly compact emission with $v > 1000$ km/s would favor an AGN-dominated explanation. This constitutes a concrete observational discriminator for this particular soliton-interpretation channel.

Implications for Paper 7 (RBH-1)

The GNSS-consistent ρ_T identified here provides the working normalization for the analysis of RBH-1 in the companion paper (Smaufield 2025h). By adopting $\rho_T \approx 20$ g/cm³ (with stated uncertainty), the soliton interpretation for RBH-1 becomes tightly constrained. The wake structure, velocity jump, and thermal emission properties can then be compared against the interaction of the wake with a soliton of characteristic radius $R \approx 7.8 \times 10^7$ km. Deviations would primarily constrain the object-specific soliton

interpretation and/or the assumed mapping between ρ_T and the effective radius in this environment, rather than the broader TEP framework.

Cross-Domain Constraint Network

Tier	Role	Channels	Relation to ρ_T
A	Direct / quasi-direct calibration	GNSS, SPARC/MW, RBH-1, Planetary Spin/MOI, condensed-matter EOS	Enters ρ_T normalization and dynamical scale checks directly
B	Screening / response-transfer test	Pulsars (10), Cepheids (11), wide binaries (13), LLR (17), Earth flyby (15), Outer System (36)	Constrains $S_\Sigma(\mathcal{E})$ and transfer functions, not independent ρ_T
C	Phantom Mass / conformal transport	Lensing (4), multiply-imaged SNe (19), JWST (12)	Tests the mechanism; not a ρ_T calibration
D	Orthogonal architecture test	J0437 (16), hi_class (18), LVK (22), QF (23), C0 (26), TH (27), BH (28), BBN (29)	Tests deep TEP structure; does not calibrate ρ_T

Only Tier A contributes to the present ρ_T normalization. Tiers B–D constrain the environmental operator, response hierarchy, and theoretical viability of the same TEP architecture.

The TEP programme now spans thirty papers across terrestrial, galactic, compact-object, lensing, and cosmological domains. It is essential to distinguish which of these constrain ρ_T directly and which constrain the broader TEP architecture. The following tiered classification makes this dependence explicit and prevents the misreading that every paper independently confirms the same density number.

Tier A — Direct or Quasi-Direct ρ_T Constraints

These channels calibrate the macroscopic saturation scale most directly:

- **GNSS λ_T (Papers 1–3):** The primary empirical anchor. The 4,200 km correlation length is conditionally identified with the projected covariance scale associated with $R_T(M_\oplus)$, up to an order-unity transfer factor, yielding the operational estimate $\rho_T \approx 20 \text{ g/cm}^3$.
- **SPARC/Milky Way onset scaling (this work):** Quasi-direct scaling consistency. The $M^{1/3}$ scaling of dark-matter onset radii across 168 SPARC galaxies (out of 175) is consistent with the saturation-radius construction, but does not independently calibrate ρ_T . The Milky Way test uses the SPARC-calibrated normalization and is therefore not an independent measurement.
- **RBH-1 crossover (Paper 7):** The runaway-black-hole candidate tests the extrapolation of $R_T(M)$ to $M \sim 10^7 M_\odot$, with wake structure providing an object-specific consistency test.
- **Planetary rotational kinematics, substellar period floor, and Solar Model S prototype (this work, Section 4.7):** Zero-parameter dynamical test combining $\rho_T = 20 \text{ g/cm}^3$ ($\omega_T = 2.365 \times 10^{-3} \text{ rad s}^{-1}$) with the Solar-Circle Galactic temporal depth $u_{\text{gal}} = (v_{\text{gal}}/c)^2$ to reproduce unbraked outer-giant-planet equatorial speeds ($v_{\text{obs}}/v_{\text{TEP}} = 1.000 \pm 0.080$, $\hat{\rho}_T^{\text{spin}} = 19.9 \text{ g/cm}^3$ across Uranus, Neptune, and Saturn; 1.016 ± 0.073 with lunar-braked Earth, with residuals tracking $r_*(M)/a$ at $r = +0.960$), the one-parameter ($\eta_0 = 0.129$, candidate benchmark $\eta_{\text{sec}} = 1/8$) two-branch + S_A extension across $N = 8$ unbraked outer-planet and substellar systems ($v_{\text{obs}}/v_{2\text{br}} = 1.001 \pm 0.060$, $\hat{\rho}_T^{\text{rot}} = 20.0 \text{ g/cm}^3$; 1.008 ± 0.060 across $N = 9$ with Earth, including SIMP J0136 and the 1.172 hr [IQR 0.98–1.24 hr] brown-dwarf period floor), interior moments of inertia ($2k_{\text{MOI}}/S = 1.039 \pm 0.027$ and $\rho_T^{\text{MOI,terr}} = 17.9 \pm 1.4 \text{ g/cm}^3$ across Mercury, Venus, and Earth; 1.17 ± 0.14 across all eight planets at 1-bar radii, and $2k_{\text{MOI}}/S_{\text{cond}} = 1.042 \pm 0.094$, $\rho_T^{\text{MOI,cond}} = 17.9 \text{ g/cm}^3$ across all seven condensed/giant planets after Lane–Emden H/He envelope de-inflation), Wesson's angular-momentum constant $p_{\text{TEP}} = G/(2v_{\text{gal}}) = 1.52 \times 10^{-15} \text{ cm}^2 \text{ g}^{-1} \text{ s}^{-1}$, and the exploratory Solar Model S convection-zone base ($\sqrt{3}R_T(M_\odot) = 0.715 R_\odot$ vs. $0.713 R_\odot$, $\rho_T^{(\text{cz})} = 20.15 \text{ g/cm}^3$; 5-channel look-elsewhere coincidence null $\hat{\rho}_T = 19.59 \pm 1.10 \text{ g/cm}^3$, $p = 2.9 \times 10^{-4}$, and $\hat{\rho}_T^{\text{no-GNSS}} = 19.68 \pm 1.25 \text{ g/cm}^3$ [$\pm 6.4\%$] across the four external astrophysical channels alone with GNSS dropped).
- **Condensed-matter EOS coincidence (this work, Section 5):** The proximity of $\rho_T \approx 20 \text{ g/cm}^3$ to warm-dense-matter / degeneracy-onset scales provides a coincidence-level physical plausibility check, not an independent empirical calibration.

Tier B — Screening and Response-Transfer Constraints

These channels do not independently calibrate ρ_T , but they constrain the environmental operator $S_\Sigma(\mathcal{E})$ and the transfer functions that connect ρ_T to observables:

- **Globular-cluster pulsars (Paper 10):** The suppressed density scaling of spin-down excess ($\Gamma \approx 0.39 \text{ dex/dex}$, vs. Newtonian 0.75) constrains how dense environments screen the temporal-field response. The observable coefficient $\tilde{\kappa}_{\text{MSP}}$ is a downstream projection, not a direct ρ_T measurement.

- **Cepheids (Paper 11):** The Cepheid clock-bias response — $0.665\text{--}1.275 \times 10^6$ mag empirically ($(0.665 \pm 0.362) \times 10^6$ mag multi-block and $(1.206 \pm 0.528) \times 10^6$ mag redshift-only WLS at canonical $\sigma_v = 250$ km/s, $(1.275 \pm 0.370) \times 10^6$ mag at $\sigma_v = 150$ km/s), bracketing the 0.96×10^6 mag benchmark and matching the Step 62 envelope-density prediction 1.196×10^6 mag at the unit-transport benchmark $\lambda_0 = 1$ ($|\beta_A| = 1$) to 0.5% at the central estimate ($(1.191 \pm 0.480) \times 10^6$ mag empirical fit) — in galactic disks bridges the weakly screened regime. It agrees with $\tilde{\kappa}_{\text{MSP}}$ after environmental transfer factors are applied, demonstrating that response coefficients are channel-specific projections.
- **Wide binaries (Paper 13):** Gaia DR3 reveals a weak-field screening transition at a few thousand AU with environmental ordering (high- $|Z|$ vs. midplane). This is an independent *morphology* test of Temporal Shear recovery, not an independent ρ_T calibration: the paper explicitly states that imported ρ_T and g_{TEP} enter only as cross-scale consistency checks.
- **Lunar Laser Ranging (Paper 17):** Reports a compactness-dependent residual channel at high significance, but remains at the "candidate residual" level pending integrator-level ephemeris refits. Classified as a *guardrail*: it suggests the local screened regime may still contain residual-channel structure, but does not yet provide a completed ρ_T likelihood term.
- **Earth flyby (Paper 15):** Uses the UCD $\rho_T \approx 20$ g/cm³ as a *shared input*, not an independent validation. Valuable as a downstream posterior predictive test: if the terrestrial screening profile is correct, flyby anomalies should organize under the restricted trajectory model.
- **Outer Solar System boundary (Paper 36):** Tests the directional proper-time domain boundary across detached extreme TNOs ($q > 30$ AU, $a > 150$ AU) and long-period comet transits, whose radial detachment and decorrelation thresholds match Neptune's individual screening shell ($r_*(M_{\text{Nep}}) = 29.97$ AU) and the combined giant-planet screening shell ($r_*(M_{\text{giants}}) = 152.6$ AU; Section 4.7).

Tier C — Phantom Mass / Conformal Transport Tests

These channels test the *mechanism* by which TEP generates apparent mass discrepancies, without directly calibrating ρ_T :

- **Gravitational lensing (Paper 4):** Under the canonical unstretched-space temporal metric (Paper 0 §2.2; Paper 4 §6.9, Step 02), intermediate temporal depth refracts photons at half the dynamical scalar potential weight ($\Phi_{\text{lens}} = \Phi_N + \frac{1}{2}c^2\beta_A u_{\text{grav}}$ vs $\Phi_{\text{dyn}} = \Phi_N + c^2\beta_A u_{\text{grav}}$, whereas on the historical isotropic-conformal benchmark $\tilde{g}_{\mu\nu} = A^2 g_{\mu\nu}$, $A^2(\phi)$ cancels out of null rays), supplying both the baryon-sourced Bullet Cluster convergence and a testable lensing-to-dynamical mass ratio $M_{\text{lens}}^{\text{u.s.}}/M_{\text{dyn}} = (1 + \beta_A^2 y_{\text{eff}})/(1 + 2\beta_A^2 y_{\text{eff}})$. This provides *mechanistic support* for the TEP interpretation of the dark sector, not an independent ρ_T calibration.
- **Multiply-imaged supernovae (Paper 19):** SN Refsdal provides directional evidence from blind model residuals for temporal transport in the lensing sector, while the action-level unstretched-space temporal refraction solve on the selected two-branch sector $P_{,X} = k\sqrt{\xi} + 2\xi$ ($k = 16.03$, Step 64) reproduces the $+330.9$ d sheet and $+221.6\text{--}443.2$ d 3-D NFW ray Fermat delay and arrival order from $f_b = 0.16$ baryons alone alongside the phenomenological magnification proxy.
- **JWST high-redshift galaxies (Paper 12):** Tests potential-depth response at $z > 5$ using an external response prior. The signal is not an independent ρ_T measurement; it probes whether the same conformal architecture organizes high-redshift compact galaxies.

Tier D — Orthogonal TEP-Architecture Tests

These papers do not calibrate ρ_T at all, but they test deep structural claims of the TEP framework:

- **J0437 scintillation (Paper 16):** Reports non-zero phase-closure signal in millisecond pulsar scintillation, rejecting standard scalar-delay ISM models ($\psi = 0$) at high confidence. This tests *non-integrable time transport* / synchronization holonomy, one of the deepest TEP claims. It does not calibrate ρ_T .
- **hi_class cosmology (Paper 18):** Demonstrates that the conformal temporal-transport geometry preserves CMB acoustic structure, sound-horizon behaviour, and stable linear perturbations. A *global consistency constraint*, not a ρ_T term.
- **C0 supernova conformal redshift (Paper 26):** Shows that Pantheon+ distance-redshift data do not uniquely require primitive spatial expansion. Supports the temporal-transport interpretation of cosmological redshift.
- **TH temporal horizon (Paper 27):** The apparent $a_{\text{eff}} \rightarrow 0$ limit becomes a temporal conformal boundary rather than a physical singularity. Provides *theoretical closure* for the late-time cosmology.
- **QF quantum tangent limit (Paper 23):** Shows that the screened quantum tangent limit is consistent with the TEP architecture. A *theoretical compatibility* check, not empirical ρ_T evidence.

Summary: Only Tier A enters the present ρ_T normalization argument directly. Tiers B–D constrain the environmental operator, response hierarchy, and theoretical viability of the same TEP architecture. This tiered dependence is stronger than a flat pile of evidence because it acknowledges exactly where each paper contributes and where it does not.

9. Conclusion

A Universal Organizing Parameter

This paper identifies the saturation scale $\rho_T \approx 20 \text{ g/cm}^3$ as a candidate organizing parameter for gravitational phenomena across planetary, galactic, compact-object, and RBH-scale regimes. The convergence of terrestrial GNSS timing structure, SPARC galaxy onset scaling, Milky Way inner mass-discrepancy structure, and the density-ordered screening hierarchy motivates the hypothesis that ρ_T is a real physical scale in the temporal-field topology, not a system-specific fitting parameter. The screening hierarchy is not treated as an independent discovery of the $1/3$ exponent; rather, it demonstrates that the same ρ_T coherently recovers GR-like behavior in dense systems while permitting active-shear scalar phenomenology in diffuse regimes.

Key Results

The primary findings are:

- GNSS Calibration:** Distance-structured correlations in atomic clocks yield $L_c \approx 4200 \text{ km}$, corresponding to $\rho_T \approx 20 \text{ g/cm}^3$ under the conditional identification $L_c \leftrightarrow R_T(M_\oplus)$. The underlying GNSS correlation scale L_c exhibits 25-year stability, multi-center consistency, and survives raw RINEX validation.
- Galactic Scaling:** SPARC rotation curves yield $\alpha_{\text{SPARC}} = 0.352 \pm 0.043 \text{ (stat)} \pm 0.07 \text{ (definition)}$ on the discrete grid and 0.417 ± 0.050 on the $n = 75$ resolved crossings, where an *a priori* Freeman-disk baryonic slope split ($\gamma_{\text{Freeman}} = 0.210$) separates rising-baryon 3-D core crossings ($\alpha_{\text{rising}} = 0.256 \pm 0.069$ vs. $1/3$) from concentrated Keplerian-tail crossings ($\alpha_{\text{tail}} = 0.581 \pm 0.086$ vs. $1/2$; $\Delta\alpha = +0.325 \pm 0.110$, $z = 2.95$, $p = 0.0031$).
- Deterministic Regime Map:** Evaluating $S \equiv R_T(M)/R_{\text{phys}} \equiv (\bar{\rho}/\rho_T)^{1/3}$ across 26 objects spanning ~ 15 orders of magnitude in density defines a deterministic algebraic regime map explaining why compact-body GR tests pass (binary pulsars: $S \sim 29,500$) while galactic dynamics reside in the unscreened active-shear regime ($S \sim 10^{-9}$ at $\rho \sim 10^{-24} \text{ g/cm}^3$).
- Milky Way Test:** The SPARC-calibrated $M^{1/3}$ relation predicts a dark-matter onset radius $R_{\text{DM}} \approx 3 \text{ kpc}$ for the Milky Way, consistent with the observed transition from baryonic to dark-matter-dominated rotation at $R \sim 3\text{--}5 \text{ kpc}$, providing a local scale-consistency check.
- Planetary Rotational Kinematics & Solar Prototype:** Combining the universal saturation clock frequency $\omega_T = \sqrt{4\pi G \rho_T/3} = 2.365 \times 10^{-3} \text{ rad s}^{-1}$ ($\tau_T = 44.3 \text{ min}$) with the Solar-Circle Galactic temporal depth $u_{\text{gal}} = (v_{\text{gal}}/c)^2$ yields the zero-parameter ambient equatorial rotation law $v_{\text{eq}}/c = u_T(M)/\sqrt{u_{\text{gal}}}$ ($v_{\text{obs}}/v_{\text{TEP}} = 1.000 \pm 0.080$ and $\hat{\rho}_T^{\text{spin}} = 19.9 \text{ g/cm}^3$ across the three unbraked outer giant planets Uranus, Neptune, and Saturn; 1.016 ± 0.073 and 20.9 g/cm^3 with lunar-braked Earth, where residuals track $r_*(M)/a$ at $r = +0.960$, $p = 0.040$ and volumetric/deep-interior periods give 1.031 ± 0.041), its one-parameter ($\eta_0 = 0.129$, candidate benchmark $\eta_{\text{sec}} = 1/8$) two-branch + S_A extension across the $N = 8$ unbraked outer-planet and substellar speed-limit systems ($v_{\text{obs}}/v_{2\text{br}} = 1.001 \pm 0.060$, $\hat{\rho}_T^{\text{rot}} = 20.0 \text{ g/cm}^3$; 1.008 ± 0.060 across $N = 9$ with Earth, matching SIMP J0136 within -2.8% [-0.3% at η_{sec}] and the brown-dwarf period floor $P_{\text{min}}^{\text{TEP}} = (k_{\text{MOI}}/\eta_0)\tau_T = 1.172 \text{ hr}$ [IQR $0.98\text{--}1.24 \text{ hr}$; 1.21 hr at η_{sec}] vs. observed $1.150 \pm 0.075 \text{ hr}$), the self-compressed planetary gyration scaling $k_{\text{MOI}} \simeq \frac{1}{2}S$ ($2k_{\text{MOI}}/S = 1.039 \pm 0.027$ and $\hat{\rho}_T^{\text{MOI,terr}} = 17.9 \pm 1.4 \text{ g/cm}^3$ across Mercury, Venus, and Earth; 1.17 ± 0.14 across all eight planets at 1-bar radii, and $2k_{\text{MOI}}/S_{\text{cond}} = 1.042 \pm 0.094$, $\rho_T^{\text{MOI,cond}} = 17.9 \text{ g/cm}^3$ across all seven condensed/giant planets after Lane–Emden envelope de-inflation), the zero-parameter Wesson constant $p_{\text{TEP}} = G/(2v_{\text{gal}}) = 1.52 \times 10^{-15} \text{ cm}^2 \text{ g}^{-1} \text{ s}^{-1}$ ($\chi_{\text{TEP}} = 681.3$), and the exploratory Standard Solar Model helioseismic prototype (nuclear core $\rho_\odot = \rho_T$ at $0.249 R_\odot$, convection-zone base $\sqrt{3}R_T(M_\odot) = 0.715 R_\odot$ vs. $0.713 R_\odot$ [$\rho_T^{(\text{cz})} = 20.15 \text{ g/cm}^3$], prolate tachocline $r_c = 0.688\text{--}0.708 R_\odot$ vs. $0.693\text{--}0.717 R_\odot$, equatorial rate 456.5 vs. 460.7 nHz , and differential shear $S_\odot/\sqrt{2} = 0.292$ vs. 0.289 ; across all five independent ρ_T -calibrating channels, $\hat{\rho}_T = 19.59 \pm 1.10 \text{ g/cm}^3$ [$\sigma_{\log_{10} \rho} = 0.025 \text{ dex}$], rejecting random-density look-elsewhere coincidence at $p = 2.9 \times 10^{-4}$ [3.6σ]).

Dark Matter as Phantom Mass

The TEP framework reinterprets dark-matter observations as violations of the isochrony axiom—the assumption that spatially separated clocks at the same gravitational potential tick at the same rate. On the selected two-branch kinetic sector $P_{,X} = k\sqrt{\xi} + 2\xi$ ($k = 16.03$ fixed externally by a_0 , zero per-galaxy free parameters), the coupled-gradient additive force law $a_\phi(a_\phi + g_N) = g_N a_{\text{eff}} \implies g_{\text{obs}} = \frac{1}{2}g_N + \sqrt{(g_N/2)^2 + g_N a_{\text{eff}}}$ derives the fiducial $V_{\text{obs}}/V_{\text{bar}} = \sqrt{(1 + \sqrt{5})/2} = 1.272 \approx 1.3$ onset threshold directly from the action and reproduces the SPARC radial-acceleration relation at the canonical $\Upsilon_d = 0.50$, $\Upsilon_b = 0.70$ with 0.1330 dex rms (61.1% within $\pm 0.1 \text{ dex}$, matching the empirical 0.1328 dex MOND benchmark with zero per-galaxy refits on the quality-controlled $N = 2,700$ SPARC RAR sample, vs 0.1522 dex uncoupled and 0.1394 dex quadrature; 0.1994 dex across all $3,389$ raw points vs 0.2812 dex for the unit-term baseline, Step 10) and the outer-edge BTFR ($v_{\text{obs}}/v_{\text{pred}} = 0.956$, 0.0807 dex rms at $\Upsilon_d = 0.50$, and 1.014 , 0.0785 dex at $\Upsilon_d = 0.35$, beating the 0.0812 dex quadrature comparator, Step 13), while the high-gradient quadratic branch satisfies Solar-System screening bounds. Evaluated under the canonical unstretched-space temporal metric (Paper 0 §2.2, Step 80), this same zero-parameter kinetic sector also closes the optical lensing sector across cluster, strong-lensing, and CMB scales from baryons alone (Papers 4, 18, 19).

This reinterpretation offers a candidate common account of several dark-sector puzzles:

- **Universal scaling:** The $M^{1/3}$ relation follows naturally from the saturation-radius construction.

- **Dark-matter-deficient UDGs (DF2/DF4):** Hyperbolic scalar propagation ($v_{\parallel} \in [1, \sqrt{3}] c$) fixes the field re-equilibration time across $R_e = 2.2$ kpc to $\tau_{\text{eq}} = 4.14\text{--}7.18$ kyr $\ll \tau_{\text{dyn}}$, while external-field kinetic saturation in the NGC 1052 host potential ($g_{\text{ext}} \simeq 1.17 \times 10^{-10} \text{ m s}^{-2}$) suppresses the predicted velocity dispersion from 19.1 km s^{-1} (isolated) to $\sigma_{\text{los}}^{\text{EFE}} = 8.91 \text{ km s}^{-1}$, matching the observed $8.5^{+2.3}_{-3.1} \text{ km s}^{-1}$ (Step 14).
- **Core-cusp structure:** Saturated temporal-field cores offer a possible route to flat effective density profiles.
- **Baryonic Tully–Fisher behaviour:** On the selected two-branch kinetic sector $P_{,X} = k\sqrt{\xi} + 2\xi$ ($k = 16.03$), flux conservation derives the deep-tail asymptote $v_{\text{flat}}^4 = GM_b a_{\text{eff}}$ with $a_{\text{eff}} = 4\sqrt{2} g_t/k = 1.20 \times 10^{-10} \text{ m s}^{-2}$ and a finite-radius slope $d \log M_b/d \log V_{\text{flat}} = 3.53\text{--}3.64$ across the SPARC outer-edge regime ($g_N/g_t \sim 0.02$), matching the SPARC outer-edge BTFR ($v_{\text{obs}}/v_{\text{pred}} = 0.956, 0.0807$ dex rms at canonical $\Upsilon_d = 0.50$ under coupled-gradient additive closure, vs 1.0109 quadrature and 0.910 uncoupled additive, with inverse-OLS slope $3.76\text{--}3.86$ on the quality-filtered sample, Step 13) with zero galaxy-specific free parameters.

Astrophysical Applications

The GNSS-consistent value $\rho_T \approx 20 \text{ g/cm}^3$ enables specific applications for astrophysical systems. The companion paper (Smawfield 2025h, Paper 7) applies this calibration to the RBH-1 runaway black hole candidate, yielding a saturation radius $R_T \approx 7.8 \times 10^7 \text{ km}$. This provides a testable extrapolation: if the RBH-1 mass is revised by a factor of 3, the expected radius would change by $\sim 44\%$, offering a clear consistency check with observations.

Future applications include:

- **JWST high-redshift galaxies:** "Little Red Dots" and massive quiescent galaxies at $z > 10$ may exhibit soliton signatures.
- **EHT polarimetry:** M87* and Sgr A* polarization patterns could reveal scalar field structure near supermassive compact objects.
- **Gravitational waves:** Binary black hole mergers may produce scalar radiation detectable by LISA.
- **Strong lensing time delays:** The phantom-mass sector can act on time-delay cosmography both through the mass-model normalization — where the single-halo dynamical–lensing mass difference biases the velocity-dispersion prior that breaks the mass-sheet degeneracy in the direction that elevates the inferred H_0 (Paper 19, Step 58 channel audit; 0.0 d propagated static-conformal residual on the isotropic-conformal benchmark) — and directly through the canonical unstretched-space temporal metric (Paper 0 §2.2; Paper 19, Step 64), where intermediate temporal depth on the two-branch sector $P_{,X} = k\sqrt{\xi} + 2\xi$ reproduces the SN Refsdal Fermat delay ($+330.9$ d sheet and $+221.6\text{--}443.2$ d 3-D NFW ray) from $f_b = 0.16$ baryons alone.

Theoretical Implications

The convergence of terrestrial, compact, galactic, and cosmological constraints on a single density scale suggests a fundamental organization principle governing gravitational phenomenology across many orders of magnitude in mass. This organization is formalized in the Temporal Equivalence Principle (Smawfield 2025a, Paper 0), which posits that gravitational phenomena arise from a temporal scalar field $\phi(x^\mu)$ coupled to matter via the two-branch k-essence action:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + P(X, \phi) \right] + S_m[\tilde{g}_{\mu\nu}], \quad P(X, \phi) = P_{\text{kin}}(X) - V(\phi), \quad (43)$$

where $\tilde{g}_{\mu\nu} = -A_{\text{loc}}^2(u_{\text{grav}})U_\mu U_\nu + A_{\text{sp}}^2(u_{\text{drift}})\Pi_{\mu\nu}$ in the canonical unified unstretched-space temporal metric ($\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}$ on the historical isotropic-conformal benchmark) with $A(\phi) = \exp(\beta_A \phi/M_{\text{Pl}})$.

The saturation scale ρ_T emerges as the scale at which the scalar field ϕ reaches its self-interaction threshold, saturating the field. Under a soliton interpretation, this would correspond to soliton formation; regardless of microscopic realization, this provides a natural explanation for the $M^{1/3}$ scaling and the dynamical phantom-mass phenomenon.

Systematic Uncertainties and Robustness

The primary uncertainty is the GNSS length scale determination. Four distinct contributions are reported: (i) the 25.3-year CODE GPS-only per-measurement and cross-sector dispersion ($4,201 \pm 1,967 \text{ km}$, $\pm 47\%$, with azimuth-averaged isotropic $\lambda = 3,210 \pm 389 \text{ km}$; Paper 2); (ii) the GPS-only PPP inter-center systematic spread across CODE, IGS, and ESA ($3,330\text{--}4,549 \text{ km}$, CV 18.2%, $\sim \pm 15\%$ half-range; Paper 1, and $4,767 \pm 835 \text{ km}$ on the raw-RINEX ionofree channel; Paper 3), with the caveat that centers share underlying data so this spread is not an independent ensemble; (iii) the fractional window/chain scatter and multi-GNSS combined-filter transfer offset established by the executed MGEX stress test (Paper 14 Steps 3.1 and 3.5): on the 4-constellation combined MGEX product, quarterly windows scatter from $1,050$ to $1,935 \text{ km}$ ($\sim 30\text{--}40\%$ fractional scatter around the $1,862 \pm 155 \text{ km}$ pooled mean), matched-day independent chains CODE/GPSEST and CNES/GINS (GRG) return $2,744$ vs. $1,933 \text{ km}$ ($\sim 30\%$ offset, GRG within 4% of the pooled $1,862 \text{ km}$), and a factorial estimator decomposition on identical station-days

(Paper 14 §4.5b, Step 3.5) proves that both GPS-only IGS-final and 4-constellation MGEX recover the identical band-ordered decay profile, with the 4-constellation combined filter compressing the fitted scale by $\sim 2\times$ relative to GPS-only PPP; and (iv) the $\mathcal{O}(1)$ datum-projection prefactor established by the 4-layer PREM BVP integration ($L_c^{\text{PREM}}/R_T(M_\oplus) = 0.967$; Section 2, Step 14). Applying the $\sim 30\text{--}40\%$ fractional window/chain uncertainty around the 25-year GPS-only anchor ($L_c = 4,200$ km) yields the primary systematic band $\rho_T \in [8, 70]$ g/cm³ around $\rho_T = 20 \pm 7$ g/cm³, while adopting the unadjusted 4-constellation MGEX scale ($\lambda = 1,862\text{--}2,744$ km) at unit prefactor defines an asymmetric upper sensitivity bracket of $\rho_T \sim 69\text{--}220$ g/cm³. Quantitative sensitivity to the numerical value of ρ_T resides in the GNSS anchor, the RBH-1 crossover ($M_\times \propto \rho_T^{-1/2}$), and the planetary v_{eq} and k_{MOI} factors ($\propto \rho_T^{\pm 1/3}$, which independently favor $\rho_T \approx 20$ g/cm³ over the unadjusted ~ 220 g/cm³ MGEX value by $2.2\times$), whereas the SPARC/Milky Way onset scaling, the algebraic $S(\rho)$ hierarchy, and the product J_{TEP} test structural exponents and scale cancellation.

The robustness of the result is demonstrated by:

- **Multi-center consistency:** CODE, IGS, and ESA yield consistent GNSS correlation structure across independent processing centres.
- **Temporal stability:** 25-year consistency disfavors transient effects.
- **Raw data validation:** RINEX analysis constrains processing artifacts.
- **Cross-scale agreement:** Five independent saturation-density channels ($\hat{\rho}_T = 19.59 \pm 1.10$ g/cm³; 19.68 ± 1.25 g/cm³ GNSS-free) plus compact-to-SMBH regime tests spanning 18 orders of magnitude in mass ($10^{20}\text{--}10^{38}$ kg).

Future Directions

The identification of $\rho_T \approx 20$ g/cm³ as a candidate cross-scale saturation parameter opens several avenues for future research:

1. **Non-Spherical 3-D Azimuthal Kinetic PDE Integration for Earth:** Extending the completed 4-layer PREM canonical-kinetic ($L_c^{\text{PREM}} = 4,010$ km, $L_c^{\text{vol}} = 3,213$ km) and 1-D spherical nonlinear Picard ($L_c^{\text{NL-PREM}} = 3,957$ km, $L_c^{\text{NL,vol}} = 3,170$ km, $\|\Delta u^{(100)}\|_\infty/\|u\|_\infty = 2.67 \times 10^{-14}$; Step 14) BVP solutions of Eq. (62) to a non-spherical 3-D azimuthal solver incorporating Earth's heliocentric orbital velocity vector to model directional EW/NS clock-covariance anisotropy from first principles.
2. **Independent Third-Party Geodesy & Optical Time-Transfer Replication:** Having executed the 4-quarter 2023 CDDIS RINEX3 single-constellation GPS, Galileo, and GLONASS reprocessing and station-geometry resampling nulls on public archives in Step 14 ($\bar{L}_c = 3,853$ km $\sqrt{N}$ -weighted and $\bar{L}_c^{\text{CME-rem}} = 3,155 \pm 181$ km [CV = 5.7%] before 4-constellation ISB compression to 1,478 km), the primary metrological target is independent replication of the public CDDIS/ `RTKLIB` recipe by external geodesy groups alongside continental optical-fiber and two-way satellite time-transfer links.
3. **Exoplanet and Brown-Dwarf Rotational Population Tests:** Testing the soliton-capacity breakup ceiling ($v_{\text{eq}}/v_{\text{breakup}} \leq (\eta_0/k_{\text{MOI}})\sqrt{\min(1, S)}$ for $M > M_{\text{cap}} = 0.61 M_{\text{Jup}}$, satisfied by 27/27 objects in Bryan et al. 2020; Step 15) against expanded $v \sin i$ catalogs of directly imaged super-Jupiters and brown dwarfs and TESS/Kepler stellar hosts.
4. **High-Redshift Galaxies & Gravitational-Wave Cosmology:** Testing soliton-core kinematics in $z > 10$ JWST galaxies (Paper 12) and bi-metric propagation in standard sirens (Paper 22).

Concluding Remarks

This paper identifies $\rho_T \approx 20$ g/cm³ as a candidate macroscopic saturation/proximity scale in the TEP environmental-response hierarchy. GNSS clock correlations provide the primary terrestrial timing anchor for the Earth-scale covariance length. SPARC onset scaling, Milky Way structure, RBH-1 crossover behaviour, compact-object screening, and condensed-matter/EOS considerations then test whether the same scale organizes otherwise disconnected gravitational regularities without retuning. Channel-specific quantities such as ρ_{trans} , R_s , κ_{Cep} , κ_{MSP} , and lensing response coefficients are not independent measurements of ρ_T , but downstream projections of $S_\Sigma(\mathcal{E})$.

This paradigm shift—from dark matter as substance to dark matter as temporal shear—provides a candidate unified organizing framework for the $M^{1/3}$ scaling, screening hierarchy, and dynamical phantom-mass phenomenon, to the extent permitted by lensing–dynamics consistency. The GNSS-consistent value of ρ_T enables specific testable applications, including the RBH-1 case study (Paper 7), positioning the Temporal Equivalence Principle as an empirically grounded candidate alternative to the particle dark matter paradigm in the dynamical sector, with the dynamical–lensing mass comparison (Paper 19) as its decisive open test.

References

- Abedi, J., Dykaar, H., & Afshordi, N. 2017, *Phys. Rev. D*, 96, 082004 (arXiv:1612.00266)
- Allen, M. G., Groves, B. A., Dopita, M. A., Sutherland, R. S., & Kewley, L. J. 2008, *ApJS*, 178, 20 (arXiv:0805.0204)
- Babichev, E., Deffayet, C., & Ziour, R. 2009, *Int. J. Mod. Phys. D*, 18, 2147 (DOI: 10.1142/S0218271809016107; arXiv:0905.2943; k-Mouflage gravity)
- Barreira, A., Brax, P., Clesse, S., Li, B., & Valageas, P. 2015, *Phys. Rev. D*, 91, 123522 (DOI: 10.1103/PhysRevD.91.123522; arXiv:1504.01493; K-mouflage gravity models that pass Solar System and cosmological constraints)

- Barro, G., et al. 2025, *From "The Cliff" to "Virgil": Mapping the Spectral Diversity of Little Red Dots with JWST/NIRSpec* (arXiv:2512.15853)
- Bekenstein, J. D. 2004, *Phys. Rev. D*, 70, 083509 (DOI: 10.1103/PhysRevD.70.083509; arXiv:astro-ph/0403694; Relativistic gravitation theory for the MOND paradigm)
- Bland-Hawthorn, J., & Gerhard, O. 2016, *ARA&A*, 54, 529 (Milky Way mass model)
- Brosche, P. 1963, *Z. Astrophys.*, 57, 143 (Mass–angular momentum diagram of astronomical bodies)
- Bryan, M. L., Ginzburg, S., Chiang, E., Morley, C., Bowler, B. P., Xuan, J. W., & Knutson, H. A. 2020, *ApJ*, 905, 37 (DOI: 10.3847/1538-4357/abc0ef; arXiv:2010.07315; As the Worlds Turn: Constraining Spin Evolution in the Planetary-mass Regime)
- MacDonald, G. J. F. 1964, *Rev. Geophys.*, 2, 467 (DOI: 10.1029/RG002i003p00467; Tidal friction and planetary angular momentum scaling)
- Campanelli, M., Lousto, C. O., Zlochower, Y., & Merritt, D. 2007, *Phys. Rev. Lett.* (arXiv:gr-qc/0702133)
- Cao, H., Dougherty, M. K., Hunt, G. J., et al. 2020, *Icarus*, 344, 113541 (DOI: 10.1016/j.icarus.2019.113541; The landscape of Saturn's internal magnetic field from the Cassini Grand Finale)
- Cardoso, V., Hopper, S., Macedo, C. F. B., Palenzuela, C., & Pani, P. 2016, *Phys. Rev. D*, 94, 084031 (arXiv:1608.08637)
- Castillo-Rogez, J. C., Johnson, T. V., Thomas, P. C., et al. 2012, *Icarus*, 219, 86 (DOI: 10.1016/j.icarus.2012.02.002; Geophysical evolution of Saturn's satellite Phoebe, a large planetesimal in the outer Solar System)
- Chen, K., Li, Z., Inayoshi, K., & Ho, L. C. 2025, *ApJ Lett.* (DOI: 10.3847/2041-8213/ae1955; arXiv:2505.22600)
- Colpi, M., & Dotti, M. 2011, *Adv. Astron.*, 2011, 1 (arXiv:0906.4339)
- Colpi, M., Geppert, U., & Page, D. 2000, *ApJ*, 529, L29 (DOI: 10.1086/312448; arXiv:astro-ph/9912066)
- Donato, F., Gentile, G., Salucci, P., et al. 2009, *MNRAS*, 397, 1169 (DOI: 10.1111/j.1365-2966.2009.15004.x; arXiv:0904.4054)
- Davies, M. E., Abalakin, V. K., Bursa, M., et al. 1995, *Celest. Mech. Dyn. Astron.*, 63, 127 (Report of the IAU/IAG/COSPAR Working Group on Cartographic Coordinates and Rotational Elements)
- Debras, F., & Chabrier, G. 2019, *A&A*, 632, A97 (DOI: 10.1051/0004-6361/201935682; New models of Jupiter in the context of Juno and Galileo)
- Desch, M. D., & Kaiser, M. L. 1981, *Nature*, 292, 739 (DOI: 10.1038/292739a0; Voyager measurement of the rotation period of Saturn's magnetic field)
- Desmond, H. 2023, *MNRAS*, 526, 3342 (DOI: 10.1093/mnras/stad2762; arXiv:2303.10057; The underlying radial acceleration relation)
- Dong, D., Fang, P., Bock, Y., Webb, F., Prawirodirdjo, L., Kedar, S., & Jamason, P. 2006, *J. Geophys. Res.*, 111, B03405 (DOI: 10.1029/2005JB003806; Spatiotemporal filtering using principal component analysis and Karhunen–Loeve expansion approaches for regional GPS network analysis)
- Delvecchio, I., et al. 2025, *A&A* (DOI: 10.1051/0004-6361/202557164; arXiv:2509.07100)
- Dougherty, M. K., Cao, H., Khurana, K. K., et al. 2018, *Science*, 359, 46 (DOI: 10.1126/science.aat5434; Saturn's magnetic field revealed by the Cassini Grand Finale)
- Emelyanov, N. V. 2005, *A&A*, 438, L33 (DOI: 10.1051/0004-6361:200500143; The mass of Himalia from the perturbations on other satellites)
- Event Horizon Telescope Collaboration. 2019, *ApJ Lett.*, 875, L1 (DOI: 10.3847/2041-8213/ab0ec7; M87* image)
- Event Horizon Telescope Collaboration. 2021, *ApJ Lett.*, 910, L12 (DOI: 10.3847/2041-8213/abe71d; M87* polarization)
- Event Horizon Telescope Collaboration. 2021, *ApJ Lett.*, 910, L13 (DOI: 10.3847/2041-8213/abe71e; EHT Paper VII)
- Event Horizon Telescope Collaboration. 2022, *ApJ Lett.*, 930, L12 (DOI: 10.3847/2041-8213/ac6674; Sgr A* image)
- Heeck, J., et al. 2021, *Phys. Rev. D*, 103, 115004 (arXiv:2009.08463)
- Hinterbichler, K., & Khoury, J. 2010, *Phys. Rev. Lett.*, 104, 231301 (DOI: 10.1103/PhysRevLett.104.231301; arXiv:1001.4525)
- Hong, J., Tu, R., Gao, Y., Zhang, R., Fan, L., Zhang, P., & Liu, J. 2019, *Adv. Space Res.*, 63, 3777 (DOI: 10.1016/j.asr.2019.02.037; Characteristics of inter-system biases in Multi-GNSS with precise point positioning)
- Khoury, J., & Weltman, A. 2004, *Phys. Rev. Lett.*, 93, 171104 (DOI: 10.1103/PhysRevLett.93.171104; arXiv:astro-ph/0309300)
- Kiss, C., Pál, A., Farkas-Takács, A. I., et al. 2016, *MNRAS*, 457, 2908 (DOI: 10.1093/mnras/stw081; Nereid from space: rotation, size and shape analysis from K2, Herschel and Spitzer observations)
- Hui, L., Ostriker, J. P., Tremaine, S., & Witten, E. 2017, *Phys. Rev. D*, 95, 043541 (DOI: 10.1103/PhysRevD.95.043541; arXiv:1610.08297)
- Iess, L., Militzer, B., Kaspi, Y., et al. 2019, *Science*, 364, eaat2965 (DOI: 10.1126/science.aat2965; Measurement and implications of Saturn's gravity field and ring mass)
- Komossa, S. 2012, *Recoiling black holes: electromagnetic signatures, candidates, and astrophysical implications* (arXiv:1202.1977)
- Coleman, S. 1985, *Nucl. Phys. B*, 262, 263 (DOI: 10.1016/0550-3213(85)90286-X)
- Gleiser, M. 1994, *Phys. Rev. D*, 49, 2978 (DOI: 10.1103/PhysRevD.49.2978; arXiv:hep-ph/9308279)

- Fowler, W. A., & Hoyle, F. 1964, *ApJS*, 9, 201 (DOI: 10.1086/190103; Neutrino Processes and Pair Formation in Massive Stars and Supernovae)
- Freeman, K. C. 1970, *ApJ*, 160, 811 (DOI: 10.1086/150474; On the Disks of Spiral and S0 Galaxies)
- Fuller, J., Luan, J., & Quataert, E. 2016, *MNRAS*, 458, 3867 (DOI: 10.1093/mnras/stw609; arXiv:1601.05804; Resonance locking as the source of rapid tidal migration in the Jupiter and Saturn moon systems)
- Gaia Collaboration. 2023, *A&A*, 674, A1 (DOI: 10.1051/0004-6361/202243940; Gaia Data Release 3)
- Galanti, E., Kaspi, Y., Miguel, Y., et al. 2019, *Geophys. Res. Lett.*, 46, 616 (DOI: 10.1029/2018GL078087; Saturn's deep atmospheric flows revealed by the Cassini Grand Finale gravity measurements)
- Galopeau, P. H. M., & Lecacheux, A. 2000, *J. Geophys. Res.*, 105, 13089 (DOI: 10.1029/2000JA900012; Variations of Saturn's radio rotation period measured at kilometre wavelengths)
- Goldreich, P., & Soter, S. 1966, *Icarus*, 5, 375 (DOI: 10.1016/0019-1035(66)90051-0; Q in the Solar System)
- Gong, Y., Papantonopoulos, E., & Yi, Z. 2018, *Eur. Phys. J. C*, 78, 738 (DOI: 10.1140/epjc/s10052-018-6227-9; arXiv:1711.04102)
- Gronke, M., & Oh, S. P. 2018, *MNRAS*, 480, L111 (arXiv:1806.02729)
- Gurnett, D. A., Kurth, W. S., Hospodarsky, G. B., et al. 2005, *Science*, 307, 1255 (DOI: 10.1126/science.1105356; Radio and plasma wave observations at Saturn from Cassini's approach and first orbit)
- Gurnett, D. A., Lecacheux, A., Kurth, W. S., et al. 2009, *Geophys. Res. Lett.*, 36, L16102 (DOI: 10.1029/2009GL039621; Discovery of a north–south asymmetry in Saturn's radio rotation period)
- Hviding, R. E., et al. 2025, *A&A* (DOI: 10.1051/0004-6361/202555816; arXiv:2506.05459)
- Seidel, E., & Suen, W.-M. 1991, *Phys. Rev. Lett.*, 66, 1659 (DOI: 10.1103/PhysRevLett.66.1659)
- Serra, D., Dimare, L., Tommei, G., & Milani, A. 2019, *Geophys. Res. Lett.*, 46, 14292 (DOI: 10.1029/2019GL086572; Jupiter's gravity field halfway through the Juno mission and comparison with Cassini Grand Finale empirical accelerations)
- Kurth, W. S., Gurnett, D. A., Lecacheux, A., et al. 2008, *Geophys. Res. Lett.*, 35, L02204 (DOI: 10.1029/2007GL029393; An update to a Saturnian longitude system based on kilometric radio emissions)
- Kusenko, A. 1997, *Phys. Lett. B*, 404, 285 (arXiv:hep-th/9704073)
- Lainey, V., Arlot, J.-E., Karatekin, Ö., & Van Hoolst, T. 2009, *Nature*, 459, 957 (DOI: 10.1038/nature08108; Strong tidal dissipation in Io and Jupiter from astrometric observations)
- Lainey, V., Casajus, L. G., Fuller, J., et al. 2020, *Nature Astronomy*, 4, 1053 (DOI: 10.1038/s41550-020-1120-5; Resonance locking in giant planets indicated by the rapid orbital expansion of Titan)
- Lelli, F., McGaugh, S. S., & Schombert, J. M. 2016, *AJ*, 152, 157 (DOI: 10.3847/0004-6256/152/5/157; SPARC Database)
- Li, P., Lelli, F., McGaugh, S., & Schombert, J. 2018, *A&A*, 615, A3 (DOI: 10.1051/0004-6361/201732547; arXiv:1803.00022; Fitting the Radial Acceleration Relation to Individual SPARC Galaxies)
- Lin, Z., Seager, S., & Weiss, B. P. 2025, *Planet. Sci. J.*, 6, 27 (DOI: 10.3847/PSJ/ada124; Interior and Gravity Field Models for Uranus Suggest a Mixed-composition Interior)
- Mankovich, C., Marley, M. S., Fortney, J. J., & Movshovitz, N. 2019, *ApJ*, 871, 1 (DOI: 10.3847/1538-4357/aaf798; Cassini Ring Seismology as a Probe of Saturn's Interior. I. Rigid Rotation)
- Mankovich, C. R., & Fuller, J. 2021, *Nature Astronomy*, 5, 1103 (DOI: 10.1038/s41550-021-01448-3; A diffuse core in Saturn revealed by ring seismology)
- Margot, J.-L., Campbell, D. B., Giorgini, J. D., et al. 2021, *Nature Astronomy*, 5, 676 (DOI: 10.1038/s41550-021-01339-7; Spin state and moment of inertia of Venus)
- McGaugh, S. S., Lelli, F., & Schombert, J. M. 2016, *Phys. Rev. Lett.*, 117, 201101 (DOI: 10.1103/PhysRevLett.117.201101; arXiv:1609.05917)
- McGaugh, S. S., Schombert, J. M., Bothun, G. D., & de Blok, W. J. G. 2000, *ApJ Lett.*, 533, L99 (DOI: 10.1086/312628; arXiv:astro-ph/0003001)
- Milgrom, M. 1983, *ApJ*, 270, 365 (DOI: 10.1086/161130; A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis)
- Militzer, B., & Hubbard, W. B. 2023, *Planet. Sci. J.*, 4, 95 (DOI: 10.3847/PSJ/acd2ed; Relation of Gravity, Winds, and the Moment of Inertia of Jupiter and Saturn)
- Montenbruck, O., Steigenberger, P., Prange, L., Deng, Z., Zhao, Q., Perosanz, F., Romero, I., Noll, C., Stürze, A., Weber, G., Schmid, R., MacLeod, K., & Schaer, S. 2017, *Adv. Space Res.*, 59, 1671 (DOI: 10.1016/j.asr.2017.01.011; The Multi-GNSS Experiment (MGEX) of the International GNSS Service)
- Nauenberg, M. 1972, *ApJ*, 175, 417 (DOI: 10.1086/151568; Analytic Approximations to the Mass–Radius Relation and Energy of Zero-Temperature Stars)
- Morf, L., & Helled, R. 2025, *A&A*, 704, A183 (DOI: 10.1051/0004-6361/202556911; Icy or rocky? Convective or stable? New interior models of Uranus and Neptune)

- Ogiya, G., & Nagai, D. 2023, *ApJ Lett.*, 958, L5 (arXiv:2309.09031)
- Poggianti, B. M., et al. 2019, *ApJ*, 874, 140 (arXiv:1810.05164)
- Pilcher, F., Mottola, S., & Denk, T. 2012, *Icarus*, 219, 741 (DOI: 10.1016/j.icarus.2012.03.021; Photometric lightcurve and phase curve of Bonnie / Himalia)
- Read, P. L., Dowling, T. E., & Schubert, G. 2009, *Nature*, 460, 608 (DOI: 10.1038/nature08194; Saturn's rotation period from its atmospheric planetary-wave configuration)
- Rybicki, G. B., & Lightman, A. P. 1979, *Radiative Processes in Astrophysics* (Wiley-VCH)
- Sanchez Almeida, J., Montes, M., & Trujillo, I. 2023, *A&A* (DOI: 10.1051/0004-6361/202346430; arXiv:2304.12344)
- Schive, H.-Y., Chiueh, T., & Broadhurst, T. 2014, *Phys. Rev. Lett.*, 113, 261302 (DOI: 10.1103/PhysRevLett.113.261302; arXiv:1407.7762)
- Sofue, Y. 2020, *PASJ*, 72, 81 (DOI: 10.1093/pasj/psaa065; arXiv:2006.15008)
- Sutherland, R. S., & Dopita, M. A. 1993, *ApJS*, 88, 253 (DOI: 10.1086/191823)
- TDCOSMO Collaboration. 2025, *A&A* (DOI: 10.1051/0004-6361/202555801; arXiv:2506.03023)
- Thomas, P. C., Armstrong, J. W., Asmar, S. W., et al. 2007, *Icarus*, 190, 573 (DOI: 10.1016/j.icarus.2007.03.012; Hyperion's sponge-like appearance and Cassini Phoebe flyby shape and mass)
- van Dokkum, P., et al. 2023, *ApJ Lett.*, 946, L50 (arXiv:2302.04888)
- van Dokkum, P., et al. 2025, *JWST Confirmation of a Runaway Supermassive Black Hole via its Supersonic Bow Shock* (arXiv:2512.04166)
- Taylor, J. H., & Weisberg, J. M. 1982, *ApJ*, 253, 908 (DOI: 10.1086/159690)
- Vainshtein, A. I. 1972, *Phys. Lett. B*, 39, 393 (DOI: 10.1016/0370-2693(72)90147-5)
- Wahl, S. M., Hubbard, W. B., Militzer, B., et al. 2017, *Geophys. Res. Lett.*, 44, 4649 (DOI: 10.1002/2017GL073160; Comparing Jupiter interior structure models to Juno gravity measurements and the role of a dilute core)
- Weisberg, J. M., & Taylor, J. H. 2005, *Binary Radio Pulsars*, ASP Conf. Ser., 328, 25 (arXiv:astro-ph/0407149)
- Wesson, P. S. 1979, *A&A*, 80, 296 (Self-similarity and the angular momentum–mass relation $J = pM^2$)
- Wesson, P. S. 1981, *Phys. Rev. D*, 23, 1730 (DOI: 10.1103/PhysRevD.23.1730; Scale-invariant gravity and astrophysical angular momentum scaling)
- Westerweck, J., et al. 2018, *Phys. Rev. D*, 97, 124037 (arXiv:1712.09966)
- Will, C. M. 2014, *Living Rev. Relativ.*, 17, 4 (DOI: 10.12942/lrr-2014-4; arXiv:1403.7377)
- Williams, J. G., Turyshev, S. G., & Boggs, D. H. 2012, *Class. Quantum Grav.*, 29, 184004 (DOI: 10.1088/0264-9381/29/18/184004; arXiv:1203.2150)
- Williams, J. G., & Boggs, D. H. 2016, *Celest. Mech. Dyn. Astron.*, 126, 89 (DOI: 10.1007/s10569-016-9702-3; Secular tidal changes in lunar orbit and Earth rotation)
- Wisdom, J., Dbouk, R., Militzer, B., et al. 2022, *Science*, 377, 1285 (DOI: 10.1126/science.abn1234; Loss of a satellite could explain Saturn's obliquity and young rings)
- Wisdom, J., Peale, S. J., & Mignard, F. 1984, *Icarus*, 58, 137 (DOI: 10.1016/0019-1035(84)90032-0; The chaotic rotation of Hyperion)
- Woosley, S. E. 2017, *ApJ*, 836, 244 (DOI: 10.3847/1538-4357/836/2/244; arXiv:1608.08939; Pulsational Pair-Instability Supernovae)
- Younes, G., et al. 2020, *ApJ Lett.*, 896, L42 (DOI: 10.3847/2041-8213/ab9a48; arXiv:2006.02814)
- Younes, G., et al. 2022, *Nature Astronomy*, 7, 339 (arXiv:2210.11518)
- Zhang, H., et al. 2025, *Polarization Images of Solitonic Boson Stars* (arXiv:2508.11992)
- Zebker, H. A., Marouf, E. A., & Tyler, G. L. 1985, *Icarus*, 64, 531 (DOI: 10.1016/0019-1035(85)90074-0; Saturn's rings: particle size distributions for thin layer model)
- Zhang, Z., Hayes, A. G., Janssen, M. A., et al. 2017, *Icarus*, 281, 297 (DOI: 10.1016/j.icarus.2016.07.020; Cassini microwave observations of Saturn's C ring and buried silicate-rich rubble belt)
- Zumalacárregui, M., & Bellini, E. 2018, *JCAP*, 2018, 054 (DOI: 10.1088/1475-7516/2018/10/054; arXiv:1705.05302; hi_class cosmology solver)
- Pintore, F., et al. 2016, *MNRAS*, 458, 2088 (DOI: 10.1093/mnras/stw449; arXiv:1602.05950)
- Ostriker, J. P., & Peebles, P. J. E. 1973, *ApJ*, 186, 467 (DOI: 10.1086/152513; A Numerical Study of the Stability of Flattened Galaxies: or, Can Cold Galaxies Survive?)
- Peale, S. J. 1999, *Annu. Rev. Astron. Astrophys.*, 37, 533 (DOI: 10.1146/annurev.astro.37.1.533; Origin and Evolution of the Natural Satellites)
- Ray, J., Dong, D., & Altamimi, Z. 2008, *GPS Solut.*, 12, 55 (DOI: 10.1007/s10291-007-0067-6; Anomalous harmonics in the spectra of GPS position estimates)

- Senior, K. L., Ray, J. R., & Beard, R. L. 2008, *J. Geophys. Res.*, 113, B08412 (DOI: 10.1029/2007JB005408; Characterization of periodic variations in the IGS time scale)
- Ray, P. S., et al. 2019, *ApJ*, 879, 130 (DOI: 10.3847/1538-4357/ab24d8)
- Tuo, Y. L., et al. 2024, *ApJ*, 966, 80 (DOI: 10.3847/1538-4357/ad2fb6; arXiv:2403.12137)
- Kramer, M., et al. 2006, *Science*, 314, 97 (DOI: 10.1126/science.1132305)
- van Dokkum, P., et al. 2018, *Nature*, 555, 629 (DOI: 10.1038/nature25767)
- van Dokkum, P., et al. 2019, *ApJ Lett.*, 874, L5 (DOI: 10.3847/2041-8213/ab0d92)
- van Dokkum, P., et al. 2022, *Nature*, 605, 435 (DOI: 10.1038/s41586-022-04665-6)
- Jiao, Y., et al. 2023, *A&A*, 678, A208 (DOI: 10.1051/0004-6361/202347513; arXiv:2309.00048)
- Archibald, R. F., et al. 2013, *Nature*, 497, 591 (DOI: 10.1038/nature12159)
- Archibald, R. F., et al. 2017, *ApJ*, 829, L21 (DOI: 10.3847/2041-8205/829/1/L21; arXiv:1608.01007)
- de Graaff, A., et al. 2025, *A&A* (arXiv:2503.01891)
- Dib, R., & Kaspi, V. M. 2014, *ApJ*, 784, 37 (DOI: 10.1088/0004-637X/784/1/37; arXiv:1401.5738)
- Benomar, O., Bazot, M., Nielsen, M. B., et al. 2018, *Science*, 361, 1239 (DOI: 10.1126/science.aao6571; Asteroseismic detection of latitudinal differential rotation in 13 Sun-like stars)
- Chandrasekhar, S. 1969, *Ellipsoidal Figures of Equilibrium* (New Haven, CT: Yale Univ. Press)
- Charbonneau, P., Christensen-Dalsgaard, J., Henning, R., et al. 1999, *ApJ*, 527, 445 (DOI: 10.1086/308050; Helioseismic Constraints on the Structure of the Solar Tachocline)
- Courteau, S., Dutton, A. A., van den Bosch, F. C., et al. 2007, *ApJ*, 671, 203 (DOI: 10.1086/522193; Scaling Relations of Spiral Galaxies)
- Lange, R., Driver, S. P., Robotham, A. S. G., et al. 2015, *MNRAS*, 447, 2603 (DOI: 10.1093/mnras/stu2467; Galaxy And Mass Assembly (GAMA): mass–size relations of $z < 0.1$ galaxies)
- Christensen-Dalsgaard, J., Däppen, W., Ajukov, S. V., et al. 1996, *Science*, 272, 1286 (DOI: 10.1126/science.272.5266.1286; The Current State of Solar Modeling)
- Dziewonski, A. M., & Anderson, D. L. 1981, *Phys. Earth Planet. Inter.*, 25, 297 (DOI: 10.1016/0031-9201(81)90046-7; Preliminary Reference Earth Model)
- Gagné, J., Faherty, J. K., Burgasser, A. J., et al. 2017, *ApJ Lett.*, 841, L1 (DOI: 10.3847/2041-8213/aa70e2; SIMP J013656.5+093347 is a planetary-mass object in Carina-Near)
- Karoff, C., Metcalfe, T. S., Santos, A. R. G., et al. 2018, *ApJ*, 852, 46 (DOI: 10.3847/1538-4357/aaa026; Differential rotation and magnetic activity in the Kepler solar analog HD 173701 / KIC 8006161)
- Lamy, L., Prangé, R., Berthier, J., et al. 2025, *Nature Astronomy* (DOI: 10.1038/s41550-025-02492-z; A new rotation period and longitude system for Uranus)
- Nicolis, A., Rattazzi, R., & Trinchini, E. 2009, *Phys. Rev. D*, 79, 064036 (DOI: 10.1103/PhysRevD.79.064036; arXiv:0811.2197)
- Olausen, S. A., & Kaspi, V. M. 2014, *ApJS*, 212, 6 (DOI: 10.1088/0067-0049/212/1/6; arXiv:1309.4167)
- Şaşmaz Muş, S., et al. 2014, *MNRAS*, 440, 2916 (DOI: 10.1093/mnras/stu445; arXiv:1402.6054)
- Schou, J., Antia, H. M., Basu, S., et al. 1998, *ApJ*, 505, 390 (DOI: 10.1086/306146; Helioseismic Studies of Differential Rotation in the Solar Envelope by SOI-MDI)
- Snellen, I. A. G., Brandl, B. R., de Kok, R. J., et al. 2014, *Nature*, 509, 63 (DOI: 10.1038/nature13253; Fast spin of the young extrasolar planet β Pictoris b)
- Tannock, M. E., Metchev, S., Heinze, A., et al. 2021, *AJ*, 161, 224 (DOI: 10.3847/1538-3881/abeb67; Weather on Other Worlds. V. The Three Most Rapidly Rotating Ultra-Cool Dwarfs)

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Revision note (v0.12): added the 7-planet unscreened-interior nonlinear Picard solve (Step 16), Saturn seven-clock kronoseismology/PV/SKR evaluation (Table 4.4), Roche-limit compositional ring stratigraphy, saturation self-energy ring mass and grain-size window, Cassini Division ambient co-rotation analysis, universal ambient-branch satellite orbital expansion law ($t_{\text{mig}}^{\text{TEP}} = \frac{3}{4}t_H = 10.48 \text{ Gyr}$) across Earth–Moon and Saturn’s six regular moons, Saturnian satellite screening-shell boundary mapping across Prometheus, Pandora, and chaotically tumbling Hyperion, and the outer non-tidally-locked satellite soliton-capacity test across Phoebe, Himalia, and Nereid (§4.7, Steps 15–16).

Appendix A: GNSS Calibration — Summary of Evidence

The claim that terrestrial clock correlations calibrate a fundamental density parameter ($\rho_T \approx 20 \text{ g/cm}^3$) is extraordinary and requires rigorous justification. This appendix summarizes the key validation results from the companion GNSS papers (Smawfield 2025b,c,d) and addresses common concerns about systematics.

A.1 Physical Mechanism

In the TEP framework, the scalar field ϕ modulates proper time via a conformal factor $A(\phi) = \exp(\beta_A \phi / M_{\text{Pl}})$. Near a gravitating mass, the field profile creates a spatial gradient in clock rates. GNSS atomic clocks, distributed across Earth’s gravitational field, sample this gradient at different radial positions. The characteristic correlation length $L_c \approx 4200 \text{ km}$ corresponds to the scale where the scalar field’s gradient becomes steep enough to produce measurable timing correlations—the saturation radius for Earth’s mass.

A.2 Why Not Ionospheric, Tropospheric, or Orbital Systematics?

Scale Separation

The 4200 km correlation length is distinct from known geodetic systematics:

- **Tropospheric correlations:** ~100–500 km (weather systems)
- **Ionospheric correlations:** ~500–2000 km (TEC structures)
- **Orbital harmonics:** ~12 hr (half-sidereal period)

Power spectral analysis shows the 4200 km feature as a persistent background covariance floor in the GPS-only clean baseline, with no counterpart in ionospheric or tropospheric delay products; combined multi-GNSS receiver-clock products (Paper 14) recover the same band-ordered correlation structure.

Multi-Center Verification

The correlation structure persists across independent clock solutions from three analysis centers:

Center	λ Median (km)	95% CI (km)	R ² (pooled)
CODE	4,181	1,198–5,918	0.920
IGS Combined	3,763	3,197–4,871	0.968
ESA Final	3,193	2,532–3,984	0.970

Table A.1: Multi-center correlation parameters. The bootstrap-median scale spans 3,193–4,181 km across centres (CV 13.4%); the corresponding exponential best-fit range is 3,330–4,549 km (CV 18.2%, Paper 1). Because centers process largely shared underlying data, this spread is not an independent ensemble average; it is treated as a systematic range rather than a statistical error reduction. λ values are bootstrap medians; R^2 is from the exponential best fit.

Null Tests

Three null tests were performed (all satisfied at $>5\sigma$):

1. **Epoch randomization:** Shuffling satellite epochs destroys the correlation ($r^2 < 0.01$, $N = 1000$ shuffles)
2. **Residual shuffling:** Shuffling clock residuals within each satellite eliminates the spatial structure
3. **White noise injection:** Replacing real data with white noise yields no coherent scale

A.3 25-Year Temporal Stability

The correlation length has been confirmed over a 25-year temporal baseline (1999–2024) using CODE final products. The signal shows:

- Decadal stability (no secular drift)
- Persistent spatial anisotropy
- Strong orbital velocity coupling ($r = -0.86$, $p = 0.002$ for Southern Hemisphere)
- Annual-phase direction recovery near Earth's aphelion velocity tangent; the directed CMB-apex template is anti-phase

A.4 Raw RINEX Validation

To strongly constrain the processing artifact hypothesis, the correlation was detected in raw RINEX observations processed with Single Point Positioning (SPP) using broadcast ephemerides only—no network solutions, no precise orbits, no clock products.

Signature	PPP (Processed)	SPP (Raw)	Status
Exponential decay	$R^2 = 0.92\text{--}0.97$	$R^2 = 0.94$	✓ Confirmed
Directional anisotropy	Detected	Detected	✓ Confirmed
Orbital velocity coupling	$r = -0.86$	Detected	✓ Confirmed
Preferred-direction recovery	$RA = 6^\circ$, $Dec = +4^\circ$; $\sim 6^\circ$ from aphelion velocity tangent; CMB apex anti-phase	Detected	✓ Confirmed

Table A.2: Raw RINEX validation results. All signatures detected in processed (PPP) data are independently confirmed in raw (SPP) observations, constraining processing artifacts as the origin of the observed signatures.

A.5 What the GNSS Measurement Does and Does Not Claim

What GNSS Measures

The GNSS analysis measures a characteristic correlation length $L_c \approx 4200$ km in atomic clock residuals. This length is operationally interpreted as the projected Temporal Topology covariance scale associated with $R_T(M_\oplus)$, the geometric saturation scale for Earth's mass under the TEP framework.

Scope of the GNSS Calibration

The GNSS analysis measures the spatial covariance scale L_c of the clock-sector residual field and uses it to anchor $\rho_T \approx 20$ g/cm³. Whether that macroscopic scale originates from a soliton-like Temporal Topology transition is tested by its out-of-sample transfer across the astrophysical mass hierarchy in the main text.

A.6 Companion Papers

Full methodology, data products, and reproducibility information are available in the companion papers:

- **Smawfield 2025b:** Multi-center validation (CODE, ESA, IGS)
- **Smawfield 2025c:** 25-year longitudinal analysis
- **Smawfield 2025d:** Raw RINEX validation
- **Smawfield 2025f:** Integrated synthesis (TEP-GTE)

Analysis code and data products are available at: github.com/matthewsmawfield/TEP-GNSS

Appendix B: Open Questions and Limitations

The TEP-UCD manuscript makes several strong empirical claims. This appendix discusses active research questions and known limitations. Addressing these is essential for the theory to mature from a consistency argument to a fully predictive framework.

B.1 The GNSS Correlation Mechanism

The operational identification of the terrestrial coherence length $L_c \approx 4200$ km with the projected covariance scale associated with the geometric saturation radius $R_T(M_\oplus) = (3M_\oplus/4\pi\rho_T)^{1/3}$ assumes that the observed exponential decay in clock residual correlations is controlled by the source-mass scale $R_T(M)$, rather than by network geometry, processing, or by the in-medium Compton wavelength $\lambda_C(\rho)$ of any specific candidate completion. This is not guaranteed. Alternative explanations include:

- **Network geometry:** The spatial distribution of GNSS stations is not random; continental clustering could imprint a characteristic scale unrelated to any physical field.
- **Common-mode clock processing:** Inter-analysis-center differences in orbit/clock estimation software may introduce correlated noise with scale-dependent structure.
- **Ephemeris systematics:** Orbital errors propagate into clock solutions with characteristic spatial signatures (e.g., along-track harmonics) that could mimic an exponential envelope.
- **Flicker noise and power-law correlations:** The null-model covariance analysis (TEP-GNSS, Paper 1) formally excludes power-law and constant kernels, but a more sophisticated non-stationary model might evade the current tests.

Mitigation: The null-model covariance analysis (`null_model_covariance.py`) systematically compares the exponential hypothesis against ionospheric dipole, power-law flicker, Gaussian, and Matérn kernels. The exponential model is preferred by AIC/BIC across all three analysis centers. In addition, Step 14 (`station_geometry_and_continental_clustering_null_audit`) executes a direct station-geometry and continental-clustering resampling audit on the actual IGS continental station coordinates ($n = 54$ global stations) to test whether continental clustering or ocean gaps imprint a spurious $L_c \approx 4,200$ km $\approx \frac{2}{3}R_\oplus$ scale: (i) injecting known exponential fields $L_{\text{true}} \in \{1,000, 2,000, 3,000, 4,200, 6,000\}$ km onto the real continental IGS coordinates recovers $\hat{L}_c = 885 \pm 82, 1,924 \pm 127, 2,783 \pm 210, 3,600 \pm 300,$ and $4,644 \pm 434$ km linearly (never manufacturing 4,200 km when $L_{\text{true}} \neq 4,200$ km, and showing that ocean gaps on a 54-station continental network mildly compress a true 4,200 km field to $\sim 3,600$ km under unweighted binning, matching the unweighted 3,423–3,766 km CDDIS RINEX3 recovery); (ii) subsampling an equal number of station pairs per distance bin ($K = 30$ pairs/bin across 200 bootstrap draws, flattening the continental pair-count histogram $N(d)$ completely) on the real 2023 CDDIS RINEX3 GPS data recovers $\hat{L}_c = 3,805.9 \pm 908.6$ km (within +1.1% of the full-sample unweighted 3,765.7 km); (iii) permuting station coordinates across the real continental network flattens the correlation profile completely (slope $+0.006$ Mm $^{-1}$, bin SD 0.063); and (iv) confronting the Karhunen–Loève / PCA common-mode-error (CME) filtering literature (Dong et al. 2006), the Funk–Hecke theorem on the 2-sphere S^2 shows that for an isotropic exponential covariance $C(\theta) = \exp(-\alpha\theta)$ with $\alpha = R_\oplus/L_c = 6,371/4,200 = 1.5169$, the exact analytical fraction of the zero-lag variance $C(0) = \sum_{\ell=0}^{\infty} f_\ell = 1$ carried by spherical-harmonic degree ℓ (with degeneracy weight $2\ell + 1$) is $f_\ell = \frac{2\ell+1}{2} \int_0^\pi e^{-\alpha\theta} P_\ell(\cos\theta) \sin\theta d\theta$, giving the closed-form analytical integrals $f_0 = (1 + e^{-\alpha\pi})/[2(1 + \alpha^2)] = 15.28\%$ (16.99% of the resolved $\ell \leq 14$ station-network band), $f_1 = 3(1 - e^{-\alpha\pi})/[2(4 + \alpha^2)] = 23.60\%$ (26.25% of $\ell \leq 14$), and $f_2 = 15.55\%$ (17.29% of $\ell \leq 14$), or 54.43% cumulative across $\ell \leq 2$ (60.53% of $\ell \leq 14$, and 95.40% of the Parseval L^2 squared-kernel norm $\sum (2\ell + 1)\lambda_\ell^2$). Removing the $\ell = 0$ global common mode on the full-day 24-hour window ($n_{\text{seg}} = 288$, matched 10–500 μHz) across all 198 CDDIS RINEX3 station-days yields $\bar{L}_c^{\text{CME-rem}} = 3,155.2 \pm 180.8$ km (CV = 5.73% across GPS 2,953.3, Galileo 3,302.1, and GLONASS 3,210.2 km, $R^2 = 0.955$ –0.969), matching the pre-computed 4-layer PREM full-volume datum-removed kernel $L_c^{\text{PREM,vol}} = 3,213.2$ km (model-calibrated BVP transfer ratio $L_c^{\text{vol}}/R_T = 0.7749 \implies R_T = 4,071.8$ km, $\hat{\rho}_T = 21.12$ g/cm ^{3} , vs. $\hat{\rho}_T = 45.40$ g/cm ^{3} if inverted at raw unit prefactor without the volume-kernel transfer ratio), whereas higher-rank $m \geq 2$ regional SVD truncation projects out the physical field's own $\ell = 1$ spherical harmonic just as 4-constellation ISB removal does, while physical broadcast-minus-precise common-mode subtraction ($D_i(t) = b_i(t) - p_i(t)$ in Paper 3 Step 2.12) removes satellite clock/orbit common modes without killing $\ell = 1$ and recovers $L_c = 4,474.8$ km ($R^2 = 0.664$).

B.2 Earth is Not a Soliton

The narrative operationally identifies L_c with the projected covariance scale associated with $R_T(M_\oplus) = (3M_\oplus/4\pi\rho_T)^{1/3}$. For Earth mass $M_\oplus$, this yields $R_{T,\oplus} \approx 4146$ km and $\rho_T \approx 20$ g/cm ^{3} . However, Earth's mean density is $\rho_\oplus \sim 5.5$ g/cm ^{3} , well below the quoted saturation scale. The planet therefore sits in the *transition* regime, not the deep-screening regime. The identification of the GNSS correlation scale with $R_T(M_\oplus)$ is phenomenologically motivated but lacks a single-potential $V(\phi)$ demonstration that both are the same parameter.

Mitigation: A fixed-input, canonical-kinetic Earth-profile consistency calculation has now been executed in Step 14 (`step_14_scale_audit.py`, `prem_earth_bvp_integration`). Holding the quartic coupling fixed at the Cassini-compatible corpus normalization $\lambda = 7.53 \times 10^{-66}$ and using the GNSS-consistent $\rho_T = 20$ g/cm ^{3} ($\lambda_c(\rho_T) = 2,478.5$ km), the boundary-value problem $\nabla^2 u = M_{\text{pl}}^{-2} V'_{\text{eff}}(u)$ is solved across a 4-layer piecewise-constant approximation to the Preliminary Reference Earth Model (PREM; Dziewonski & Anderson 1981; layer-mean densities joined by 30 km tanh transitions, total mass within 0.4% of $M_\oplus$; inner core 12.90 g/cm ^{3} , outer core 11.05 g/cm ^{3} to 3,480 km, lower mantle 4.90 g/cm ^{3} to 5,701 km, upper mantle/crust 3.55 g/cm ^{3} to 6,371 km). The solved transition gradient $r^2|du/dr|$ peaks across the core–mantle transition with radial centroid

$R_{\text{grad}} = 4,874.9 \text{ km} = 1.176 R_T(M_{\oplus})$ and transition Compton scale $\lambda_c(R_{\text{grad}}) = 7,807.9 \text{ km}$. Projecting the 3-D Yukawa spatial kernel from this transition shell onto pairs of surface stations at $R_{\oplus} = 6,371 \text{ km}$ separated by arc baselines $d \in [200, 8,000] \text{ km}$ (with network datum-mean removal) recovers an exponential spatial-kernel decay scale ($R^2 = 0.927$) of $L_c^{\text{PREM}} = 4,010.2 \text{ km}$ —within -4.5% of the GNSS-consistent length $L_c = 4,200 \text{ km}$ ($L_c^{\text{PREM}}/R_T(M_{\oplus}) = 0.967$, confirming an order-unity spatial transfer prefactor; varying the fit window $d_{\text{max}} \in \{6,000, 8,000, 10,000\} \text{ km}$ shifts L_c^{PREM} across 3,290--4,595 km and scaling $\lambda_c(\rho_T) \propto \rho_T^{-1/3}$ across $\rho_T \in \{5, 20, 50, 200\} \text{ g/cm}^3$ shifts the endpoint-clamped shell integral L_c^{PREM} from 4,105 to 3,650 km because the $1/|\mathbf{x}_A - \mathbf{x}_s|$ geometric factor and endpoint minimum-subtraction on $[200, 8,000] \text{ km}$ pin the single-shell integral to Earth's fixed R_{CMB} and $R_{\oplus}$ geometry, whereas the exact 3-D volume Yukawa convolution $\mathcal{F}^{-1}[(k^2 + m_{\text{eff}}^2)^{-2}] \propto \exp(-d_{\text{chord}}/\lambda_c(\bar{\rho}_{\oplus}; \rho_T))$ — where the $1/r$ factor cancels identically in 3-D convolution — tracks $\rho_T^{-1/3}$ across 6,329, 3,951, 2,888, and 1,796 km and the saturation-core coherence kernel $\exp(-d_{\text{chord}}/R_T(M_{\oplus}, \rho_T))$ tracks $R_T \propto \rho_T^{-1/3}$ across 6,901, 4,311, 3,152, and 1,960 km, reinforced by the independent cross-mass tests of Sections 4 and 6). Furthermore, solving the coupled 1-D spherical nonlinear two-branch kinetic operator $\nabla \cdot (P_{,X} \nabla u) = M_{\text{pl}}^{-2} V'_{\text{eff}}(u)$ (Eq. 62) via 100-iteration damped Picard relaxation converges to $\|\Delta u^{(100)}\|_{\infty}/\|u\|_{\infty} = 2.67 \times 10^{-14}$ and yields $R_{\text{grad}}^{\text{NL-PREM}} = 4,809.9 \text{ km}$ ($1.160 R_T(M_{\oplus})$), $L_c^{\text{NL-PREM}} = 3,957 \text{ km}$, and $L_c^{\text{NL,vol}} = 3,170 \text{ km}$, while the rotating-station and slant-path clock modulation yields $\delta y_{\text{rms}} = 7.0 \times 10^{-15}$ ($\delta \tau_{\text{rms}} \simeq 0.10 \text{ ns}$ at $f_{\text{sid}} = 11.61 \mu\text{Hz}$ and orbital carriers 19.7--24.7 μHz ; Appendix C.2), leaving only non-spherical 3-D azimuthal PDE integration for future work.

B.3 The SPARC $M^{1/3} \rightarrow M^{1/2}$ Transition and Disk-Scale Confound

On the discrete first-bin grid ($N = 168$), the empirical dark-matter onset exponent $\alpha_{\text{SPARC}} = 0.352 \pm 0.043 \text{ (stat)} \pm 0.07 \text{ (definition)}$ is consistent with $1/3 = 0.333$, whereas on the $n = 75$ resolved interior crossings the full-sample slope shifts to $\alpha_{\text{res}} = 0.417 \pm 0.050$, and the in-sample mass-size relation ($R_d \propto M_{\text{bar}}^{0.322}$) poses a potential disk-size confound.

Mitigation: Both issues are resolved quantitatively in Step 8 (`step_8_density_vs_size_confound.py`) and Step 14 (`step_14_scale_audit.py`). First, a left-censored Tobit maximum-likelihood regression across all $N = 168$ galaxies (retaining the 93 R_{min} -censored low-surface-brightness dwarfs as upper bounds, where R_{min} tracks R_d at $r = +0.410, p = 4.6 \times 10^{-5}$ with median $R_{\text{min}}/R_d = 0.396$ and internal slope -0.245 against the resolved crossings' median 1.236 and slope $+0.116$, creating a Simpson's-paradox cancellation on the raw discrete grid and inflating the apparent $M_{\text{bar}}^{1/3}$ scatter from 0.366 dex resolved / 0.180 dex enclosed-mass to 0.483 dex) formally rejects pure disk-scale invariance ($R_{\text{DM}}/R_d = \text{const}$) with slope $+0.323 \pm 0.034$ ($z = 9.57, p < 10^{-4}$), while on the $n = 75$ resolved crossings the non-circular fixed-radius partial correlation $r_{\text{part}}(R_{\text{DM}}, V_{\text{bar}}(2.2R_d) | R_d) = +0.547$ ($p = 3.8 \times 10^{-7}$) confirms that gravitational potential depth drives R_{DM} at fixed disk size. Second, combining the two-branch Gauss–Klein–Gordon flux equation (Eq. [13], which derives $g_{\text{crit}}(\tau = 1.30) = 1.031 \times 10^{-10} \text{ m s}^{-2}$ and $\bar{\rho}_{\text{trans}} = 3g_{\text{crit}}/(4\pi G R_{\text{DM}}) = 0.068 M_{\odot} \text{ pc}^{-3}$ [$0.030 M_{\odot} \text{ pc}^{-3}$ uncoupled] at $R_{\text{DM}} = 2.6 \text{ kpc}$) with the Freeman (1970) exponential-disk profile predicts a regime crossover mass of $M_{\text{cross}}^{\text{SPARC}} = 1.45 \times 10^{10} M_{\odot}$. Partitioning the $n = 75$ resolved galaxies *a priori* at the exact analytical Freeman velocity slope $\gamma_{\text{Freeman}} = 0.210$ (zero dependence on V_{obs} or R_{DM}) cleanly separates rising-baryon 3-D core crossings ($\gamma_V \geq 0.210, n = 38$: $\alpha_{\text{rising}} = 0.256 \pm 0.069$, consistent with $1/3$ at 1.1σ and rejecting $1/2$ at 3.5σ) from concentrated 2-D Keplerian-tail crossings ($\gamma_V < 0.210, n = 37$: $\alpha_{\text{tail}} = 0.581 \pm 0.086$, consistent with $1/2$ at 0.95σ and rejecting $1/3$ at 2.9σ ; subsample shift $\Delta\alpha = +0.325 \pm 0.110, z = 2.95, p = 0.0031$).

B.4 Screening Hierarchy: Algebraic Tautology?

The screening fit $S \propto \rho^{0.334}$ with $R^2 = 0.99995$ (11 dense objects, $\rho > \rho_T$, excluding planets and black holes) is extraordinarily tight. Because $S \equiv R_T/R_{\text{phys}}$ and both radii are defined by mass-density relations, the slope $\sim 1/3$ is algebraically expected given the model assumptions. The high R^2 confirms internal consistency; it does not constitute independent confirmation of TEP.

Mitigation: The screening hierarchy verification (`scripts/steps/step_5_ultimate_screening.py` and `scripts/verification/verify_screening_claim.py`) performs object-by-object residual analysis across the 11 dense objects ($\rho > \rho_T$), confirming $\beta_{\text{scr}} = 0.3338 \pm 0.0008$ ($R^2 = 0.99995$). While the density-ordered $S = R_T/R_{\text{phys}}$ relation is an internal consistency check by construction, the planetary rotational kinematics, moment-of-inertia, and satellite tests in Section 4.7 (`step_15_planetary_rotation.py`, `step_16_saturn_interior.py`) break this algebraic circularity by testing $R_T(M)$ and S against independent NASA/JPL ephemeris spin velocities ($v_{\text{obs}}/v_{\text{TEP}} = 1.000 \pm 0.080$ across the three unbraked outer giant planets Uranus, Neptune, and Saturn [$\rho_T^{\text{spin}} = 19.9 \text{ g/cm}^3$], and 1.016 ± 0.073 with lunar-braked Earth), moments of inertia ($2k_{\text{MOI}}/S = 1.039 \pm 0.027$ across the three core-screened terrestrial planets and $2k_{\text{MOI}}/S_{\text{cond}} = 1.042 \pm 0.094$ across all seven condensed/giant planets), outer non-tidally-locked satellite soliton capacity ($\bar{\eta}_{\text{moons}} = 0.1320 \pm 0.0113$ across Phoebe, Himalia, and Nereid, $\hat{\rho}_T^{\text{moons}} = 19.8\text{--}20.6 \text{ g/cm}^3$), Wesson's angular-momentum constant $p_{\text{TEP}} = G/(2v_{\text{gal}})$, and a 100,000-trial random-density look-elsewhere coincidence null test across five independent channels ($\hat{\rho}_T = 19.59 \pm 1.10 \text{ g/cm}^3, p = 2.9 \times 10^{-4}$; and $19.77 \pm 0.48 \text{ g/cm}^3, p = 5.2 \times 10^{-5}$ on the three-channel subset without algebraic prefactors).

B.5 Cross-Paper Parameter Drift

Multiple distinct exponents appear across the TEP literature, and their notation has been ambiguous:

Quantity	Typical Value	Source	Physical Meaning
α_{SPARC}	0.352 ± 0.043 (stat) ± 0.07 (definition)	UCD (this work)	Galactic DM onset scaling (expected 1/3)
α_{sat}	0.366	WB (Paper 13)	Wide-binary saturation boost (phenomenological)
β_{scr}	0.334	UCD (this work)	Screening exponent from $S(\rho)$ hierarchy
κ_{Cep}	$0.665\text{--}1.275 \times 10^6$ mag ($0.67\text{--}1.28 \times 10^6$) empirical; 1.196×10^6 mag derived (Step 62)	H0 (Paper 11)	Cepheid clock-bias response coefficient (derived from envelope density ratio $\rho_T/\langle\rho\rangle(10\text{ d}) = 8.45 \times 10^5$ at the unit-transport benchmark $\lambda_0 = 1$ with $ \beta_A = 1$, matching the SH0ES central estimate $(1.191 \pm 0.480) \times 10^6$ mag to 0.5%)
$\tilde{\kappa}_{\text{MSP}}$	$\sim 10^4$	COS (Paper 10)	Pulsar timing full response coefficient (suppressed observed value after dense-cluster geometric screening; bare geometric-factor estimate is $\sim 10^6\text{--}10^7$)

These are not the same α . The notation purge across all manuscripts has disambiguated them, but the proliferation of phenomenological exponents remains a vulnerability. A unified Lagrangian should predict all of them from a single coupling function $A(\phi)$.

Mitigation: The global notation purge completed in this version removes ambiguity. Step 14 (`amplitude_low_check`) proves that the $1/3 = 1/(m - 1)$ saturation exponent follows identically from the $m = 4$ quartic potential $V(u) = \frac{1}{4}\lambda M_{\text{Pl}}^4 u^4$, while Step 8 and Step 12 prove that the $1/2$ galactic tail exponent follows from the square-root branch of $P_X(\xi) = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$, and Paper 11 (Step 62) derives $\kappa_{\text{Cep}} = 1.196 \times 10^6$ mag from $\rho_T/\langle\rho\rangle(10\text{ d})$.

B.6 Series Convention

Series convention. In this corpus, ρ_T denotes the phenomenological macroscopic saturation/proximity scale calibrated from GNSS by treating L_c as the projected covariance scale associated with $R_T(M_{\oplus})$. Channel-specific quantities such as ρ_{trans} , ρ_{half} , R_s , κ_{Cep} , κ_{MSP} , and disformal response parameters are not independent measurements of ρ_T , but downstream response or environment parameters. The symbol λ denotes a generic correlation length recovered from GNSS clock-residual analysis; center- or product-specific values are distinguished by subscripts (e.g., λ_{CODE} , λ_{MGEX}).

B.7 Causal Structure and Hyperbolicity

The TEP scalar field mediates correlations over thousands of kilometres in GNSS clocks. If this field propagates at finite speed, one must verify that the observed correlations do not imply superluminal signal transfer. This appendix establishes local causal-cone and scalar-hyperbolicity consistency on the selected physical backgrounds and explains why GNSS spatial covariance arises from a static boundary-value geometry rather than instantaneous signaling. Global causal admissibility is formulated relative to the preferred TEP foliation ($\nabla_\mu \bar{\phi}$ timelike at the cosmological boundary; Paper 0 §12).

Local Causal-Cone and Hyperbolicity Analysis

Objective. Establish that on the specified static background the effective causal (matter) metric $\tilde{g}_{\mu\nu}$ and the scalar field equation of motion preserve locally Lorentz-invariant matter/photon cones and strict Cauchy hyperbolicity, and that long-range spatial correlations reflect static boundary-value geometry rather than dynamic propagation.

Step 1: The Bimetric Causal Cones

In TEP (Paper 0), gravity is governed by the Einstein-frame metric $g_{\mu\nu}$. Matter and electromagnetic fields evolve along the geodesics of the causal (Jordan-frame) metric, written here on the isotropic conformal–disformal benchmark (in the canonical unified unstretched-space decomposition of Paper 0 §2.2, $\tilde{g}_{\mu\nu} = -A_{\text{loc}}^2 U_\mu U_\nu + A_{\text{sp}}^2 \Pi_{\mu\nu} + B(\phi)\Pi_\mu^\alpha \Pi_\nu^\beta \nabla_\alpha \phi \nabla_\beta \phi$):

$$\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu} + B(\phi)\nabla_\mu \phi \nabla_\nu \phi. \quad (44)$$

To ensure causality, the inverse metric $\tilde{g}^{\mu\nu}$ must exist, and the Lorentzian signature $(-, +, +, +)$ must be strictly preserved. By the Sherman–Morrison formula, the inverse is:

$$\tilde{g}^{\mu\nu} = \frac{1}{A^2(\phi)} g^{\mu\nu} - \frac{B(\phi)}{A^2(\phi) [A^2(\phi) + B(\phi)(\nabla\phi)^2]} \nabla^\mu \phi \nabla^\nu \phi, \quad (45)$$

where $(\nabla\phi)^2 \equiv g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi$.

Condition 1 (Signature Preservation). To avoid a metric singularity and a flip in causality, the denominator must not cross zero:

$$A^2(\phi) + B(\phi)(\nabla\phi)^2 > 0. \quad (46)$$

As long as Condition 1 holds on the $B(\phi) \geq 0$, static-gradient benchmark ($\partial_t \phi = 0$, $(\nabla\phi)^2 > 0$), the causal cone of $\tilde{g}_{\mu\nu}$ remains strictly nested inside, or tangent to, the gravitational light cone of $g_{\mu\nu}$.

Step 2: Signal Velocity of Electromagnetic / Matter Fields

Computing the local speed of light c_γ (matter-frame photons) in a static spherically symmetric background. In Earth's rest frame, the field is static ($\partial_t \phi = 0$) and radially dependent $\phi = \phi(r)$. The metric components become:

$$\tilde{g}_{00} = A^2(\phi) g_{00} = -A^2 c^2, \quad \tilde{g}_{rr} = A^2(\phi) g_{rr} + B(\phi)(\partial_r \phi)^2. \quad (47)$$

For a radial null signal in the matter frame, $\tilde{g}_{\mu\nu} dx^\mu dx^\nu = 0$:

$$-A^2 c^2 dt^2 + [A^2 + B(\phi)(\partial_r \phi)^2] dr^2 = 0, \quad (48)$$

which yields the effective light speed:

$$c_\gamma^2 \equiv \left(\frac{dr}{dt} \right)^2 = \frac{A^2 c^2}{A^2 + B(\phi)(\partial_r \phi)^2}. \quad (49)$$

If $B(\phi) > 0$, then $c_\gamma < c$. The causal cone of light and matter is squeezed inside the bare gravitational cone. Information transfer is strictly subluminal relative to the Einstein frame. (As noted in Paper 0, $B(\phi)$ is deeply constrained by GW170817 multi-messenger limits in the late universe, meaning $c_\gamma \approx c$ macroscopically.)

Step 3: Propagation and Hyperbolicity of the Scalar Field

Consider perturbations $\delta\phi$ of the scalar field around a background solution $\bar{\phi}$ under the convention $X \equiv -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$. On a homogeneous cosmological background ($\bar{X} > 0$), the EFT of Dark Energy implementation (Paper 18, `hi_class` cosmology) satisfies the no-ghost ($\alpha_K > 0$) and no-gradient-instability ($c_s^2 > 0$) conditions with $c_s \leq c$. On a static spacelike-gradient background ($\bar{X} = -\frac{1}{2}(\nabla\bar{\phi})^2 < 0$, so $|X| = -X > 0$) for the selected two-branch k-essence completion $P_X(\xi) = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$ ($\xi \equiv |X|/\Lambda_X^4$, $\epsilon_{\text{reg}} = 10^{-3}$; Paper 0 §12, Step 77), varying $\nabla_\mu(P_X \nabla^\mu \phi)$ with $\nabla_\nu X = -\nabla^\mu \phi \nabla_\nu \nabla_\mu \phi$ yields the linearized hyperbolic wave equation:

$$P_X \ddot{\delta\phi} - c^2 \nabla_i \left[(P_X \delta^{ij} - P_{XX} \nabla^i \bar{\phi} \nabla^j \bar{\phi}) \nabla_j \delta\phi \right] + V_{,\phi\phi} \delta\phi = \text{source terms}, \quad (50)$$

whose transverse and longitudinal scalar characteristic speeds squared relative to the gravitational metric $g_{\mu\nu}$ are $v_\perp^2 = 1$ and

$$v_\parallel^2 = \frac{P_X + 2XP_{XX}}{P_X} = \frac{P_X - 2|X|P_{XX}}{P_X} = \frac{\epsilon_{\text{reg}} + 2k\sqrt{\xi} + 6\xi}{\epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi} \in [1, 3]$$

across the nonlinear branches ($v_\parallel^2 \rightarrow 1$ in the regulator-dominated limit $\xi \rightarrow 0$, $v_\parallel^2 \rightarrow 2$ on the $\sqrt{\xi}$ branch, and $v_\parallel^2 \rightarrow 3$ on the quadratic branch). As established in Paper 0 §12, this widened longitudinal scalar characteristic ($v_\parallel > 1$ on static spacelike gradients)

does not violate matter-sector causality: matter and electromagnetic fields propagate strictly on the locally Lorentz-invariant causal cone of $\tilde{g}_{\mu\nu}$ ($c_\gamma \leq c$, Step 2), while the cosmic time field $\bar{\phi}$ defines a preferred global foliation on which the classical Cauchy initial-value problem is strictly hyperbolic and well-posed ($P_X > 0$ and $P_X + 2XP_{XX} = P_X - 2|X|P_{XX} > 0$ everywhere).

Step 4: The Steady-State Fallacy (The 4,200 km GNSS Correlation)

Crucially, the observed GNSS spatial covariance over $L_c \approx 4,200$ km does not rely on dynamic wave propagation between clocks at all—whether luminal or characteristic-widened.

The error in the superluminality objection lies in conflating *dynamic wave communication* with a *static spatial boundary-value profile*.

The GNSS atomic clocks are not interacting *with each other* via ϕ . They are independently measuring the local magnitude of a pre-existing, steady-state geometric structure: Earth's Temporal Topology screening profile $\bar{\phi}(r)$.

Consider the Coulomb field of a proton. The electric field extends to infinity. If you place two electrons 4,200 km apart in that static field, their potential energies are perfectly "correlated" according to the geometry of $1/r$. This correlation does not require the electrons to communicate instantly; it merely requires that the central proton has existed long enough ($t > r/c$) for the static field boundary-value to settle.

In TEP, the Earth ($M_\oplus$) acts as a scalar source charge. The spatial distribution of the field is the solution to the static Klein–Gordon equation with environmental screening (the Temporal Shear transition):

$$\nabla^2 \bar{\phi} - m_{\text{eff}}^2(\rho) \bar{\phi} = \frac{\beta_A}{M_{\text{Pl}}} \rho_{\text{matter}}. \quad (51)$$

Because the Earth has been in hydrostatic equilibrium for billions of years, $\partial_t \bar{\phi} = 0$. The resulting Temporal Shear profile $\Sigma_\mu = \nabla_\mu \ln A(\bar{\phi})$ forms a rigid geometric "scaffolding" around the planet.

When a GNSS clock network orbits through this profile, the residuals exhibit a spatial correlation length $L_c \approx 4,200$ km. This is not a superluminal signal propagating between satellites. It is the geometric width of the screening transition zone in the stationary background field.

Resolution of the Causal Vulnerability

By deriving the effective matter-frame light cone c_γ^2 , establishing strict Cauchy hyperbolicity on the preferred foliation, and distinguishing the stationary boundary-value solution from the dynamic Green's function, the selected physical backgrounds satisfy the stated local matter-cone and scalar-hyperbolicity conditions, and the GNSS covariance interpretation does not require superluminal communication:

- 1. Matter-Frame Light Cone & Hyperbolicity:** Matter and electromagnetic signals propagate strictly within the locally Lorentz-invariant causal cone of $\tilde{g}_{\mu\nu}$ ($c_\gamma \leq c$), while scalar perturbations remain strictly hyperbolic on the preferred global foliation ($v_\perp^2 = 1$, $v_\parallel^2 \in [1, 3]$ on static spacelike gradients; Paper 0 §12).
- 2. Correlation vs. Communication:** The 4,200 km correlation observed in the 25-year CODE analysis is a *static structural parameter* of the scalar medium, established by the Earth's mass distribution over geological timescales, not an instantaneous interaction between the atomic clocks.

B.8 Summary of Risk Register

Vulnerability	Severity	Mitigation Status	Resolution Target
GNSS correlation transfer & null discrimination	Low–Medium	4-layer PREM BVP ($L_c^{\text{PREM}} = 4,010$ km) & 100-trial MC network injection–recovery ($\hat{L}_c^{\text{GPS}} = 4,036 \pm 113$ km; white-noise $ C(d) = 0.009$ and orbital-dipole $d^2C/dd^2 < 0$ nulls excluded; Step 14)	Resolved in Step 14 (hardware timing-perturbation test optional)
Earth not a soliton	Low–Medium	4-layer PREM Earth BVP, 1-D spherical nonlinear Picard solve ($\ \Delta u^{(100)}\ _\infty = 2.67 \times 10^{-14}$; recovers $R_{\text{grad}} = 4,875$ km [4,810 km nonlinear], $L_c^{\text{PREM}} = 4,010$ km [3,957 km nonlinear], $L_c^{\text{vol}} = 3,213$ km [3,170 km nonlinear]; Step 14), and 7-planet Lane–Emden unscreened-interior nonlinear Picard scan ($R_{\text{grad}} \approx 0.75\text{--}0.83 R_{\text{vol}}$; Step 16)	Resolved in TEP-UCD v0.11–v0.12 (Steps 14, 16; 3-D azimuthal EW/NS PDE in future work)
SPARC $M^{1/3} \rightarrow M^{1/2}$	Low	Left-censored Tobit MLE rejects $R_{\text{DM}}/R_d = \text{const}$ ($z = 9.57$, $p < 10^{-4}$); <i>a priori</i> Freeman-disk baryonic slope split	Resolved in TEP-UCD v0.11 (Step 8)

Vulnerability	Severity	Mitigation Status	Resolution Target
transition & R_d confound		around $M_{\text{cross}} = 1.45 \times 10^{10} M_{\odot}$ separates $M^{1/3}$ core crossings ($\alpha = 0.256 \pm 0.069$) from $M^{1/2}$ tail crossings ($\alpha = 0.581 \pm 0.086, p = 0.0031$; Step 8)	
Screening hierarchy algebraic	Low	Supplemented by the fixed-input ambient planetary scaling relation, the two-branch planetary/substellar/outer-moon ($\bar{\eta}_{\text{moons}} = 0.1320 \pm 0.0113$) rotational consistency tests, and the three-channel zero-prefactor convergence ($\hat{p}_T = 19.77 \pm 0.48 \text{ g/cm}^3, p = 5.2 \times 10^{-5}$; §4.7, Steps 15–16)	Resolved in TEP-UCD v0.11–v0.12 (Steps 15–16)
Cross-paper parameter drift	Low	Notation purge complete	Ongoing
Causal cone & scalar hyperbolicity	Resolved	Proved: local matter-cone nesting ($c_{\gamma} \leq c$), strict Cauchy hyperbolicity ($v_{\perp}^2 = 1, v_{\parallel}^2 \in [1, 3]$ on static spacelike gradients; Paper 0 §12 & Appendix B.7), and static BVP covariance distinction	Resolved in Appendix B.7 & Paper 0 §12
MGEX scale discrepancy (1,862 vs. 4,200 km)	Low–Medium	Pre-registered test & cross-mode RINEX/PPP de-biasing executed: scale is reproducible across independent chains (COD 2,744 vs GRG 1,933 km), and single- vs 4-constellation RINEX/PPP comparison establishes a $1.90\times$ multi-GNSS ISB filter compression factor that de-biases pooled MGEX to 3,536 km (Step 14)	Resolved in TEP-GNSS-MGEX Steps 2.8, 3.1, 3.5 & TEP-UCD Step 14 (4-quarter 2023 GPS/Galileo/GLONASS single-constellation RINEX3 reprocessing executed)

The MGEX discrepancy is the most consequential held-out tension in the corpus. Paper 14 reports $\lambda \approx 1,862 \pm 112$ km (corrected pooled; the original ~ 1 -year headline was 1,396 km) from a combined multi-constellation product, versus the $\sim 4,200$ km scale recovered from multi-year single-center analyses. Candidate explanations include: (i) combined multi-constellation clock products mixing different orbital sampling geometries into a compressed projection; (ii) the short ~ 1 -year span yielding an unstable scale estimate (consistent with the window scatter and $R^2 \approx 0.70$ on the pooled fit); or (iii) a genuinely different scale. The pre-registered discriminator is a per-constellation analysis: if GPS, Galileo, and GLONASS individually recover $\sim 4,200$ km, the projection explanation wins and the MGEX result becomes a methodological footnote. If per-constellation analysis also yields $\sim 1,900$ km, the $L_c \leftrightarrow R_T(M_{\oplus})$ identification fails as calibrated, and the Level 1/2/3 claim hierarchy (Section 8) absorbs the hit exactly as designed—Level 2 falls, Levels 1 and 3 survive. Executing the matched-day CODE/CNES comparison (Paper 14 Step 3.1), the factorial estimator decomposition (Paper 14 Step 3.5), and the 100-trial Monte Carlo ISB projection simulation in Step 14 (which proves that 4-constellation ISB removal compresses a 4,200 km field to 2,277–2,485 km, $1.62\text{--}1.77\times$, matching the empirical $1.90\times$ offset that de-biases pooled MGEX 1,862 km to 3,536 km) together with the executed 4-quarter 2023 CDDIS RINEX3 single-constellation reprocessing in Step 14 (where full-day $n_{\text{seg}} = 288$ single-constellation solutions recover 3,766 km [GPS], 3,806 km [GLONASS], and 3,653 km [Galileo 10–60 μHz ; 2,697 km full-band] with a three-constellation mean of 3,423 km inside the [3,000, 5,000] km GPS-only band, and where applying MGEX's 8-hour $n_{\text{seg}} = 96$ window truncation, per-epoch datum removal, and 4-constellation ISB estimation compresses the exact same station-days step-by-step from $3,423 \rightarrow 2,850 \rightarrow 2,156$ km [CV = 0.28% across GPS 2,163.2, Galileo 2,152.9, and GLONASS 2,152.3 km] $\rightarrow 1,478$ km, a $2.32\times$ empirical compression) resolves the multi-GNSS compression mechanism directly on real observations.

The discriminator has now been executed (TEP-GNSS-MGEX Steps 3.1 and 3.5, `TEP-GNSS-MGEX/results/outputs/step_3_1_per_center_scale.json`), with the pre-registration outcome stated plainly: the literal per-constellation station-clock split could not be executed as originally designed because MGEX station clocks are published as single combined-solution parameters (one receiver clock per station-epoch combining GPS, GLONASS, Galileo, and BeiDou with inter-system bias absorption), and space-segment satellite clocks in the combined product carry no distance-decaying within-constellation coherence (Step 2.8). The substitute discriminators executed on the retained archive establish three facts: (i) on the identical COD MGEX product, quarterly windows scatter from $\lambda = 1,050$ to 1,935 km (R^2 from -0.18 to 0.80), showing that short $\lesssim 1$ -year windows carry $\sim 30\text{--}40\%$ fractional sampling scatter around their product mean; (ii) on strictly matched, verified-independent days, two independent multi-GNSS processing chains—CODE/GPSEST and CNES/GINS (GRG)—return pooled $\lambda = 2,744$ km ($n = 43,971$ pairs) and 1,933 km ($n = 44,000$) respectively ($\sim 30\%$ apart, GRG within 4% of the pooled 1,862 km MGEX scale); and (iii) a factorial estimator decomposition on identical station-days (Paper 14 §4.5b, Step 3.5) proves that estimator settings (Welch segment length and common-mode removal) shift λ by $\lesssim \pm 25\%$, whereas both GPS-only IGS-final ($\lambda \approx 3.9\text{--}5.9 \times 10^3$ km in the tidal band, isotropic $3,210 \pm 389$ km and cross-sector $4,201 \pm 1,967$ km over 25.3 years in Paper 2; raw-RINEX ionofree $4,767 \pm 835$ km in Paper 3) and 4-constellation MGEX ($\lambda \approx 1.8\text{--}2.0 \times 10^3$ km) recover the identical band-ordered decay profile, with the MGEX combined multi-GNSS filter acting as a $\sim 2\times$ scaled-down projection of the GPS-only PPP profile. Thus, the 25.3-year temporal stability of the GPS-only CODE baseline (Paper 2) and the product-family scale offset of the 4-constellation MGEX filter (Paper 14) are distinct phenomena: a quantitative cross-mode de-biasing synthesis in Step 14 (`constellation_debiasing_synthesis`) comparing single-constellation GPS/ionofree RINEX and PPP solutions across identical station networks (mean 4,066 km; including 4,767 km raw-RINEX dual-frequency ionofree and 3,512–5,026 km across ALL STATIONS and OPTIMAL 100 in Paper 3) against 4-constellation combined multi-GNSS solutions (mean 2,141 km; 1,770–2,726 km raw-RINEX multi-GNSS and 1,862–2,744 km MGEX) establishes an empirical multi-GNSS inter-system-bias (ISB) filter

compression factor of $1.90\times$. Crucially, the Monte Carlo injection–recovery suite on a 120-station global network in Step 14 proves the direction and mechanism of this bias without circular ratio rescaling: across an injected scale grid $L_c^{\text{true}} \in \{1,000, 2,000, 3,000, 4,200, 6,000\}$ km, 1-constellation GPS-only global datum removal tracks every scale ($\hat{L}_c^{\text{GPS}} = 1,019, 1,975, 2,912, 4,036 \pm 113$, and $5,721$ km; white-noise null returns $|C(d)| = 0.009 \pm 0.001$, and a correlated rank-3 satellite-orbit/geocenter dipole plus datum common-mode null yields a concave-down correlation curve $d^2C/dd^2 = -0.024 \text{ Mm}^{-2} < 0$ whose unconstrained exponential fit hits the $\sim 24,900$ km upper bound rather than manufacturing a 4,200 km scale), whereas 4-constellation multi-GNSS common-view ISB projection compresses $L_c^{\text{true}} = 4,200$ km to $\hat{L}_c^{\text{MGEX}} = 2,277 \pm 47$ km ($1.77 \pm 0.06\times$ compression) for per-epoch ISBs (matching CNES/GRG 1,933 km) and $\hat{L}_c = 2,485 \pm 45$ km ($1.62\times$ compression) for daily-constant ISBs with rotating constellation visibility (matching CODE matched-day MGEX 2,744 km). The spatial ISB projection and temporal passband truncation both suppress long-scale covariance and may describe overlapping aspects of the combined processing response; they are not treated as independent multiplicative corrections without a joint transfer calculation, with the simulated $1.62\text{--}1.77\times$ compression reproducing the direction and approximate magnitude of the observed $1.90\times$ offset (mapping pooled MGEX 1,862 km to 3,536 km inside the 3,330–4,549 km GPS-only anchor range). Applying the $\sim 30\text{--}40\%$ fractional window/chain uncertainty around the GPS-only anchor ($L_c = 4,200$ km) under the conditional $L_c \leftrightarrow R_T(M_\oplus)$ identification yields the systematic band $\rho_T \in [8, 70]$ g/cm³, while the unadjusted 4-constellation MGEX projection ($\lambda = 1,862\text{--}2,744$ km at unit prefactor) defines a conservative upper sensitivity bracket of $\rho_T \sim 69\text{--}220$ g/cm³. With the 4-quarter 2023 CDDIS RINEX3 single-constellation GPS, Galileo, and GLONASS reprocessing now executed in Step 14 (including Galileo dual-frequency iono-free LC recovering $L_c = 4,556.7$ km across 67 `optimal_100` stations), the single- versus multi-constellation discriminator is resolved on public data, leaving independent third-party geodesy replication and optical/two-way time-transfer links as the remaining external verification targets.

Each open question in this register defines an explicit quantitative test, ensuring that the framework is falsifiable at multiple independent points across mass and density scales.

Appendix C: Candidate EFT Realizations of the Soliton Profile

The empirical saturation scale $\rho_T \approx 20 \text{ g/cm}^3$ is established phenomenologically from GNSS clock correlations and SPARC scaling. A natural question is whether standard scalar-tensor Effective Field Theory (EFT) machinery can generate a density-dependent screening transition at this scale. This appendix presents two candidate completions: the symmetron, retained as an external benchmark whose $\mathbb{Z}_2$ mechanism is standard in the modified-gravity literature but whose quadratic coupling is not the corpus's universal exponential; and a confining-potential realization constructed directly on the corpus coupling $A(\phi) = e^{-\phi/M_{\text{Pl}}} (\beta_A = -1)$, which is the corpus-internal demonstration that the observed phenomenology is *mechanistically possible* without requiring new physics beyond a self-interacting scalar sector. The completions are benchmark models, not unique derivations; the corpus's working screening realization is the selected two-branch nonlinear kinetic sector of the Paper-0 master action, $P(X, \phi) = -V(\phi) + \frac{2k}{3\Lambda_X^2} X|X|^{1/2} + X|X|/\Lambda_X^4$ (with the unit-term form $P = X - V + X|X|/\Lambda_X^4$ retained as the high-gradient control), whose response $P_{,X}(\xi) = k\sqrt{\xi} + 2\xi$ ($\xi = |X|/\Lambda_X^4$, $k = 16.03$) saturates the scalar coupling once the field gradient exceeds the kinetic scale $\Lambda_X = \sqrt{M_{\text{Pl}} H_0} \approx 1.9 \text{ meV}$ (Paper 0, A4, §7; analogous to non-canonical kinetic K-mouflage screening in the scalar-tensor literature, Babichev et al. 2009; Barreira et al. 2015, where precision Solar-System perihelion, Lunar Laser Ranging, and time-varying G_{eff} bounds complement Cassini Shapiro-delay constraints). The kinetic scale fixes the shear threshold g_t — equivalently a surface-density crossover $\Sigma_* \sim 3M_{\text{Pl}}^2 H_0 \approx 0.12 \text{ g/cm}^2$ inside sources on the unit-term interior benchmark (step 14) — not a volume-density scale: the density argument of the amplitude sector enters through the potential normalization, and the potential-sector realization below is constructed on the corpus's own quartic coupling. In the TEP framework, these mechanisms are treated as candidate microscopic completions, not as the defining ontology of the theory (Paper 0, A4). TEP screening is defined by the continuous spatial profile of the scalar time field (Temporal Topology) and its gradient (Temporal Shear), with no thin-shell boundaries.

C.1 Symmetron Screening: Density-Triggered Symmetry Breaking

The symmetron mechanism (Hinterbichler & Khoury 2010) provides a natural route to density-dependent screening. The scalar potential is:

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4, \quad (52)$$

and the conformal coupling to matter is:

$$A(\phi) = 1 + \frac{1}{2} \left(\frac{\phi}{M} \right)^2, \quad (53)$$

where M is the symmetry-breaking scale. The effective potential in the presence of ambient density ρ becomes:

$$V_{\text{eff}}(\phi) = V(\phi) + \rho[A(\phi) - 1] = -\frac{1}{2}\mu_{\text{eff}}^2\phi^2 + \frac{1}{4}\lambda\phi^4, \quad (54)$$

with effective mass parameter:

$$\mu_{\text{eff}}^2 = \mu^2 - \frac{\rho}{M^2}. \quad (55)$$

The critical behavior is immediate. When $\rho > \rho_* \equiv \mu^2 M^2$, the symmetry-breaking parameter $\mu_{\text{eff}}^2 < 0$ turns negative so the physical squared mass (curvature of V_{eff} at the origin, $m_{\text{eff}}^2(0) \equiv V_{\text{eff}}''(0) = -\mu_{\text{eff}}^2 = \rho/M^2 - \mu^2 > 0$) is positive and the field sits stably at the symmetric minimum $\phi = 0$; the scalar is massive and screened, so Temporal Shear effects are suppressed. When $\rho < \rho_*$, symmetry is spontaneously broken, the field rolls to $\phi_0 = \mu_{\text{eff}}/\sqrt{\lambda}$, and the scalar becomes light—generating long-range forces. The saturation scale is:

$$\rho_* = \mu^2 M^2. \quad (56)$$

Identifying $\rho_* \equiv \rho_T \approx 20 \text{ g/cm}^3$ fixes one relation between μ and M . The second relation comes from demanding that the Compton wavelength in the unscreened regime matches the galactic scale ($\lambda_C \sim \text{kpc}$), which yields $\mu \sim 10^{-27} \text{ eV}$ and $M \sim 10^{-3} M_{\text{Pl}}$. These values are within the phenomenologically allowed window for scalar-tensor theories. Within the corpus this completion is an external benchmark only: its quadratic coupling $A(\phi) = 1 + \phi^2/2M^2$ is not the universal exponential, and the $\mathbb{Z}_2$ structure on which the mechanism rests cannot be imposed on the linearly coupled exponential $A = e^{-\phi/M_{\text{Pl}}}$.

C.2 Confining-Potential Screening on the Corpus Coupling

Under the corpus's frozen coupling $\beta_A = -1$, the matter term enters the effective potential with negative slope, so the runaway inverse-power branch $V \propto \phi^{-n}$ used in chameleon completions is inadmissible: both $V' < 0$ and $\beta_A \rho A/M_{\text{Pl}} < 0$ for all $\phi > 0$, no minimum of V_{eff} exists, and the field runs away rather than saturating. The admissible branch is a confining potential, $V' > 0$. Writing the dimensionless field $u = \phi/M_{\text{Pl}}$, the admissible family is

$$V(u) = \frac{\lambda}{m} M_{\text{Pl}}^4 u^m, \quad m > 2, \quad (57)$$

whose coefficient is the dimensionless coupling λ — not a fourth-power energy density, and distinct from the GNSS covariance length denoted λ in Section 3. The corpus's own matter-hosting branch is the quartic member of this family, the weak-field truncation of the master potential $V(u) = V_{\text{matter}}(u)e^{-(u/u_s)^4} + V_0 e^{-(u_s/u)^4}$ (Paper 0, Section 8). The conformal coupling is the corpus exponential:

$$A(u) = e^{-u} \quad (\beta_A = -1). \quad (58)$$

The effective potential is:

$$V_{\text{eff}}(u) = V(u) + \rho[A(u) - 1] = \frac{\lambda}{m} M_{\text{Pl}}^4 u^m + \rho(e^{-u} - 1). \quad (59)$$

Its derivative $V'_{\text{eff}} = \lambda M_{\text{Pl}}^4 u^{m-1} - \rho e^{-u}$ vanishes at

$$\lambda u_{\text{min}}^{m-1} e^{u_{\text{min}}} = \frac{\rho}{M_{\text{Pl}}^4} \implies u_{\text{min}} \approx \left(\frac{\rho}{\lambda M_{\text{Pl}}^4} \right)^{1/(m-1)} \quad (u_{\text{min}} \ll 1), \quad (60)$$

so a minimum exists for every $\rho > 0$, with the field elevated inside matter ($u_{\text{min}} > 0$) and approaching zero in the ambient — the direction required by the corpus sign convention. The tracking exponent is then fixed, not free: the quartic potential derives the unclipped equilibrium tracking ratio $S(\bar{\rho}) \equiv u_{\text{min}}(\bar{\rho})/u_{\text{min}}(\rho_T) = (\bar{\rho}/\rho_T)^{1/3} = R_T(M)/R_{\text{phys}}$ (the $u_{\text{min}} \ll 1$ truncation; the full $\lambda u^3 e^u$ solve agrees to $\sim 4 \times 10^{-10}$ relative, step 14), because reproducing the $(\bar{\rho}/\rho_T)^{1/3}$ scaling requires $1/(m-1) = 1/3$, i.e. $m = 4$ — the quartic member already adopted by the master potential. Macroscopic observables then project this unclipped ratio $S(\bar{\rho}) = R_T(M)/R_{\text{phys}}$ in two complementary ways across the corpus: (i) for surface rotational and structural kinematics (Section 4.7), because a body's matter distribution terminates at its physical surface R_{phys} , the active screened radius enclosed within the body cannot exceed R_{phys} ($R_{\text{active}} = \min[R_T(M), R_{\text{phys}}]$), which imposes the geometric surface ceiling $S_A(\bar{\rho}) \equiv R_{\text{active}}/R_{\text{phys}} = \min[1, (\bar{\rho}/\rho_T)^{1/3}]$ (whereas the local equilibrium $u_{\text{min}}(\bar{\rho})$ itself continues to grow as $\bar{\rho}^{1/3}$); and (ii) for deep-interior clock/coupling drift transmission in over-saturated compact objects ($\bar{\rho} > \rho_T$, Paper 22 §6.1), the unclipped ratio $S_{\text{prog}} = u_{\text{min}}(\bar{\rho})/u_{\text{min}}(\rho_T) = (\bar{\rho}/\rho_T)^{1/3} > 1$ ($S_{\text{WD}} \simeq 46.4$ inside Chandrasekhar white dwarfs) measures the interior over-saturation depth and suppresses transmitted cosmological G_{loc} drift by $[\max(1, S_{\text{prog}})]^{-1}$. Likewise, in the unified matter metric $\tilde{g}_{\mu\nu} = -A_{\text{loc}}^2(u_{\text{grav}}(\mathbf{x}))U_\mu U_\nu + A_{\text{sp}}^2(u_{\text{drift}}(t, \mathbf{x}))\Pi_{\mu\nu} + B(\phi)(\dots)$ of Paper 0 §2.2, quasi-static local soliton dynamics at a fixed cosmic epoch t_0 ($A_{\text{sp}}(t_0) = 1$) probe spatial gradients of the lapse $A_{\text{loc}}(\mathbf{x}) = e^{-u_{\text{grav}}(\mathbf{x})}$, whereas cosmological radiation propagation along the homogeneous drift background ($A_{\text{loc}}(t) = A_{\text{sp}}(t) = A(\bar{\phi}(t))$, $B(\phi)k^\mu \partial_\mu \phi \rightarrow 0$; Paper 22 §2.2) specializes to the conformal radiation metric $\tilde{g}_{\mu\nu} = A^2(\bar{\phi})g_{\mu\nu}$ with $c_T \equiv c$ and $\Xi(z) = A_{\text{em}}/A_0 = (1+z)^{-1}$. The normalization is likewise fixed corpus-wide at the Cassini-compatible branch $\lambda = 7.5 \times 10^{-66}$ (`LAMBDA_QUARTIC_CASSINI`; the fiducial point

$\lambda_{\text{ref}} = 7.5 \times 10^{-71}$ of the same family fails the corrected linear-in- S_Σ Cassini evaluation by $\sim 250\times$, Paper 0 Appendix E step 03). As an energy scale the potential normalization is $\Lambda_V \equiv \lambda^{1/4} M_{\text{Pl}} \approx 1.3 \times 10^2 \text{ GeV}$ — fourteen orders of magnitude above the kinetic scale $\Lambda_X \approx 1.9 \text{ meV}$, and six orders above the energy-density reading $\rho_T^{1/4} \approx 96 \text{ keV}$ of the saturation density (Section 5.5). These are three distinct quantities: Λ_X is the gradient threshold of the kinetic sector, λ the amplitude-sector coupling, and ρ_T the geometric reference density of the $R_T(M)$ law. Under this normalization the equilibrium remains deeply perturbative at all matter densities: $u_{\min}(\rho_T) = 6.9 \times 10^{-9}$ and $u_{\min}(\bar{\rho}_\oplus) = 4.5 \times 10^{-9}$, matching the terrestrial share ($\sim 0.5\%$) of the nested field decomposition (Paper 0, Section 2.2).

On the canonical-kinetic equilibrium benchmark ($P_{,X} \rightarrow 1$), the field equation in a static, spherically symmetric environment is:

$$\frac{d^2 u}{dr^2} + \frac{2}{r} \frac{du}{dr} = \frac{1}{M_{\text{Pl}}^2} V'_{\text{eff}}(u), \quad (61)$$

whereas on the operative nonlinear two-branch kinetic completion $P_{,X}(\xi) = \epsilon_{\text{reg}} + k\sqrt{\xi} + 2\xi$, the spatial differential operator carries the nonlinear kinetic stiffness directly:

$$\nabla \cdot (P_{,X}(\xi) \nabla u) = \frac{1}{r^2} \frac{d}{dr} \left(r^2 P_{,X}(\xi) \frac{du}{dr} \right) = \frac{1}{M_{\text{Pl}}^2} V'_{\text{eff}}(u). \quad (62)$$

On the canonical-kinetic equilibrium benchmark, the effective mass squared around $u_{\min}(\rho)$ is:

$$m_{\text{eff}}^2 = \frac{1}{M_{\text{Pl}}^2} V''_{\text{eff}}(u_{\min}) = \frac{3\lambda M_{\text{Pl}}^4 u_{\min}^2 + \rho e^{-u_{\min}}}{M_{\text{Pl}}^2} \simeq 3\lambda M_{\text{Pl}}^2 u_{\min}^2, \quad (63)$$

where the equilibrium condition makes the curvature term dominant at $u_{\min} \ll 1$, giving $m_{\text{eff}} \propto \lambda^{1/6} \rho^{1/3}$. The screening is doubly density-dependent. On this canonical-kinetic benchmark, the Compton wavelength $\lambda_c(\rho) = 1/m_{\text{eff}}$ contracts through denser interiors, defining an order-of-magnitude equilibrium reference scale at which a body's interior becomes Compton-resolved inside its own saturation halo, $\lambda_c(\bar{\rho}) \lesssim R_T(M)$ — for the corpus normalization $\lambda_c(\rho_T) \approx 2,500 \text{ km} \approx 0.6 R_T(M_\oplus)$, and the canonical crossover density $\lambda_c(\bar{\rho}) = R_T(M_\oplus)$ sits at $\sim 4 \text{ g/cm}^3$, inside the terrestrial condensed-matter window (step 14). Deep in the interior the field sits near its local equilibrium $u_{\min}(\rho)$, while across the transition layer the nonlinear kinetic stiffness $P_X(\xi)$ (and its longitudinal combination $P_X + 2XP_{XX} = P_X - 2|X|P_{XX} = \epsilon_{\text{reg}} + 2k\sqrt{\xi} + 6\xi$ in Eq. 62) governs the actual physical transition width ΔR_T ; solving the coupled nonlinear two-branch boundary-value operator Eq. (62) directly across the 4-layer PREM Earth profile via under-relaxed Picard iteration ($\omega = 0.35$, 100 iterations; Step 14) converges to machine precision ($\|\Delta u^{(100)}\|_\infty / \|u^{(100)}\|_\infty = 2.67 \times 10^{-14}$, grid-doubling shift $N = 1,200 \rightarrow 2,400$ of 3.70 km [0.077%], and initial-guess invariance across $u^{(0)} \in \{0, u_{\text{eq}}, 2u_{\text{eq}}\}$ to $8.8 \times 10^{-11} \text{ km}$), placing the nonlinear transition-gradient centroid at $R_{\text{grad}}^{\text{NL-PREM}} = 4,809.9 \text{ km}$ ($1.160 R_T(M_\oplus)$, within -1.3% of the 4,874.9 km canonical-kinetic centroid) and recovering a projected surface decay scale of $L_c^{\text{NL-PREM}} = 3,957 \text{ km}$ and full-volume datum-removed decay scale of $L_c^{\text{NL,vol}} = 3,170 \text{ km}$ (matching the 3,155.2 km 3-constellation CME-removed mean and 3,210 km 25-year CODE baseline within 1%). For the time-dependent GNSS clock observable, because $u \equiv \phi/M_{\text{Pl}}$ is dimensionless ($A(u) = e^{-u}$), the dimensionless fractional frequency perturbation is $y_i(t) \equiv \delta\nu_i(t)/\nu_0 = d\delta\tau_i/dt = -\delta u_{\text{loc}}(\mathbf{x}_i(t), t) - c^{-1}(d/dt) \int_{\mathbf{x}_i(t)}^{\mathbf{x}_s(t)} \delta u_\oplus(r(s')) ds'$ with rate derivative $\dot{y}_i(t) = -(\partial_t u + \mathbf{V}_i(t) \cdot \nabla u) - d^2 \delta\tau_{is}^{\text{prop}}/dt^2$: while geocentric rotation alone is tangential to the spherical Earth monopole ($(\boldsymbol{\Omega}_\oplus \times \mathbf{r}_i) \cdot \nabla u_\oplus(r) \equiv 0$), the rotating station vector $\mathbf{r}_i(t) = R_\oplus \hat{\mathbf{n}}_i(\boldsymbol{\Omega}_\oplus t)$ projects onto the external heliocentric/Galactic drift gradient ∇u_{ext} via $\mathbf{r}_i(t) \cdot \nabla u_{\text{ext}} \propto \cos(\boldsymbol{\Omega}_\oplus t + \lambda_i) \cos \phi_i$ at the sidereal diurnal frequency ($f_{\text{sid}} = 11.61 \mu\text{Hz}$), while inclined satellite slant paths modulate $\delta\tau_{is}^{\text{prop}}(t) \simeq [L_{\text{eff}}/\sin E_{is}(t)] u_\oplus(R_\oplus)/c$ at the constellation-specific orbital repeat frequencies (GPS 11.97 hr/23.21 μHz , GLONASS 11.26 hr/24.67 μHz , Galileo 14.08 hr/19.74 μHz) inside the 10--60 μHz passband, producing a conformal clock-frequency modulation of $\delta y_{\text{rms}} \simeq (v_{\text{rot}}/c) u_\oplus(R_\oplus) = 7.0 \times 10^{-15}$ ($\delta\tau_{\text{rms}} \simeq 0.10 \text{ ns}$) while nonlinear kinetic stiffness $P_{,X}(\xi)$ suppresses the transmitted dynamical Temporal-Shear acceleration ratio (while the normalized Pearson correlation profile $C(d)/C(0)$ depends only on the scale-invariant spatial eigenprofile $u(r)/u(0)$) to $|\mathbf{a}_\phi|/g_\oplus \simeq a_{\text{eff}}/g_\oplus = 1.22 \times 10^{-11}$ at Earth's surface and $|\mathbf{a}_\phi|/g_\odot(1 \text{ AU}) = 2.02 \times 10^{-8}$. The conformal self-quenching of the source slope $\rho A_{,u} = -\rho e^{-u_{\min}}$ becomes operative only when the equilibrium reaches order unity, which under the corpus normalization occurs at $\rho_{\text{sat}} \sim \lambda M_{\text{Pl}}^4 \approx 6 \times 10^{25} \text{ g/cm}^3$ — far above neutron-star densities, inside the temporal-well regime where the master potential's V_0 floor activates (Paper 28). Ordinary saturation is therefore the Compton-resolution crossover, not an order-unity field excursion.

C.3 The Saturation Scale: Calibration versus Derivation

In the symmetron benchmark, $\rho_T = \mu^2 M^2$ is the symmetry-restoration density, fixed by two potential parameters. On the corpus branch the analogous question — which property of the scalar action returns ρ_T — has a sharper answer than a dimensional guess. Under the corpus normalization the equilibrium at the transition density is $u_{\min}(\rho_T) = 6.9 \times 10^{-9} \ll 1$: the saturation scale is not a field-amplitude crossing of order unity. What ρ_T parameterizes is the geometric constant of the saturation-radius law $R_T(M) = (3M/4\pi\rho_T)^{1/3}$, calibrated by the GNSS covariance length (Section 2); its dynamical proxy on the quartic branch is the Compton-resolution crossover $\lambda_c(\bar{\rho}) \sim R_T(M)$, which for Earth sits at $\bar{\rho} \sim 4 \text{ g/cm}^3$ — the terrestrial condensed-matter window in which the measured $\rho_T \approx 20 \text{ g/cm}^3$ resides.

The shortcut identification $\rho_T \equiv \Lambda_V^4$ — equating the geometric reference density with the potential coefficient — is not available on the corpus branch, and is excluded rather than merely unnecessary. Imposing $u_{\min}(\rho_T) \sim 1$ is equivalent to fixing $\lambda = \rho_T/M_{\text{Pl}}^4 \approx 2.4 \times 10^{-90}$, and that normalization fails three independent checks (step 14): it lies 3×10^{24} below the Cassini-compatible quartic normalization that Solar-System screening requires; it drives the neutron-star-core equilibrium to $u_{\min} \approx 21$, past the master-potential knee $u_s \simeq 10$ and onto the Planck-density floor — observed pulsars are not temporal wells; and its implied Compton length at the transition density, $\lambda_c(\rho_T) \approx 3.4 \times 10^7 \text{ km}$, exceeds any terrestrial body by more than three orders of magnitude, so the local equilibrium it invokes never forms within the host. The same comparison furnishes a directional lower bound: keeping neutron-star interiors below the knee, $u_{\min}(\rho_{\text{NS}}) < u_s$ at $\rho_{\text{NS}} \sim 2 \times 10^{14} \text{ g/cm}^3$, requires $\lambda \gtrsim 10^{-84}$, which the corpus branch satisfies at nineteen orders of margin. The quartic normalization is thereby bounded from below by compact objects and pinned from the side by Cassini — a constrained window, not a free scale.

The correct microscopic reading is therefore: ρ_T is the measured geometric constant of $R_T(M)$; the quartic coupling λ is the amplitude-sector normalization; and the order-unity self-quenching crossing sits at $\rho_{\text{sat}} \sim \lambda M_{\text{Pl}}^4 \approx 6 \times 10^{25} \text{ g/cm}^3$, inside the temporal-well regime of Paper 28. Meanwhile, the SPARC galactic rotation curves and outer-edge BTFR are governed directly by the two-branch kinetic sector $P_X = k\sqrt{\xi} + 2\xi$ ($k = 16.03$ fixed externally by a_0 , Steps 10 and 13), which supplies the exact $a_{\text{eff}} = a_0$ asymptote with zero per-galaxy free parameters.

C.4 Status and Caveats

The completions above demonstrate that the TEP phenomenology—density-dependent screening, a compact soliton scale, and a galactic transition—is *mathematically realizable*: the confining-potential construction realizes it within the corpus's own frozen coupling, and the symmetron provides an external benchmark from the standard EFT literature. They do not, however, constitute a proof that nature has chosen either mechanism. Key open questions include:

- **Realization uniqueness:** Within the confining family no exponent freedom remains — the corpus amplitude-response exponent selects the quartic member $m = 4$, which is the weak-field branch of the corpus's own master potential rather than one choice among many. The residual uniqueness question concerns alternative realizations (the symmetron benchmark, the wider $K(X)$ kinetic class) and is the action-closure question recorded in Paper 0 — not a freedom this appendix retains.
- **Quantum stability:** The small quartic coupling $\lambda \approx 7.5 \times 10^{-66}$ is radiatively stable in the technical sense — its quantum corrections are proportional to λ itself. The $\mu \sim 10^{-27} \text{ eV}$ mass-scale exposure belongs to the symmetron benchmark, not to the corpus realization.
- **Terrestrial transition width:** On the canonical-kinetic quartic branch ($\lambda = 7.53 \times 10^{-66}$, $\rho_T = 20 \text{ g/cm}^3$), the 4-layer PREM Earth BVP solve and 3-D Yukawa surface covariance projection (Step 14) recovers a transition gradient centroid at $R_{\text{grad}} = 4,875 \text{ km}$ ($1.18 R_T(M_{\oplus})$) and a surface-network decay length of $L_c^{\text{PREM}} = 4,010 \text{ km}$ ($L_c^{\text{vol}} = 3,213 \text{ km}$ full-volume), while the 100-iteration damped Picard solution of the coupled 1-D spherical nonlinear two-branch operator Eq. (62) converges to $\|\Delta u^{(100)}\|_{\infty}/\|u\|_{\infty} = 2.67 \times 10^{-14}$ and yields $R_{\text{grad}}^{\text{NL-PREM}} = 4,810 \text{ km}$ ($1.160 R_T(M_{\oplus})$), $L_c^{\text{NL-PREM}} = 3,957 \text{ km}$, and $L_c^{\text{NL,vol}} = 3,170 \text{ km}$ (Section C.2). Extending this radial nonlinear solve to non-spherical 3-D azimuthal orbital-motion anisotropy remains a target for future work.

The existence of a corpus-internal completion that structurally reproduces the salient phenomenology under the frozen $\beta_A = -1$ coupling — with the symmetron as an external benchmark — removes the objection that $\rho_T \approx 20 \text{ g/cm}^3$ is a numerical construct. The scale is the geometric constant of the saturation-radius law, calibrated by GNSS, whose dynamical reading on the corpus's quartic branch is the Compton-resolution crossover of the interior field — the admissible branch selected by the corpus sign convention, normalized by the Cassini-compatible coupling, and bounded from below by compact-object equilibria (step 14). It is anchored to terrestrial kinematic data at every step.